Massachusetts Institute of Technology
Department of Physics
Physics 8.022 – Fall 2002
Quiz#1

- Total points in the quiz are 100. **ALL** problems receive **equal** points (20 each). Work on problems you are more comfortable with **first!**
- This is a closed book and closed notes exam. An equations table is given to you below.
- No programmable, plotting, integration/differentiation capable calculators are allowed.

Electrostatics Formulae for Quiz #1

Conservative Field: $\int_C \vec{F} \cdot d\vec{r} = 0$ for any closed path C, $W_{ab,C} = W_{ab,C'}$ for any C, C' connecting a *and* b, $\vec{F} = -\vec{\nabla} U$, $\vec{\nabla} \times \vec{F} = 0$

Coulomb Law: $\vec{F}_{21} = \frac{q_1 q_2}{r^2} \hat{r}_{21}$ for two point charges at distance r. $\vec{F}_{12} = -\vec{F}_{21}$, and for charges dq_1 and dq_2 that make part of continuous charge distributions 1 and 2, $d\vec{F}_{21} = \frac{dq_1 dq_2}{r^2} \hat{r}_{21}$

Electric Field: *at point 2* due to q_1 $\vec{E}_1 = \frac{q_1}{r^2} \hat{r}_{21}$. If q_1 is not a point charge but part of a continuous distribution, $d\vec{E} = \frac{dq}{r^2} \hat{r}$

Principle of Superposition: Two or more electric fields acting at a given point P add vectorially: $\vec{E}_P = \vec{E}_1 + \vec{E}_2 + ... + \vec{E}_n$

Electrostatic Field is Conservative: $\vec{\nabla} \times \vec{E} = 0$ and thus there exists scalar function ϕ such that $\vec{E} = -\vec{\nabla} \phi$ where $d\phi = \frac{dq}{r}$

Electrostatic potential: The potential at $\vec{x}$ with respect to a *ref* point is $\phi(\vec{x}) - \phi(ref) = -\int_{ref}^{\vec{x}} \vec{E} \cdot d\vec{r} = -\frac{W_{ref \to \vec{x}}}{q}$

Gauss Law: $\int_S \vec{E} \cdot d\vec{a} = 4\pi \int_V \rho dV$ where S is a closed surface and V is its corresponding volume (integral form) or $\vec{\nabla} \cdot \vec{E} = 4\pi\rho$ (differential form).

Poisson Eqn: $\nabla^2 \phi = -4\pi\rho$, Laplace Eqn: $\nabla^2 \phi = 0$

Energy: $U = \frac{1}{2} \int_V dV \int_{V'} dV' \frac{\rho(\vec{x})\rho(\vec{x}')}{|\vec{x} - \vec{x}'|} = \frac{1}{2} \int_V \rho\phi dV = \frac{1}{8\pi} \int_V E^2 dV$

Electric Force on Conductors: $\frac{dF}{da} = 2\pi\sigma^2 = \frac{E^2}{8\pi}$ Current Density: $\vec{J}(\vec{x}) = \rho(\vec{x})\vec{v}(\vec{x})$, Conservation Law/Continuity: $\vec{\nabla} \cdot \vec{J} = -\frac{\partial \rho}{\partial t}$ Capacitance: $Q = CV$, $U = \frac{1}{2} CV^2$

Problem 1 (20 points): A Very Simple Capacitor

(a) Find the potential at the surface of a conducting sphere of radius a which carries charge $+Q$.

A simple capacitor is made by placing two spheres of radius a a distance $d >> a$ apart. The spheres are far enough apart so the charge distributions are uniform on each.

(b) Find the potential difference ϕ between the spheres.
(c) Find the capacitance of the system of the two spheres.
(d) Find the stored energy of the system which is equal to $U = \int \frac{1}{8\pi} E^2 dV$; you do not need to integrate the field to find U.

Problem 2 (20 points): Charges and Conductors: Method of Images

A point charge $+Q$ sits at a distance h infront of a grounded conducting plane.

(a) Find the field $\mathbf{E}$ on the conducting plane. Express it in terms of Q, h and r, the distance of any point on the plane from the projection of the point charge onto the plane.
(b) What is the work needed to move the charge $+Q$ from its position h infront of the plane out to an infinite distance?

Problem 3 (20 points): Field and Potential of Symmetric Charge Distributions

A spherical charge distribution ρ ($\rho = \frac{dq}{dV}$ where $dV = rsin\theta d\theta r d\phi dr$) extends from $r = 0$ to $r = a$ as follows: $\rho = \rho_o(1 - \frac{r^2}{a^2})$ for $r \leq a$ and $\rho = 0$ for $r > a$.

(a) Calculate the total charge Q.
(b) Find the electric field $\mathbf{E}$ and the potential ϕ outside the charge distribution.
(c) Find the electric field $\mathbf{E}$ and the potential ϕ inside the charge distribution.

Problem 4 (20 points): Coulomb Force Between Charges

Two uniform charge distributions extend on straight lengths $AB = l_1$ and $CD = l_2$ as shown in the figure 1, i.e., lying on the same line and with their ends B and C at a distance l. Each one has constant line charge density λ_1 and λ_2 ($\lambda = \frac{dq}{dl}$).

(a) What is the Coulomb Force $\mathbf{F}$ between the two charge distributions?
(b) Show that for $l >> l_1$ and $l >> l_2$ the expression from (a) reduces to the Coulomb Force between two point charges.

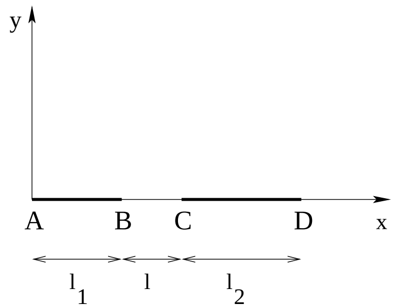

Figure 1

Problem 5 (20 points): Forces on Conductors

A conducting spherical shell of outer radius a and inner radius $\frac{3a}{4}$ is cut in two pieces via a horizontal plane at distance $\frac{a}{2}$ above the center of the spherical shell as shown in figure 2. Let us label "A" the upper part of the shell and "B" the lower part of the shell. The shell is initially uncharged and the two pieces that result from the cutting procedure remain in perfect electrical contact. A new conducting sphere of radius $\frac{a}{16}$ and total charge $+Q$ is inserted in the shell and it is centered on the shell's center as shown in the same figure.

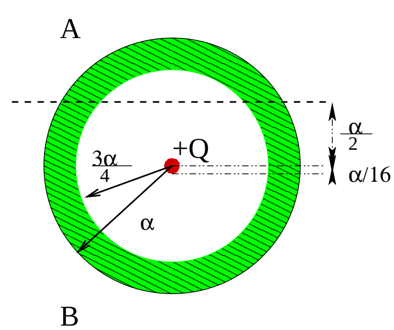

Figure 2

(a) Are there any charge densities on the inner ($r = 3a/4$) and outer ($r = a$) surfaces of the shell as well as within it? If yes, derive them.
(b) What (if any) is the force per unit area acting on the inner and outer surfaces of the shell?
(c) From now on we focus only on the "A" part of the shell: Set up an integral that will yield the net force acting on the **outside** of the "A" part of the shell. What is its direction? Identify over which variables you are integrating and what are the limits of integration. You are **not** asked to perform the integration!
(d) Do the same as (c) for the **inside** of the "A" part of the shell.
(e) Do the same as (c) for the "A" part as a **whole**.