

Lecture 23: Outline

Hour 1:

Concept Review / Overview

PRS Questions – possible exam questions

Hour 2:

Sample Exam

Yell if you have any questions

7:30-9 pm Tuesday

Exam 2 Topics

- DC Circuits
 - Current & Ohm's Law (Macro- and Microscopic)
 - Power
 - Kirchhoff's Loop Rules
 - Charging/Discharging Capacitor (RC Circuits)
- Magnetic Fields
 - Force due to Magnetic Field (Lorentz Force)
 - Magnetic Dipoles
 - Generating Magnetic Fields
 - Biot-Savart Law & Ampere's Law

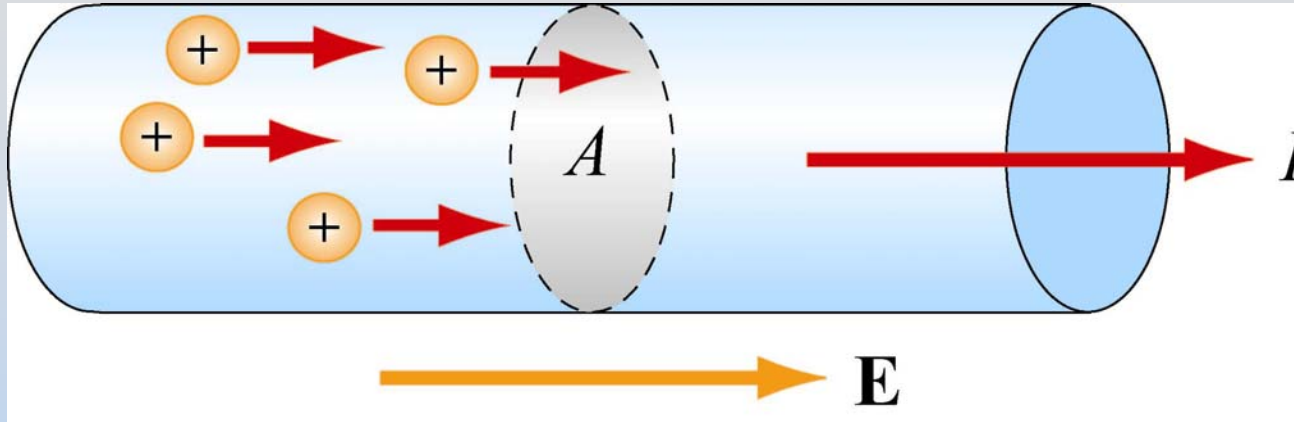
General Exam Suggestions

- You should be able to complete every problem
 - If you are confused, ask
 - If it seems too hard, you aren't thinking enough
 - Look for hints in other problems
 - If you are doing math, you're doing too much
- Read directions completely (before & after)
- Write down what you know before starting
- Draw pictures, define (label) variables
 - Make sure that unknowns drop out of solution
- Don't forget units!

What You Should Study

- Review Friday Problem Solving (& Solutions)
- Review In Class Problems (& Solutions)
- Review PRS Questions (& Solutions)
- Review Problem Sets (& Solutions)
- Review PowerPoint Presentations
- Review Relevant Parts of Study Guide
(& Included Examples)

Current & Ohm's Law



$$I = \frac{dQ}{dt}$$

$$\vec{\mathbf{J}} \equiv \frac{I}{A} \hat{\mathbf{I}}$$

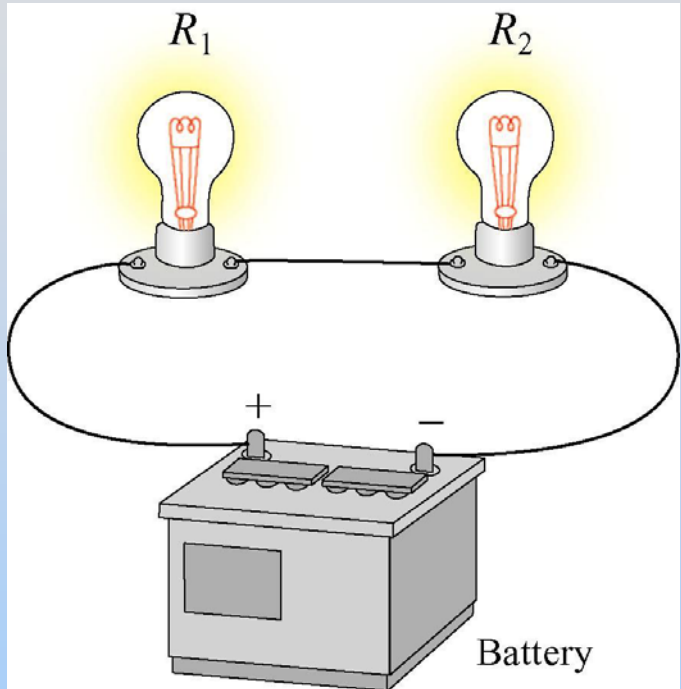
Ohm's Laws

$$\vec{\mathbf{E}} = \rho \vec{\mathbf{J}} = \left(\frac{1}{\sigma} \right) \vec{\mathbf{J}}$$

$$\Delta V = IR$$

$$R = \frac{\rho l}{A}$$

Series vs. Parallel

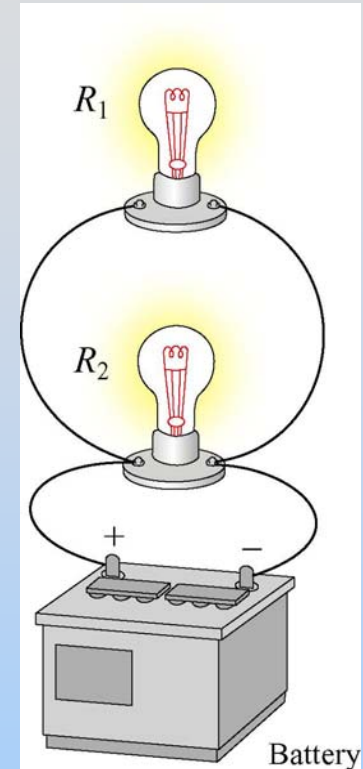


Series

- Current same
- Voltages add

$$R_s = R_1 + R_2$$
$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2}$$
$$C_p = C_1 + C_2$$



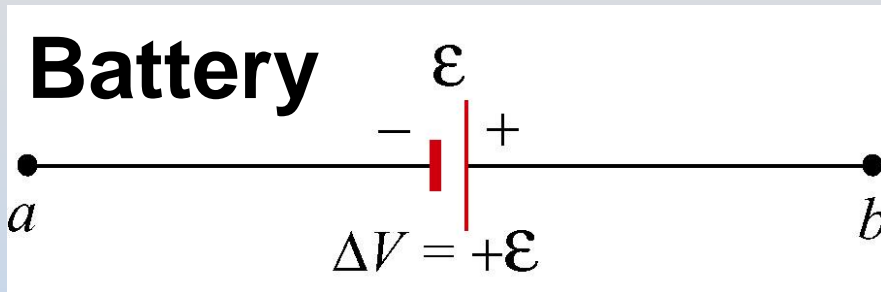
Parallel

- Currents add
- Voltages same

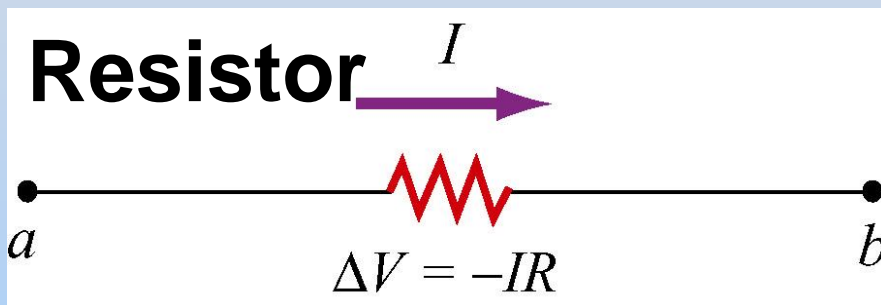
PRS Questions: Light Bulbs

Class 10

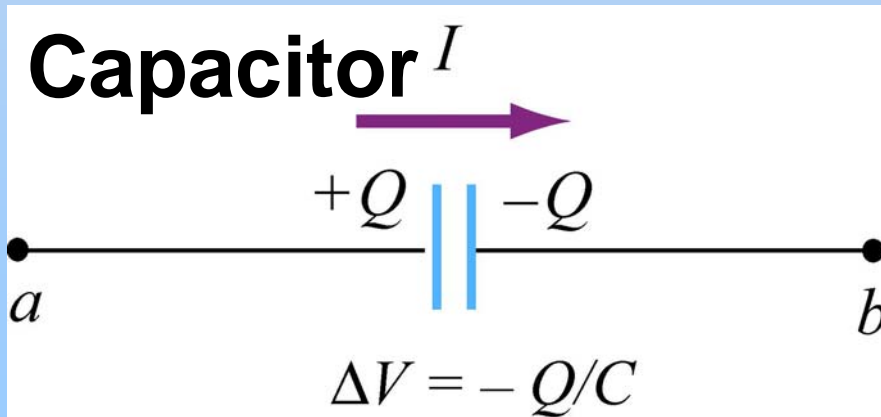
Current, Voltage & Power



$$P_{\text{supplied}} = I \Delta V = I \epsilon$$



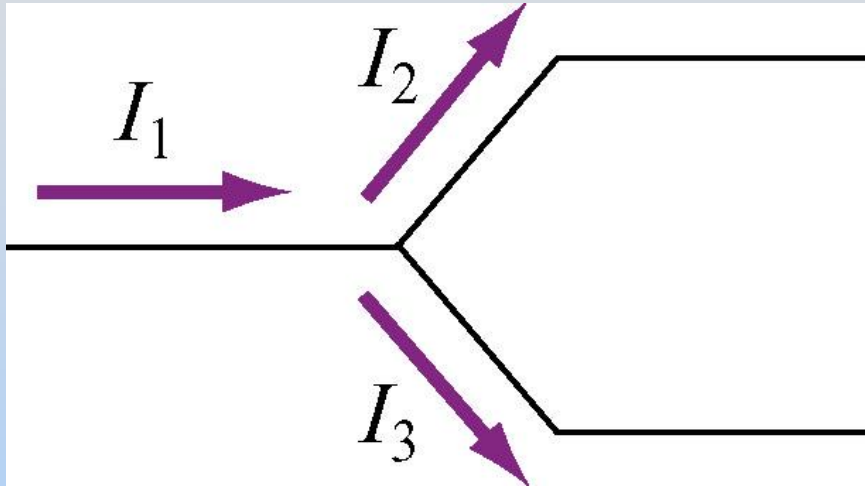
$$P_{\text{dissipated}} = I \Delta V = I^2 R = \frac{\Delta V^2}{R}$$



