MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF PHYSICS
8.022 FALL 2004
ASSIGNMENT 5: SPECIAL RELATIVITY; MAGNETIC FIELD
DUE DATE: FRIDAY, OCT 22TH

1. Lorentz invariance (10 pts). A quantity that is left unchanged by Lorentz transformations is a called a "Lorentz invariant".
a)Consider two events described in the laboratory frame by (t_1, x_1, y_1, z_1) and (t_2, x_2, y_2, z_2). Show that

$$\Delta s^2 = -(c\Delta t)^2 + (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2. \tag{1}$$

is a Lorentz invariant.
b)Energy-momentum identity. Show that

$$m^2c^4 = E^2 - p^2c^2 \tag{2}$$

where $p = |\vec{p}|$ is Lorentz invariant.

2. Transforming velocities (10 pts): A bullet is fired with velocity $\vec{u'}$ in the (x', y') plane of a moving frame F'. Frame F' moves with speed v in the $+x$ direction with respect to the laboratory frame F.
(a) Find the angle that the velocity vector makes with x axis of the lab frame.
(b) What is this angle in the limit $|\vec{u'}| = c$? Does anything weird happen?
(c) Show that when $|\vec{u'}| = c$, $|\vec{u}| = c$ – the speed of light is the same in both frames.

3. Galilean tranformations (10 pts): Prior to special relativity, people related coordinates between different frames with the "Galilean transformation" – clocks in different reference frame tick at the same rate, spatial positions are shifted by a term that depends on the relative velocity just as you would expect. For example, for frames that are moving with respect to each other in the x direction, we would have

$$\begin{aligned} t' &= t \\ x' &= x - vt \end{aligned} \tag{3}$$

Using the binomial expansion on γ, show that for small v/c the Lorentz tranformations reduce to the Galilean tranformations. At what value of v does the next term in the expansion change the x transformation by 1%?

4. Relativistic collisions (15 pts). A particle of rest mass m_1 and velocity v_1 collides with a stationary particle of rest mass m_2 and is absorbed by it. What is the velocity and the rest mass of the final compound system?

5. "Beating the speed of light" (15 pts). A friend of yours says he can beat the speed of light. His idea is as follows. He will make a cart roll across the floor with speed v. and

then puts a smaller cart on top of that cart, and roll it with speed v with respect to the first cart, and in the same direction as the first cart. he puts a third cart on this second cart; it rolls with speed v with respect to the second cart. He puts a fourth cart ... you get the idea. He claims that there is some n at which the cart must be going faster than the speed of light.
(a) Prove him wrong. Using mathematical induction, prove that if $v < c$, then $v_n < c$, where v_n is the velocity of the nth cart. Show that this holds even for extremely large n.
(b) Calculate the value of v_n given v and n.

6. Purcell 6.13 Helmholtz coil (15 pts).

7. Ampere's law (15 pts). A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field, both inside and outside the wire if
a) The current is uniformly distributed over the outside surface of the wire.
b) The current is distributed in such a way that J is proportional to s, the distance from the axis.

8. Fields of a thick sheet (10 pts).

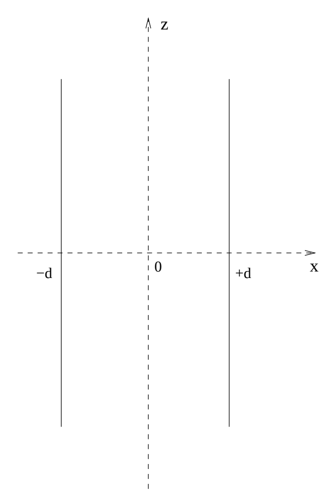

Figure 1: An infinitely large sheet.

A very large and thick sheet of conducting material carries a non-uniform current density

$$\vec{J} = J_0(\frac{x}{d})^2\hat{y} \tag{4}$$

The slab extends form $x = -d$ to $x = d$, and is infinite in y and z. Find the magetic field inside and outside of the slab. (y axis points into the paper)