

MASSACHUSETTS INSTITUTE OF TECHNOLOGY  
Department of Physics

**Problem Solving 10: Double-Slit Interference**

**OBJECTIVES**

1. To introduce the concept of interference.
2. To find the conditions for constructive and destructive interferences in a double-slit interference experiment.
3. To compute the intensity of the interference pattern.

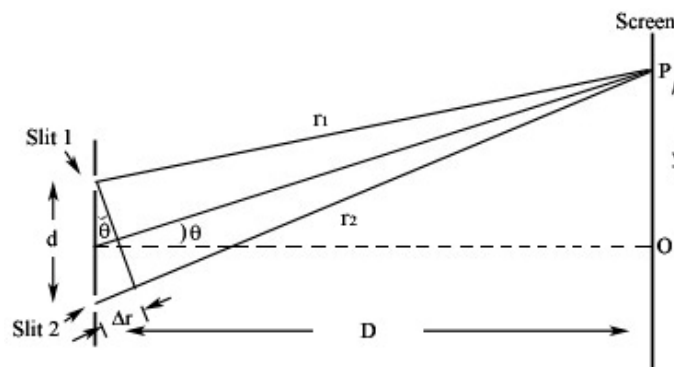
**REFERENCE:** Sections 14.1 – 14.3, 8.02 Course Notes.

**Introduction**

When ordinary light is emitted from two different sources and passes through two narrow slits, the plane waves do not maintain a constant phase relation and the light shows no interference pattern in the region beyond the openings. In order for an interference pattern to develop, the incoming light must satisfy two conditions:

- The light sources must be coherent. This means that the plane waves from the sources must maintain a constant phase relation.
- The light must be monochromatic. This means that the light has just one wavelength.

When a coherent monochromatic laser light falls on two slits separated by a distance  $d$ , the emerging light will produce an interference pattern on a viewing screen a distance  $D$  away from the center of the slits. The geometry of the double slit interference is shown in the figure below.



**Figure 1** Double slit interference

Consider light that falls on the screen at a point  $P$  a distance  $y$  from the point  $O$  that lies on the screen a perpendicular distance  $D$  from the double slit system. The light from the slit 2 will travel an extra distance  $r_2 - r_1 = \Delta r$  to the point  $P$  than the light from slit 1. This extra distance is called the path length.

**Question 1:** Explain why constructive interference will appear at the point  $P$  when the path length is equal to an integral number of wavelengths of the monochromatic light.

$$\Delta r = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \dots \quad \text{constructive interference}$$

**Answer:**

We place the screen so that the distance to the screen is much greater than the distance between the slits,  $D \gg d$ . In addition we assume that the distance between the slits is much greater than the wavelength of the monochromatic light,  $d \gg \lambda$ . Then the angle  $\theta$  is very small, so that

$$\sin \theta \approx \tan \theta = y/D.$$

**Question 2:** Based on the geometry of the double slits, show that the condition for constructive interference becomes

$$d \sin \theta = m\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \dots \quad \text{constructive interference.}$$

**Answer:**

**Question 3:** Explain why destructive interference will appear at the point  $P$  when the path length is equal to an odd integral number of half wavelengths

$$d \sin \theta = \left(m + \frac{1}{2}\right)\lambda, \quad m = 0, \pm 1, \pm 2, \pm 3, \dots \quad \text{destructive interference.}$$

**Answer:**

**Question 4:** Let  $y$  be the distance between the point  $P$  and the point  $O$  on the screen. Find a relation between the distance  $y$ , the wavelength  $\lambda$ , the distance between the slits  $d$ , and the distance to the screen  $D$  such that a constructive interference pattern will occur at the point  $P$ .

**Answer:**

**Question 5:** Find a similar relation such that destructive interference fringes will occur at the point  $P$ .

**Answer:**

### Intensity of Double-Slit Interference

Suppose that the waves are emerging from the slits are sinusoidal plane waves. The slits are located at the plane  $x = -D$ . The light that emerges from slit 1 and slit 2 at time  $t$  are in phase. Let the screen be placed at the plane  $x = 0$ . Suppose the component of the electric field of the wave from slit 1 at the point  $P$  is given by

$$E_1 = E_0 \sin(\omega t).$$

Let's assume that the plane wave from slit 2 has the same amplitude  $E_0$  as the wave from slit 1. Since the plane wave from slit 2 has to travel an extra distance to the point  $P$  equal to the path length, this wave will have a phase shift  $\phi$  relative to the wave from slit 1,

$$E_2 = E_0 \sin(\omega t + \phi).$$

**Question 6:** Why are the phase shift  $\phi$ , the wavelength  $\lambda$ , the distance between the slits, and the angle related  $\theta$  by

$$\phi = \frac{2\pi}{\lambda} d \sin \theta.$$

As a hint how are the ratio of the phase shift  $\phi$  to  $2\pi$  and the ratio of the path length  $\Delta r = d \sin \theta$  to wavelength  $\lambda$ , related?

**Answer:**

**Question 7:** The total electric field at the point  $P$  is the superposition of these two fields  $E = E_1 + E_2$ . Using the trigonometric identity

$$\sin A + \sin(B) = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right),$$

show that the total component of the electric field is

$$E = E_1 + E_2 = 2E_0 \sin\left(\omega t + \frac{\phi}{2}\right) \cos\left(\frac{\phi}{2}\right).$$

**Answer:**

The *intensity* of the light is equal to the time-averaged Poynting vector

$$I = \langle \vec{S} \rangle = \frac{1}{\mu_0} \langle \vec{E} \times \vec{B} \rangle.$$

Since the amplitude of the magnetic field is related to the amplitude of the electric field by  $B_0 = E_0/c$ . The intensity of the light is proportional to the square of the electric field,

$$I \sim \langle E^2 \rangle = 4E_0^2 \cos^2\left(\frac{\phi}{2}\right) \left\langle \sin^2\left(\omega t + \frac{\phi}{2}\right) \right\rangle = 2E_0^2 \cos^2\left(\frac{\phi}{2}\right).$$

Let  $I_{\max}$  be the amplitude of the intensity. Then the intensity of the light at the point  $P$  is

$$I = I_{\max} \cos^2\left(\frac{\phi}{2}\right)$$

**Question 8:** Show that the intensity is maximal when  $d \sin \theta = m\lambda$ ,  $m = 0, \pm 1, \pm 2, \pm 3, \dots$

**Answer:**

**Question 9:** Graph the intensity pattern on the screen as a function of distance  $y$  from the point  $O$  for the case that  $D \gg d$  and  $d \gg \lambda$ .

**Answer:**

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Tear off this page and turn it in at the end of class !!!!

Note:

Writing in the name of a student who is not present is a Committee on Discipline offense.

Group \_\_\_\_\_ (e.g. 6A Please Fill Out)

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**Answer:**

**Question 5:** Find a similar relation such that destructive interference fringes will occur at the point  $P$ .

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**Answer:**

**Question 7:** Using the trigonometric identity  $\sin A + \sin(B) = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$ , show that the total component of the electric field is

$$E = E_1 + E_2 = 2E_0 \sin\left(\omega t + \frac{\phi}{2}\right) \cos\left(\frac{\phi}{2}\right).$$

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