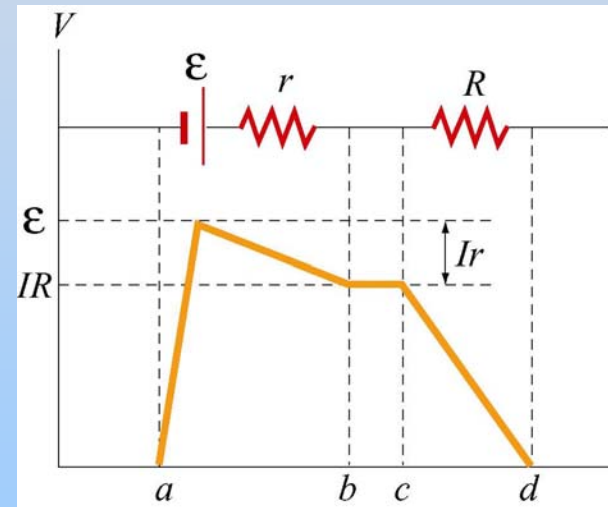
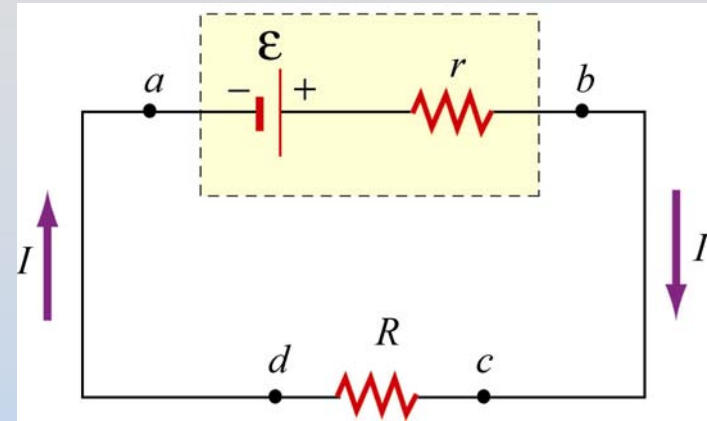
$$P_{\text{absorbed}} = I \Delta V = \frac{dQ}{dt} \frac{Q}{C}$$

$$= \frac{d}{dt} \frac{Q^2}{2C} = \frac{dU}{dt}$$

Kirchhoff's Rules



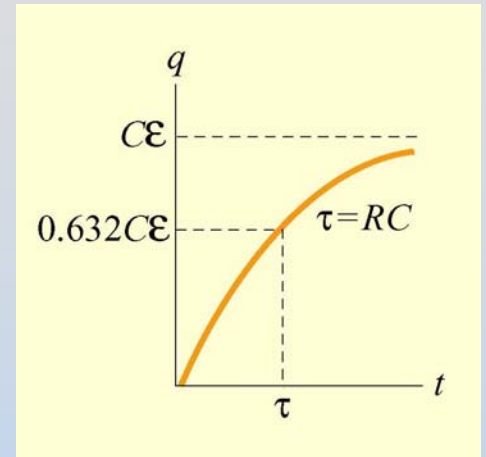
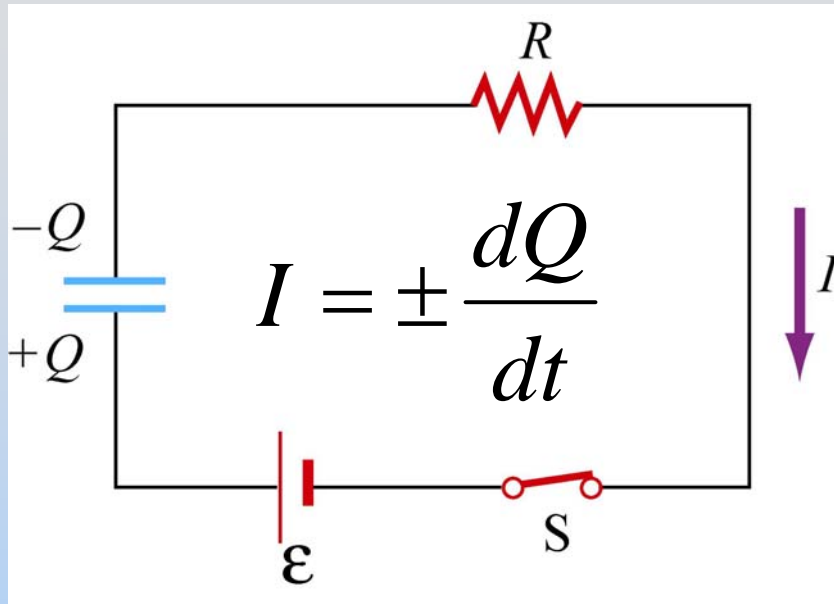
$$I_1 = I_2 + I_3$$



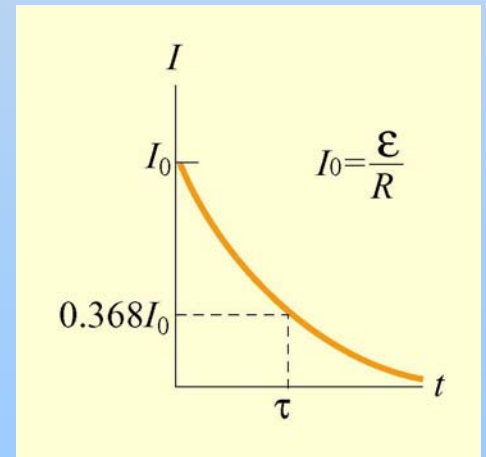
$$\Delta V = - \oint_{\text{Closed Path}} \vec{\mathbf{E}} \cdot d\vec{\mathbf{s}} = 0$$

Closed Path

(Dis)Charging A Capacitor



$$Q = C\mathcal{E} \left(1 - e^{-t/RC} \right)$$



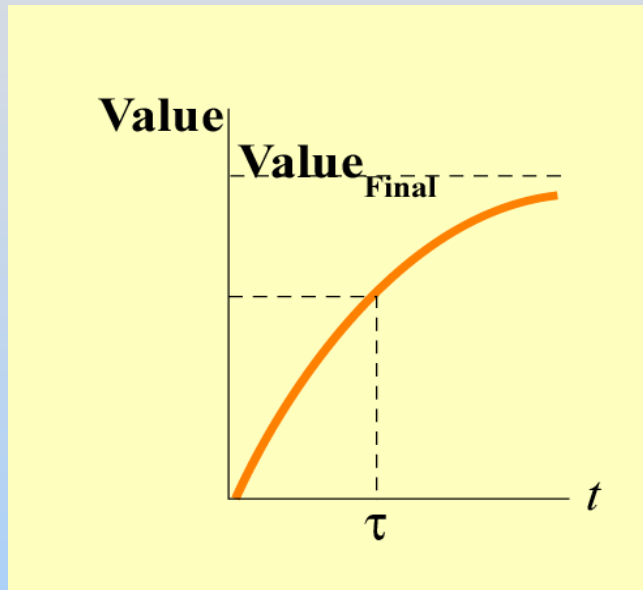
$$I = \frac{dQ}{dt} = \frac{\mathcal{E}}{R} e^{-t/RC}$$

$$\sum_i \Delta V_i = \mathcal{E} - \frac{Q}{C} - IR = 0$$

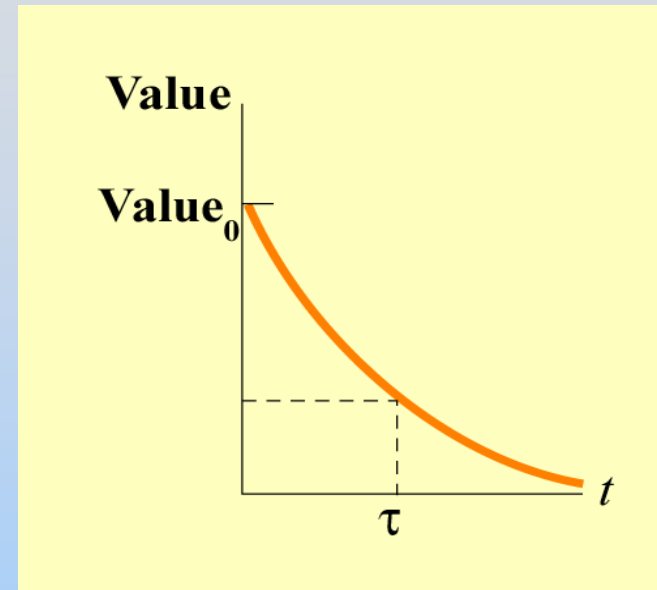
$$C\mathcal{E} - Q - RC \frac{dQ}{dt} = 0$$

