

Massachusetts Institute of Technology
Department of Physics
Physics 8.022 - Fall 2002

Assignment #7
Spacetime in Special Relativity
Forces, Momenta, Fields, Energies in Special Relativity
Fields of Moving Charges

Reading Purcell: Chapter 5 and Appendix I (pages 451-458), handouts I and II (there is a **second** one posted!) on Special Relativity by Prof. Fisher.

Problem Set #7

Work on **all** problems. Not all problems receive equal points. Total points for this set is 100.

- **(15 points) [1]** Transforming velocities.

A bullet is fired with velocity \vec{u}' in the $x'y'$ plane of a moving frame S' and at an angle θ' with respect to x' . Frame S' moves with velocity v with respect to the laboratory frame S along the x axis. Find the angle the velocity vector of the bullet makes with the x axis of the laboratory frame. What if the bullet is a photon (light)?

- **(20 points) [2]** "Traveling circus beats the speed of light!"

A circus company advertises its arrival in Cambridge, Massachusetts with big headlines in local newspapers describing the first show ever that is **plausible** to beat the speed of light. The spectacle is described as follows. First, they set up a long enough train that travels at velocity v ($v < c$) with respect to the audience. Then, a number of small carts are added one on top of each other and on top of the train. Each cart moves at velocity v with respect to previous one and in the same direction with the previous one, i.e., the first cart rolls in the same direction with the train and with velocity v relative to the train; the second cart rolls on the first cart in the same direction with it and with a velocity v with respect to it and *so on and so forth*.

The company claims to put as many carts as it takes so that the speed (as seen by the circus audience) of the final cart on top of this pyramid to **exceed** the speed of light and calls all Cambridge citizens to come witness this unique moment.

Will they be able to ever succeed their claim? Explain.

Hint: use *induction* and you should have your answer in less than 1 line.

Optional: Find the speed of the n th cart. Notice that $\lim_{n \rightarrow \infty} \left(\frac{1-\beta}{1+\beta}\right)^n$ tends to 0 when $n \rightarrow \infty$.

Find the maximum attainable speed the Circus company can ever succeed reaching.

- **(20 points) [3]** Lorentz invariants (part-II).

We have already seen scalar quantities that are invariant under Lorentz transformations: $s^2 = (ct)^2 - x^2 - y^2 - z^2$ or $(m_0 c)^2 = \left(\frac{E}{c}\right)^2 - p_x^2 - p_y^2 - p_z^2$, for example. These are the "squares" (taken in the underlying Minkowski metric: $\text{diag}\{1, -1, -1, -1\}$ of special relativity) of the 4-vectors corresponding to position and momentum. Another celebrated scalar is the product of the 4-momentum and 4-position $\Phi = Et - \vec{p}\vec{r}$ (where \vec{p}, \vec{r} are the ordinary 3-vector momentum and position) which we will identify in the last 2 weeks of our course as proportional to the phase of a plane wave of a propagation vector $\vec{k} = \frac{\vec{p}}{\hbar}$ and angular frequency $\omega = \frac{E}{\hbar}$.

Show that Φ as defined above is a Lorentz invariant.

- **(15 points) [4]** High energy protons.

A proton in Fermilab's Tevatron accelerator is accelerated to about 1TeV. In all the following, approximate proton's rest mass energy to 1GeV.

- What is the γ and β factor of the proton?
- What is its total energy?
- What is its momentum?
- What is its relativistic mass?
- *Optional:* Check out in **real time** the status of Fermilab's Tevatron accelerator by visiting: <http://www-bd.fnal.gov/notifyervlet/www?project=outside>

- **(15 points) [5]** Relativistic collisions.

A particle of rest mass m_1 and velocity v_1 collides with a stationary particle of rest mass m_2 and is absorbed by it. What is the velocity and the rest mass of the final compound system?

- **(15 points) [6]** *Purcell* Problem 5.1 (p.200): Relativistic capacitor.

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