General Comment: RC

All Quantities Either:



$$\text{Value}(t) = \text{Value}_{\text{Final}} \left(1 - e^{-t/\tau}\right)$$



$$\text{Value}(t) = \text{Value}_0 e^{-t/\tau}$$

τ can be obtained from differential equation
(prefactor on d/dt) e.g. $\tau = RC$

PRS Questions: DC Circuits with Capacitors

Class 12

Right Hand Rules

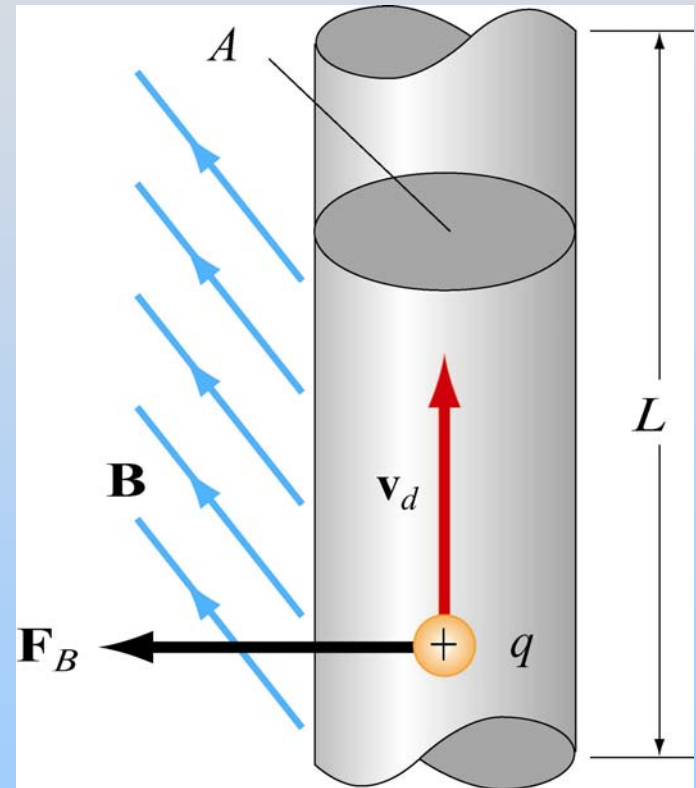
1. Torque: Thumb = torque,
Fingers show rotation
2. Feel: Thumb = I ,
Fingers = B ,
Palm = F
3. Create: Thumb = I
Fingers (curl) = B
4. Moment: Fingers (curl) = I
Thumb = Moment (= B inside loop)

Magnetic Force

$$\vec{\mathbf{F}}_B = q\vec{\mathbf{v}} \times \vec{\mathbf{B}}$$

$$d\vec{\mathbf{F}}_B = Id\vec{\mathbf{s}} \times \vec{\mathbf{B}}$$

$$\vec{\mathbf{F}}_B = I \left(\vec{\mathbf{L}} \times \vec{\mathbf{B}} \right)$$

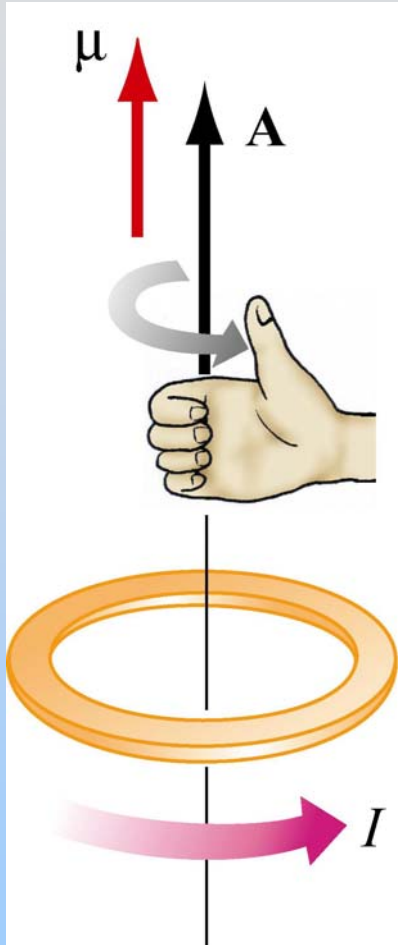


PRS Questions: Right Hand Rule

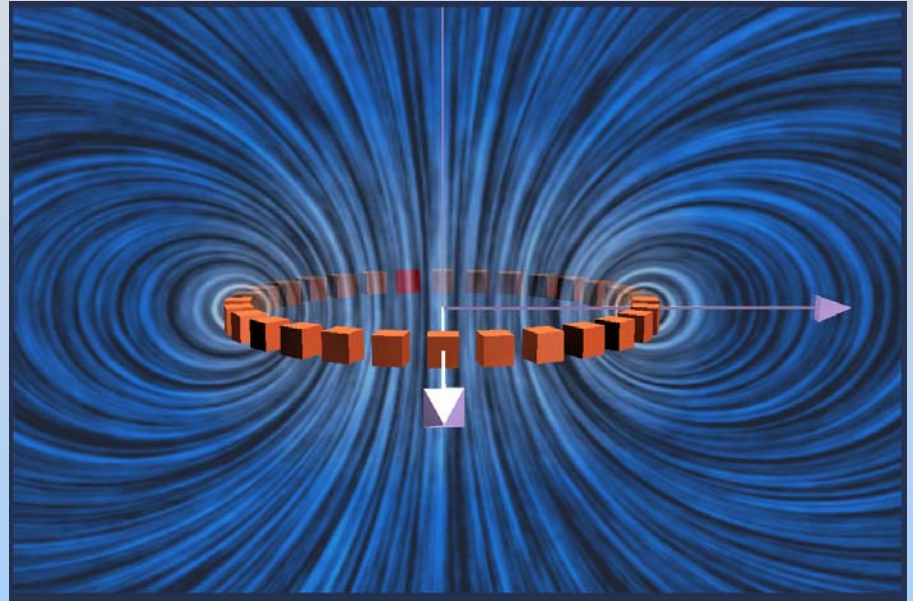
Class 14

Magnetic Dipole Moments

$$\vec{\mu} \equiv IA\hat{n} \equiv I\vec{A}$$



Generate:



Feel:

- 1) Torque to align with external field
- 2) Forces as for bar magnets

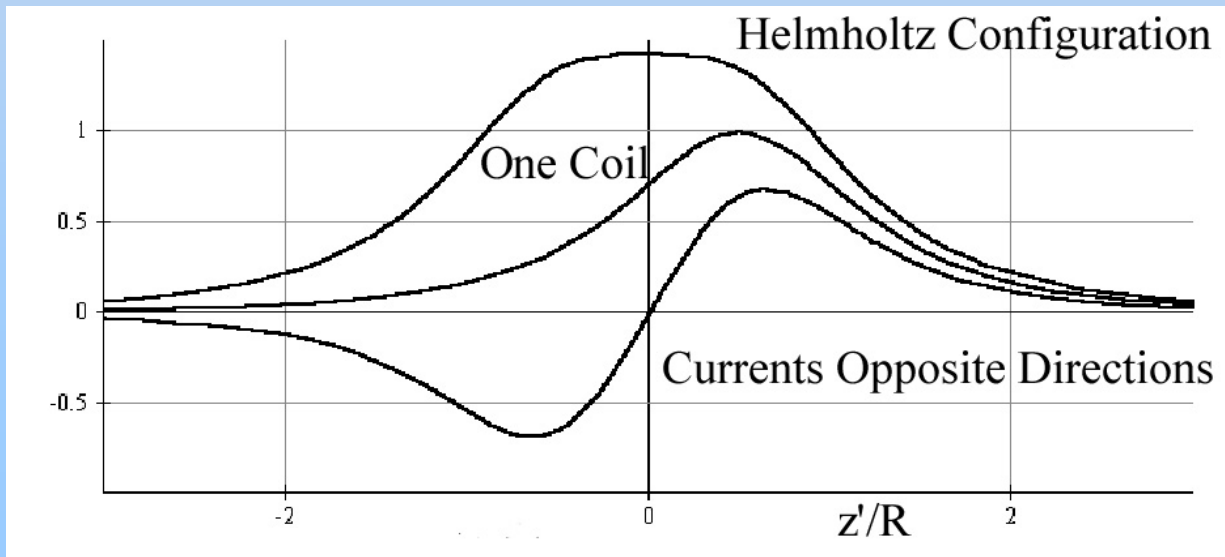
Helmholtz Coil



Common Concept Question

Parallel (Helmholtz) makes uniform field (torque, no force)

Anti-parallel makes zero, non-uniform field (force, no torque)

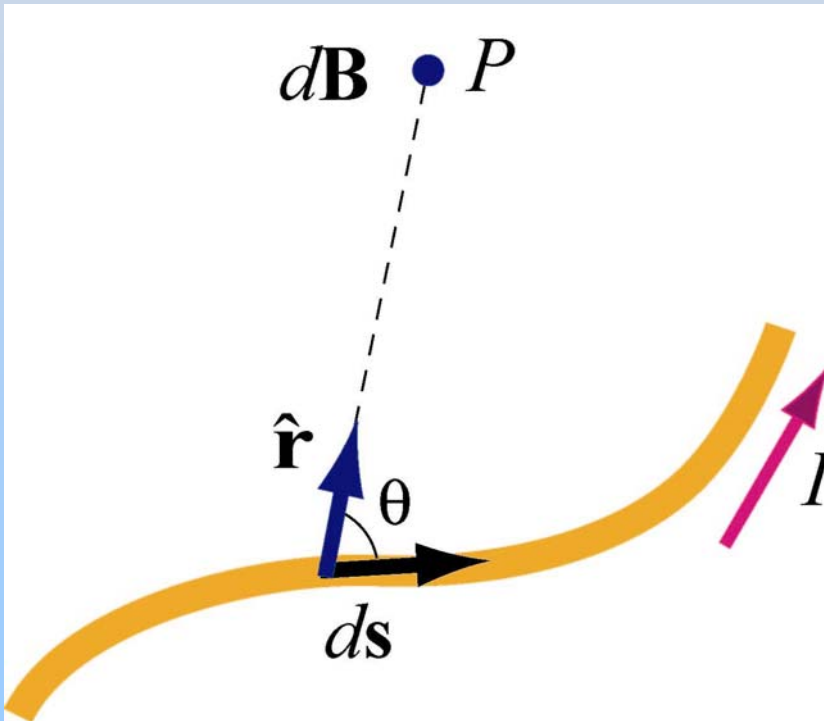


PRS Questions: Magnetic Dipole Moments

Class 17

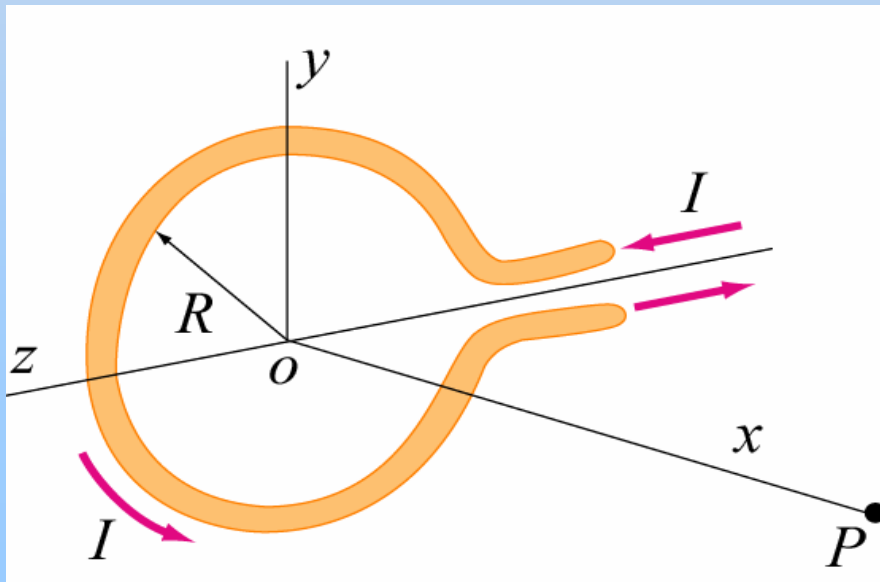
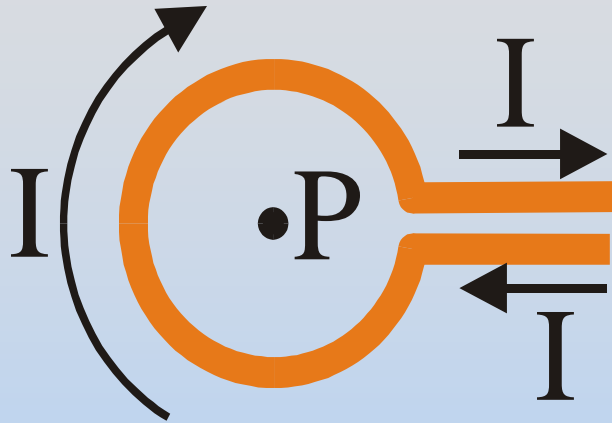
The Biot-Savart Law

Current element of length ds carrying current I
(or equivalently charge q with velocity v)
produces a magnetic field:



$$\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{q \vec{\mathbf{v}} \times \hat{\mathbf{r}}}{r^2}$$
$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{I d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2}$$

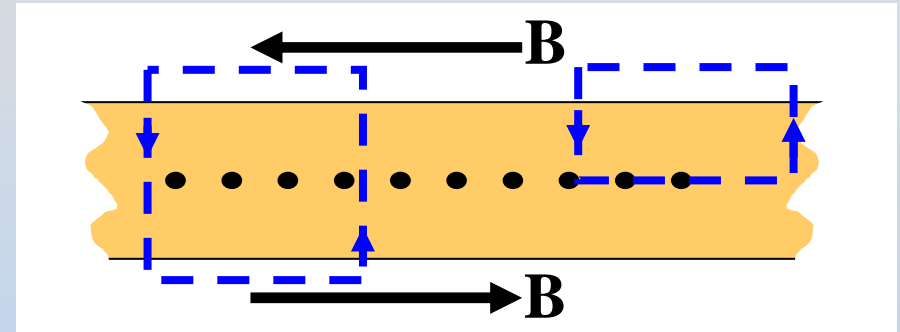
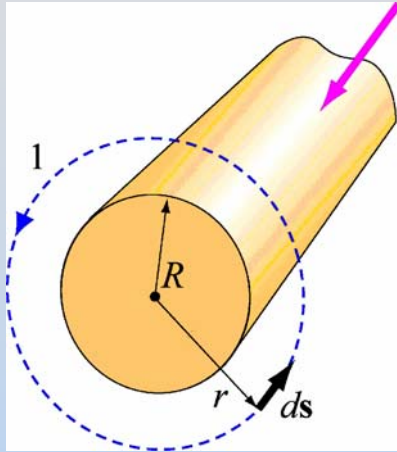
Biot-Savart: 2 Problem Types



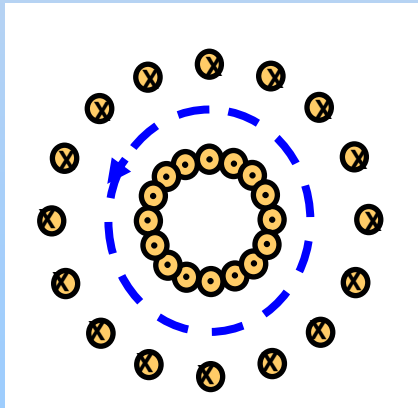
Notice that r is the same for every point on the loop. You don't really need to integrate (except to find path length)

Ampere's Law: $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$

Long
Circular
Symmetry

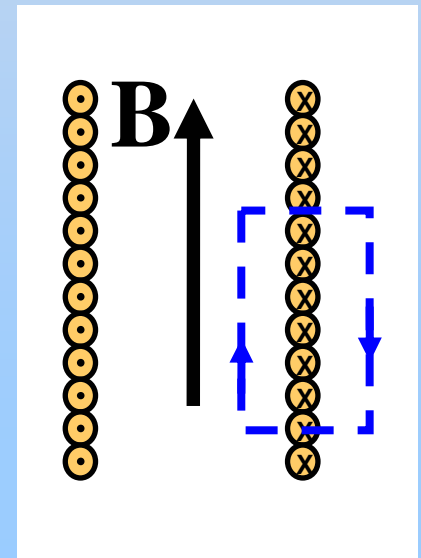
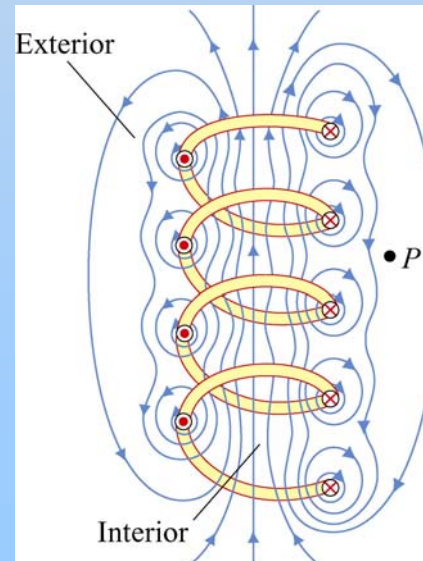


(Infinite) Current Sheet



Torus/Coax

Solenoid
=
2 Current
Sheets

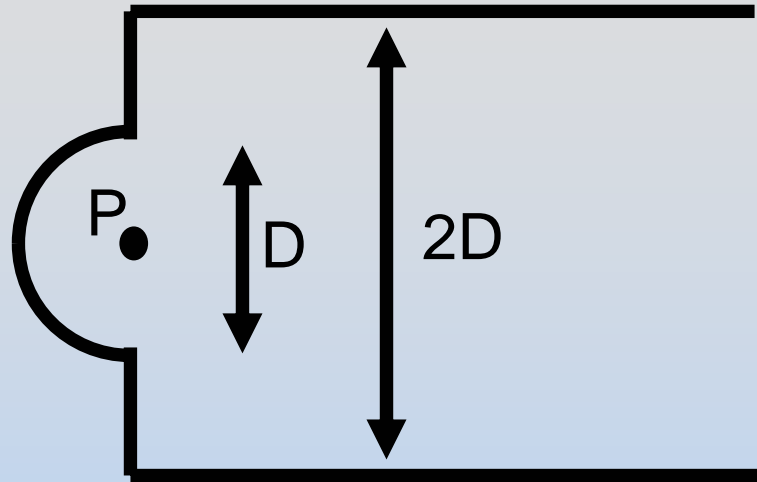


PRS Questions: Making B Fields

Classes 14-19

SAMPLE EXAM:

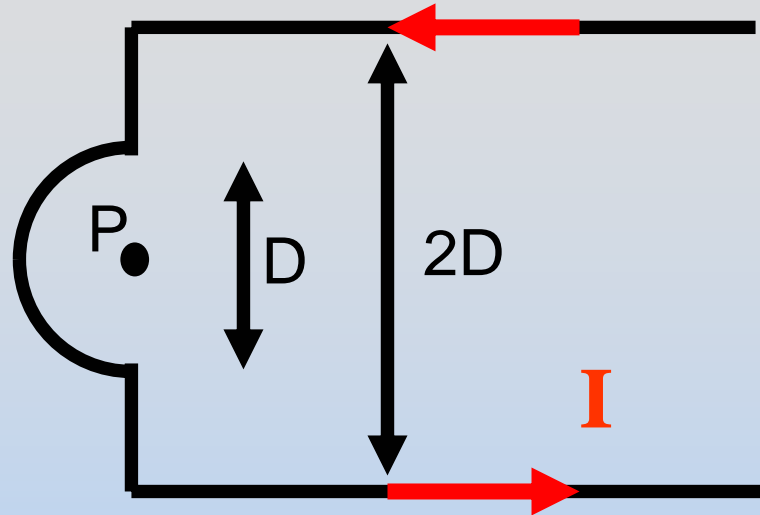
Problem 1: Wire Loop



A current flowing in the circuit pictured produces a magnetic field at point P pointing out of the page with magnitude B .

- What direction is the current flowing in the circuit?
- What is the magnitude of the current flow?

Solution 1: Wire Loop



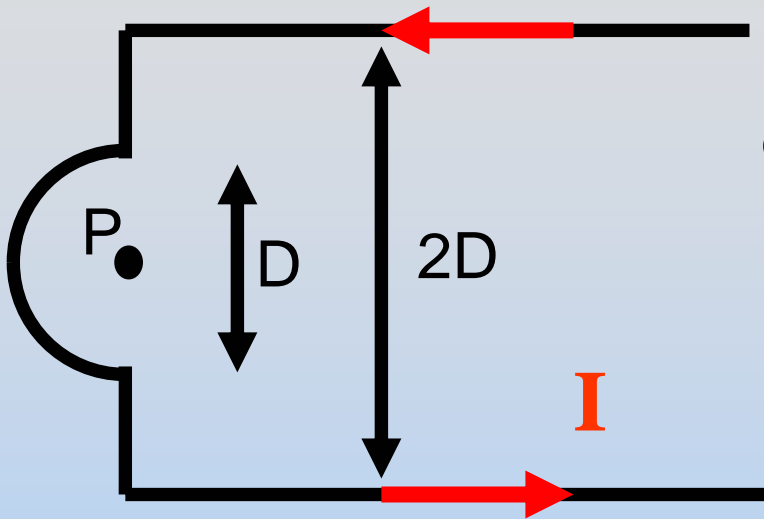
a) The current is flowing counter-clockwise, as shown above

b) There are three segments of the wire: the semi-circle, the two horizontal leads, and the two vertical leads.

The two vertical leads do not contribute to the B field ($ds \parallel r$)

The two horizontal leads make an infinite wire a distance D from the field point.

Solution 1: Wire Loop



For infinite wire use Ampere's Law:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} \Rightarrow B \cdot 2\pi D = \mu_0 I$$

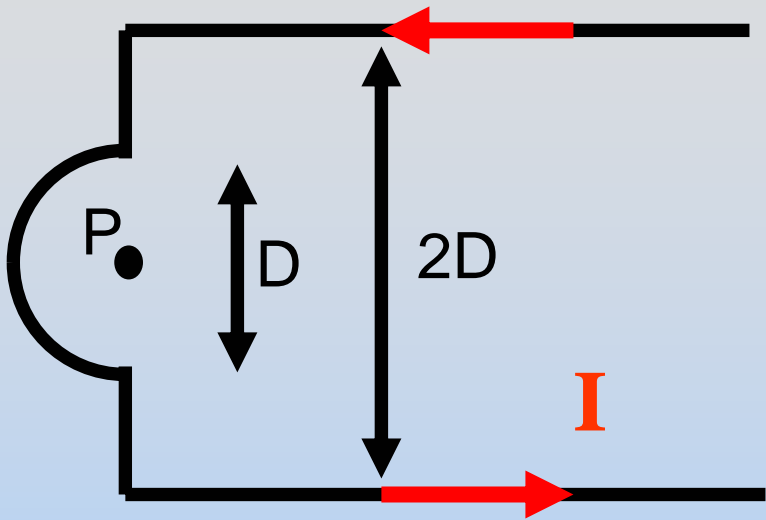
$$B = \frac{\mu_0 I}{2\pi D}$$

For the semi-circle
use Biot-Savart:

$$d\vec{\mathbf{B}} = \frac{\mu_0}{4\pi} \frac{I d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2} \quad r = \frac{D}{2} \text{ and } d\vec{\mathbf{s}} \perp \hat{\mathbf{r}}$$

$$\begin{aligned} B &= \int dB = \int \frac{\mu_0}{4\pi} \frac{I d\vec{\mathbf{s}} \times \hat{\mathbf{r}}}{r^2} \\ &= \frac{\mu_0}{4\pi} \frac{I}{r^2} (\pi r) = \frac{\mu_0 I}{4r} = \frac{\mu_0 I}{2D} \end{aligned}$$

Solution 1: Wire Loop



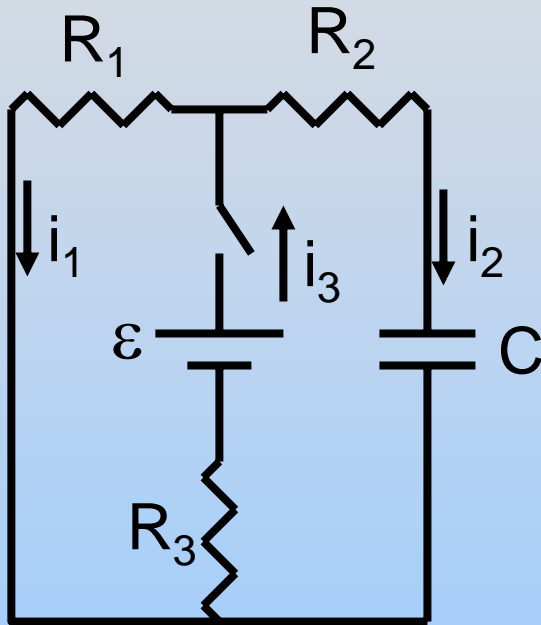
Adding together the two parts:

$$B = \frac{\mu_0 I}{2\pi D} + \frac{\mu_0 I}{2D} = \frac{\mu_0 I}{2D} \left(1 + \frac{1}{\pi} \right)$$

They gave us B and want I to make that B:

$$I = \frac{2DB}{\mu_0 \left(1 + \frac{1}{\pi} \right)}$$

Problem 2: RC Circuit



Initially C is uncharged.

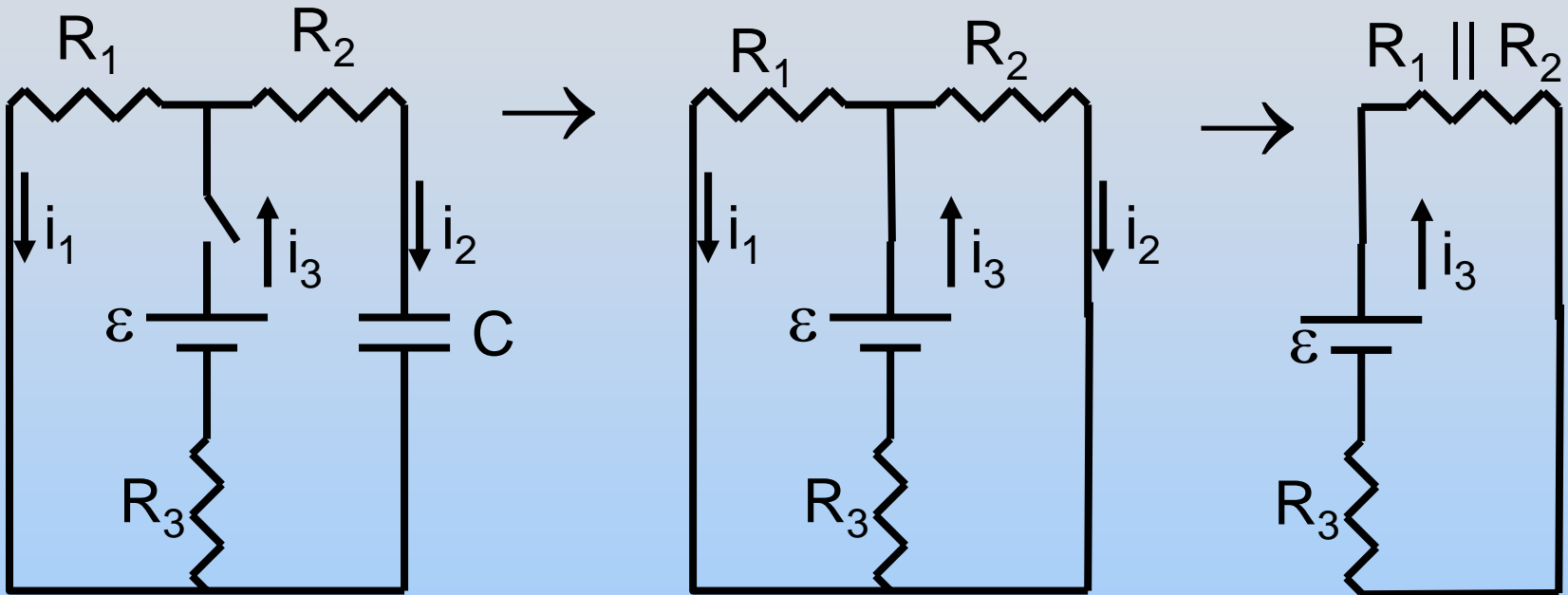
1. When the switch is first closed, what is the current i_3 ?
2. After a very long time, how much charge is stored on the capacitor?
3. Obtain a differential equation for the charge on the capacitor
(Here only, let $R_1=R_2=R_3=R$)

Now the switch is opened

4. Immediately after opening the switch, what is i_1 ? i_2 ? i_3 ?
5. How long before i_2 falls to $1/e$ of this initial value?

Solution 2: RC Circuit

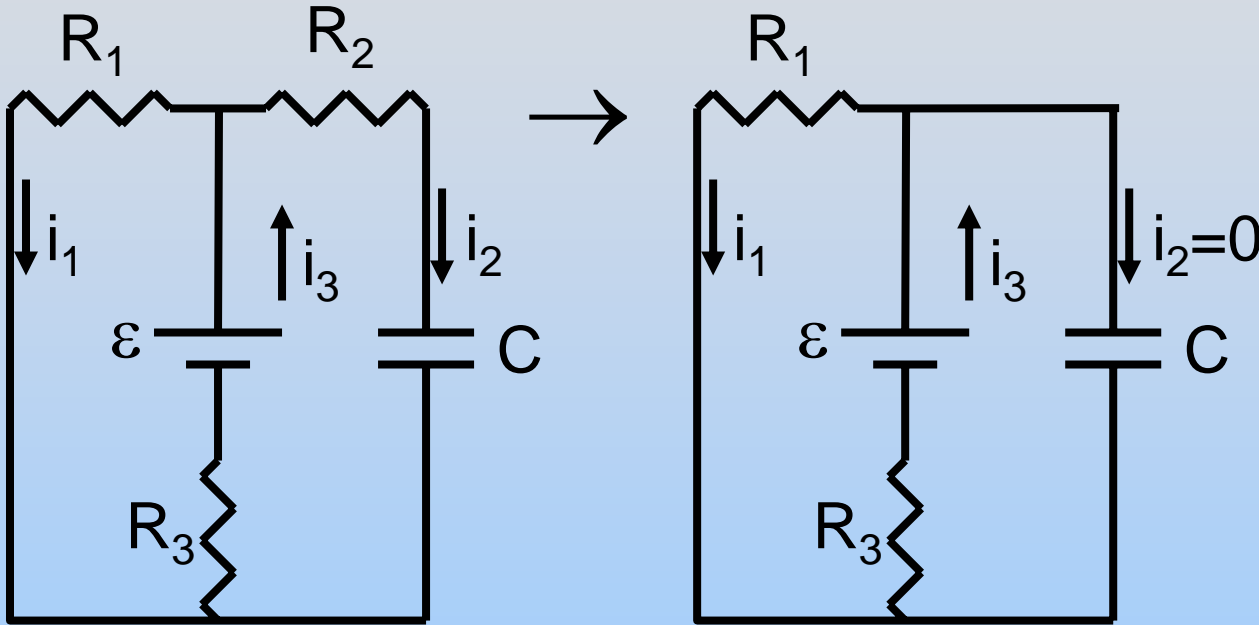
Initially C is uncharged \rightarrow Looks like short



$$R_{\text{eq}} = R_3 + \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} \Rightarrow i_3 = \frac{\varepsilon}{R_{\text{eq}}}$$

Solution 2: RC Circuit

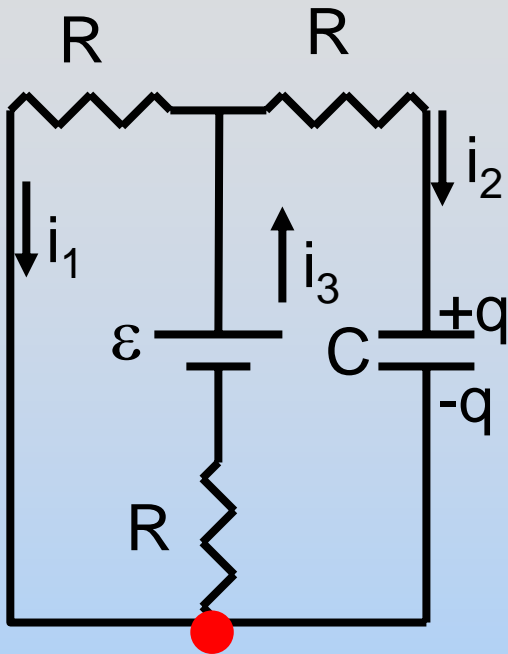
After a long time, C is full $\rightarrow i_2 = 0$



$$i_1 = i_3 = \frac{\varepsilon}{R_1 + R_3}$$

$$Q = CV_C = C(i_1 R_1) = \boxed{C\varepsilon \frac{R_1}{R_1 + R_3}}$$

Solution 2: RC Circuit



Kirchhoff's Loop Rules

$$\text{Left: } -i_3 R + \varepsilon - i_1 R = 0$$

$$\text{Right: } -i_3 R + \varepsilon - i_2 R - \frac{q}{c} = 0$$

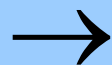
$$\text{Current: } i_3 = i_1 + i_2$$

Want to have i_2 and q only ($L-2R$):

$$0 = -(i_1 + i_2)R + \varepsilon - i_1 R + 2(i_1 + i_2)R - 2\varepsilon + 2i_2 R + \frac{2q}{c}$$

$$= 3i_2 R - \varepsilon + \frac{2q}{c}$$

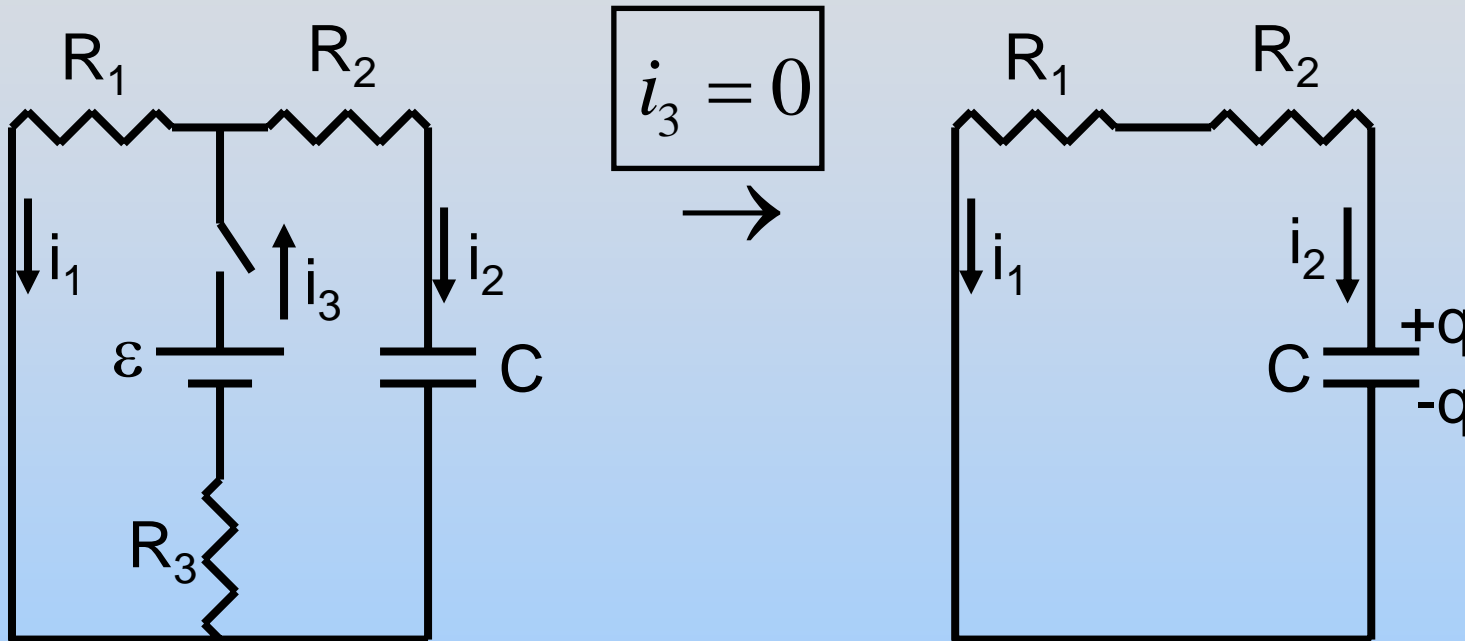
$$i_2 = + \frac{dq}{dt}$$



$$\frac{dq}{dt} = \frac{\varepsilon}{3R} - \frac{2q}{3RC}$$

Solution 2: RC Circuit

Now open the switch.



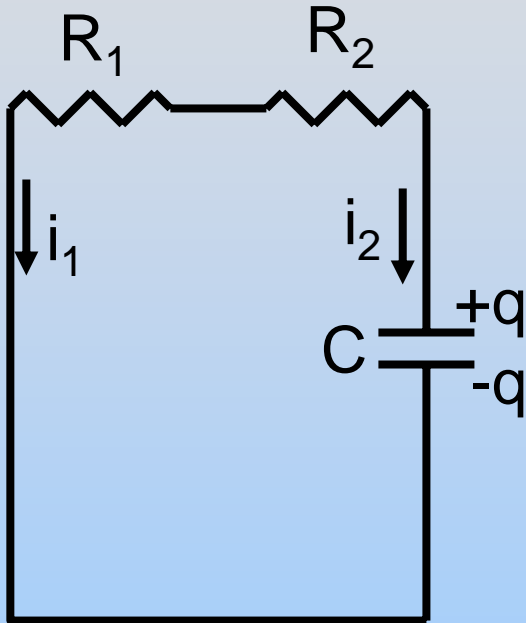
Capacitor now like a battery, with:

$$V_C = \frac{Q}{C} = \varepsilon \frac{R_1}{R_1 + R_3}$$

$$i_1 = -i_2 = \frac{V_C}{R_1 + R_2} = \varepsilon \frac{R_1}{R_1 + R_3} \frac{1}{R_1 + R_2}$$

Solution 2: RC Circuit

How long to fall to 1/e of initial current? The time constant!

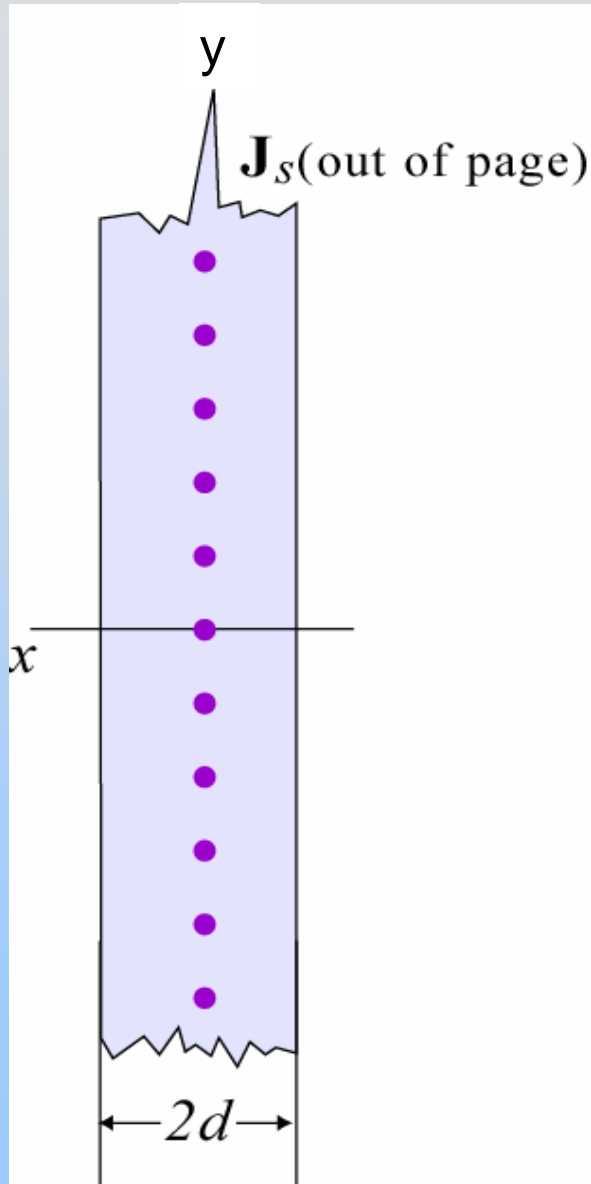


This is an easy circuit since it just looks like a resistor and capacitor in series, so:

$$\tau = (R_1 + R_2)C$$

Notice that this is different than the charging time constant, because there was another resistor in the circuit during the charging

Problem 3: Non-Uniform Slab

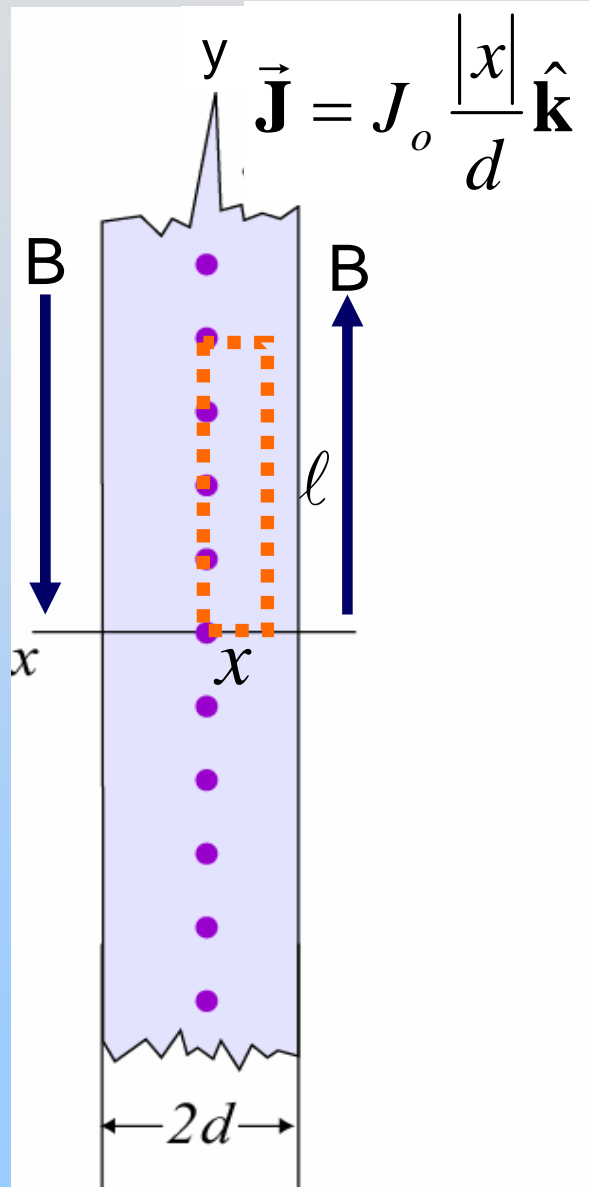


Consider the slab at left with non-uniform current density:

$$\vec{\mathbf{J}} = J_o \frac{|x|}{d} \hat{\mathbf{k}}$$

Find \mathbf{B} everywhere

Solution 3: Non-Uniform Slab



Direction: Up on right, down on left

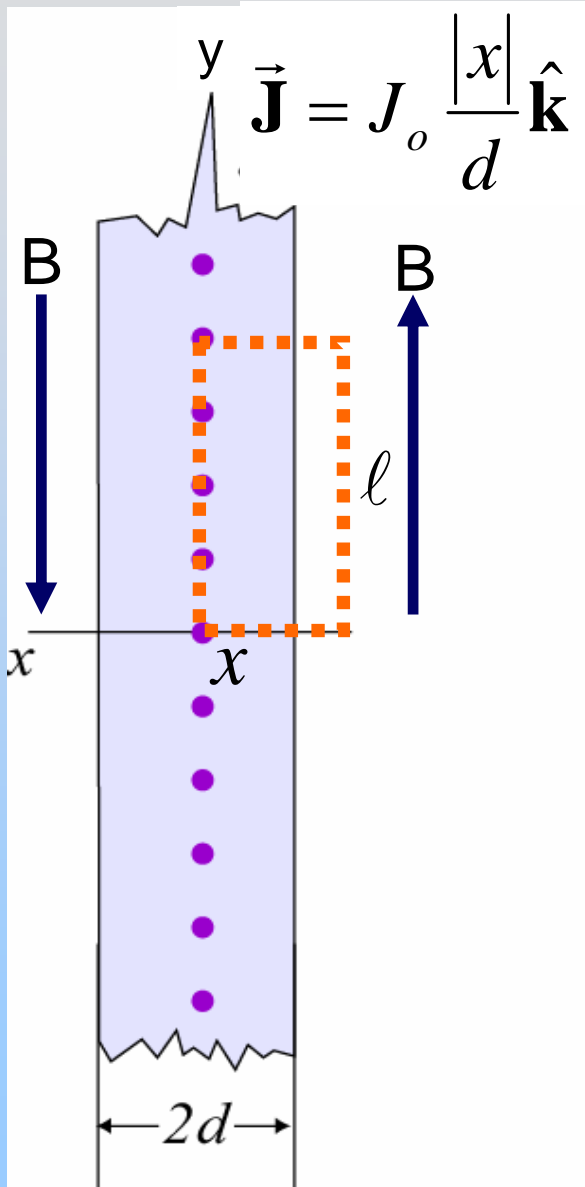
Inside: (at $0 < x < d$): $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$

$$\oint \vec{B} \cdot d\vec{s} = B\ell + 0 + 0 + 0$$

$$\begin{aligned} \mu_0 I_{enc} &= \mu_0 \iint \vec{J} \cdot d\vec{A} = \mu_0 \int_0^x \frac{J_0 x}{d} \ell dx \\ &= \mu_0 \frac{J_0 \ell}{d} \frac{x^2}{2} \end{aligned}$$

$$B = \mu_0 \frac{J_0}{d} \frac{x^2}{2} \quad \text{up}$$

Solution 3: Non-Uniform Slab



Direction: Up on right, down on left

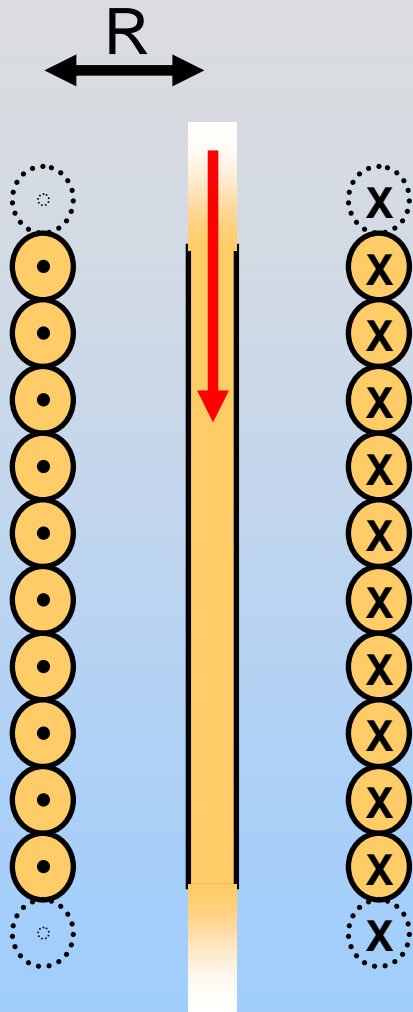
Outside: ($x > d$): $\oint \vec{B} \cdot d\vec{s} = \mu_0 I_{enc}$

$$\oint \vec{B} \cdot d\vec{s} = B\ell + 0 + 0 + 0$$

$$\begin{aligned} \mu_0 I_{enc} &= \mu_0 \iint \vec{J} \cdot d\vec{A} = \mu_0 \int_0^d \frac{J_0 x}{d} \ell dx \\ &= \mu_0 \frac{J_0 \ell}{d} \frac{d^2}{2} \end{aligned}$$

$$B = \frac{1}{2} \mu_0 J_0 d \text{ up}$$

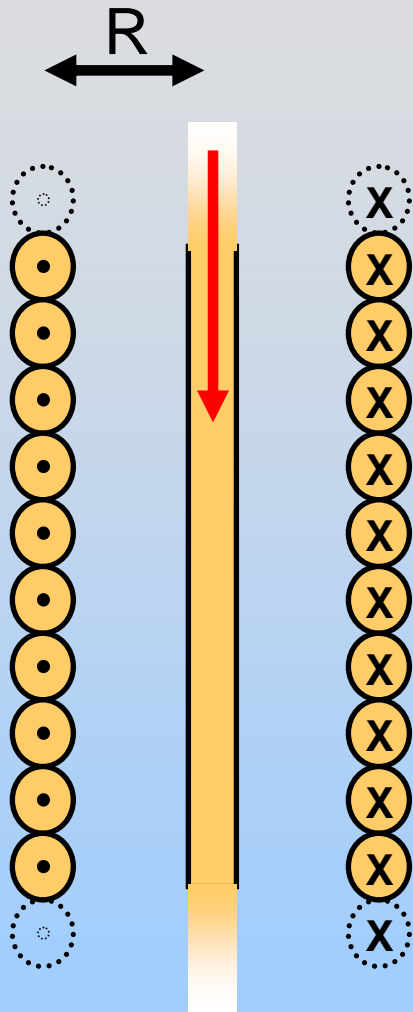
Problem 4: Solenoid



A current I flows up a very long solenoid and then back down a wire lying along its axis, as pictured. The wires are negligibly small (i.e. their radius is 0) and are wrapped at n turns per meter.

- What is the force per unit length (magnitude and direction) on the straight wire due to the current in the solenoid?
- A positive particle (mass m , charge q) is launched inside of the solenoid, at a distance $r = a$ to the right of the center. What velocity (direction and non-zero magnitude) must it have so that the field created by the wire along the axis never exerts a force on it?

Solution 4: Solenoid



SUPERPOSITION

You can just add the two fields from each part individually

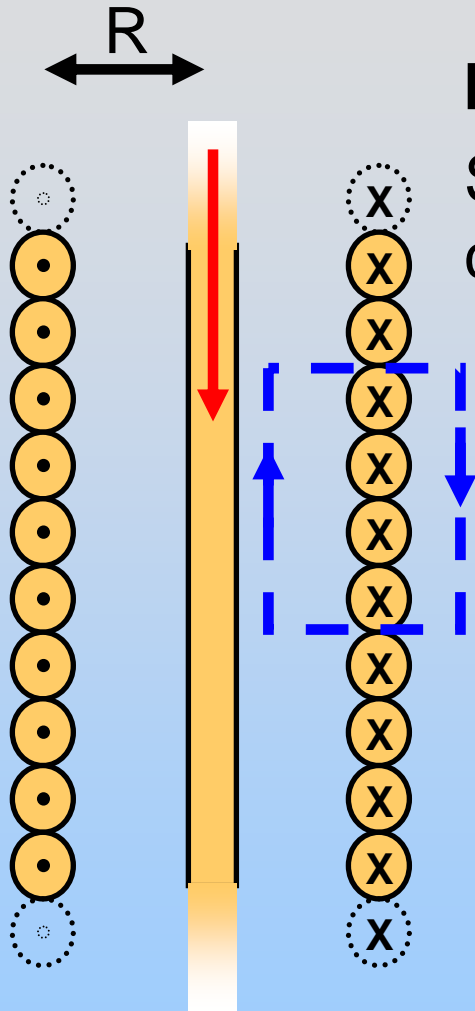
a) Force on wire down axis

Since the current is anti-parallel to the field produced by the solenoid, there is no force ($F=0$) on this wire

b) Launching Charge q

The central wire produces a field that wraps in circles around it. To not feel a force due to this field, the particle must always move parallel to it – it must move in a circle of radius a (since that is the radius it was launched from).

Solution 4: Solenoid



b) Launching Charge q

So first we should use Ampere's law to calculate the field due to the solenoid:

$$\oint \vec{\mathbf{B}} \cdot d \vec{\mathbf{s}} = Bl = \mu_0 NI$$

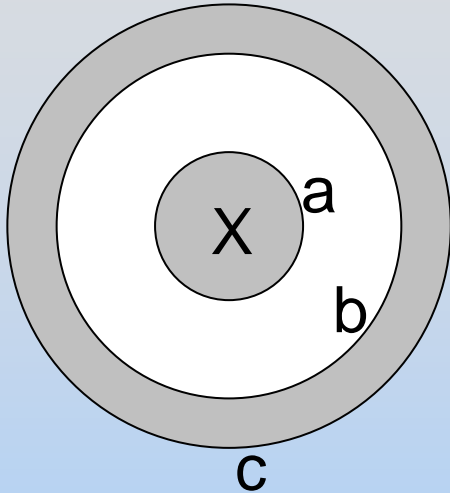
$$B = \frac{\mu_0 NI}{l} = \mu_0 nI \text{ up the solenoid}$$

Now we just need to make a charge q move in a circular orbit with $r = a$:

$$\vec{\mathbf{F}}_B = q \vec{\mathbf{v}} \times \vec{\mathbf{B}} = qvB = m \frac{v^2}{r} = \frac{mv^2}{a}$$

$$v = \frac{qBa}{m} = \frac{q\mu_0 nIa}{m} \text{ out of the page}$$

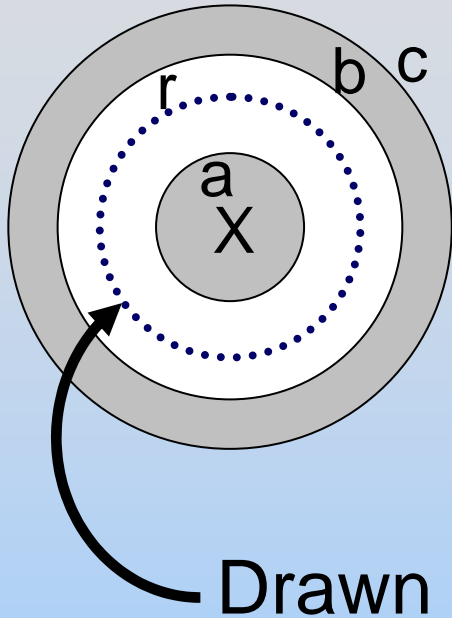
Problem 5: Coaxial Cable



Consider a coaxial cable of with inner conductor of radius a and outer conductor of inner radius b and outer radius c . A current I flows into the page on the inner conductor and out of the page on the outer conductor.

What is the magnetic field everywhere (magnitude and direction) as a function of distance r from the center of the wire?

Solution 5: Coaxial Cable



Everywhere the magnetic field is clockwise. To figure out the magnitude use Ampere's Law:

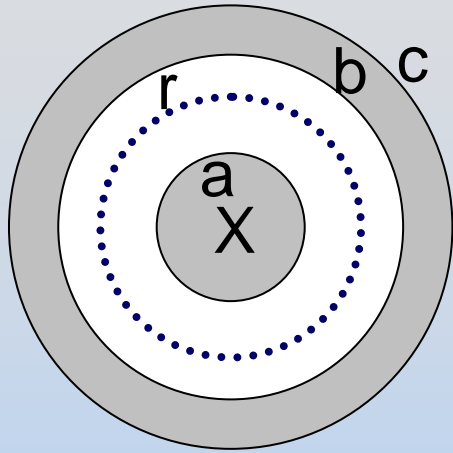
$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I_{enc} \Rightarrow B \cdot 2\pi r = \mu_0 I_{enc}$$
$$\Rightarrow B = \frac{\mu_0 I_{enc}}{2\pi r}$$

Drawn for $a < r < b$

The amount of current penetrating our Amperian loop depends on the radius r :

$$r \leq a: I_{enc} = I \frac{r^2}{a^2} \Rightarrow B = \frac{\mu_0 I r}{2\pi a^2} \text{ clockwise}$$

Solution 5: Coaxial Cable



Remember: Everywhere

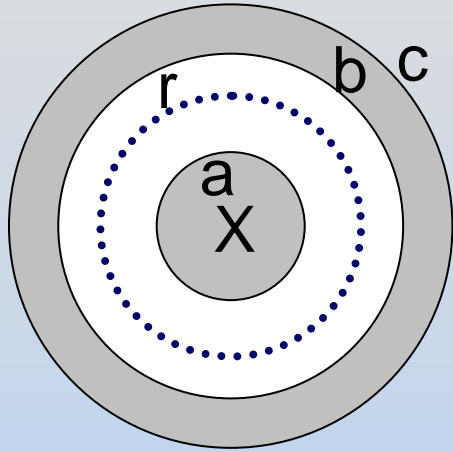
$$B = \frac{\mu_0 I_{enc}}{2\pi r} \text{ clockwise}$$

$$a \leq r \leq b: I_{Encl} = I \Rightarrow B = \frac{\mu_0 I}{2\pi r} \text{ clockwise}$$

$$b \leq r \leq c: I_{Encl} = I \left(1 - \frac{r^2 - b^2}{c^2 - b^2} \right)$$

$$\Rightarrow B = \frac{\mu_0 I}{2\pi r} \left(1 - \frac{r^2 - b^2}{c^2 - b^2} \right) \text{ clockwise}$$

Solution 5: Coaxial Cable



Remember: Everywhere

$$B = \frac{\mu_0 I_{enc}}{2\pi r} \text{ clockwise}$$

$$r \geq c: I_{Encl} = 0 \quad \Rightarrow \quad \boxed{B = 0}$$