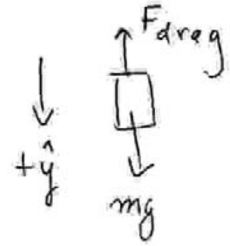
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

DEPARTMENT OF PHYSICS

8.01X FALL 2001

Practice Exam 3 Solutions

Problem 1:



$$\begin{array}{c|c} F_y & = m\,a_y \\ \hline mg - F_{drag} & = 0 \end{array}$$

$mg = F_{drag}$

$a_y = 0$ since $(v_y)_{term}$ is a constant!

a) $d = v_{term}\, t \Rightarrow t = \dfrac{d}{v_{term}} = \dfrac{1.6\ km}{5.0 \times 10^1\ m/s} = 32\ s$

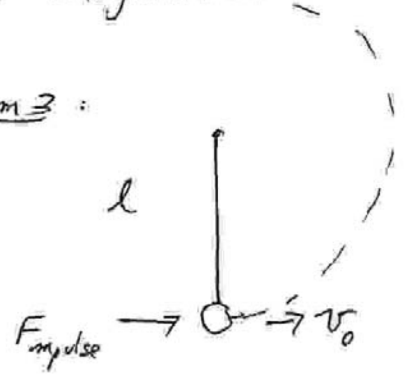
b) $W_{grav} = mgd = (8.0 \times 10^1\ kg)(9.8 \frac{m}{s^2})(1.6 \times 10^3\ m) = 1.25 \times 10^6\ J$

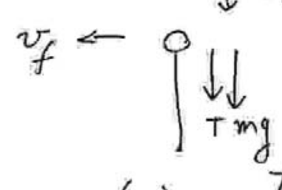
c) $W_{drag} = -F_{drag}\, d = -mgd = -1.25 \times 10^6\ J$

d) $P_{drag} = \dfrac{W_{drag}}{t} = -\dfrac{mgd}{t} = \dfrac{-1.25 \times 10^6\ J}{3.2 \times 10^1\ s} = -3.92 \times 10^4\ W$

e) $E_{drag} = W_{drag} = -mgd = -3.92 \times 10^4\ W$. This energy shows up as heat; heating both the skydiver and the air.

Problem 3:





$$\begin{array}{c|c} F_{rad} = m & a_{radial} \\ \hline (1) \quad T + mg = & \dfrac{m v_f^2}{\ell} \end{array}$$

a)

Mech. Energy is conserved since T is always perpendicular to the displacement

$$\Delta K + \Delta P.E. = W_{n.c.}$$

$$\frac{1}{2} m(v_f^2 - v_0^2) + mg(2l) = 0$$

solving for $v_f^2 = v_0^2 - 2g(2l)$

$$v_f = \left((7.0\,m/s)^2 - (2)(9.8\,\tfrac{m}{s^2})(2)(.5\,m) \right)^{1/2} = 5.4\,\frac{m}{s}$$

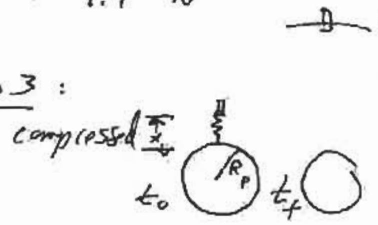
b) $T = \dfrac{m v_f^2}{l} - mg = \dfrac{m}{l}\left(v_0^2 - 2g(2l)\right) - mg$

$$T = \frac{m v_0^2}{l} - 5mg = \frac{(.1\,kg)(7.0\,\frac{m}{s})^2}{(.5\,m)} - (5)(.1\,kg)(9.8\,\tfrac{m}{s^2})$$

$$T = 4.9\,N$$

Problem 3:

the pen should have zero velocity at ∞



a) Energy is conserved

$x_0 \neq 0 \quad x_f = 0$

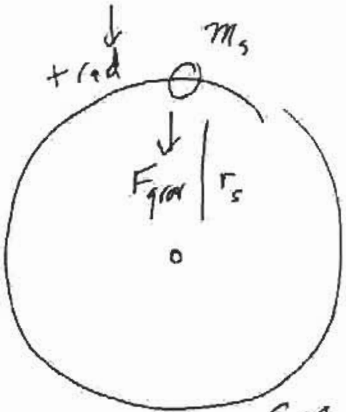
$v_0 = 0 \quad v_f = 0$

$$\Delta K + \Delta P.E. = W_{n-c}$$

$$0 + \frac{1}{2}k(x_f^2 - x_0^2) - Gm_1m_2\left(\frac{1}{\infty} - \frac{1}{R_p}\right) = 0$$

$$-\frac{1}{2}kx_0^2 + \frac{Gm_1m_2}{R_p} = 0 \quad \Rightarrow \quad x_0^2 = \frac{2Gm_1m_2}{R_p k}$$

$$x_0 = \left(\frac{(2)(6.67\times10^{-11}\,\frac{N\cdot m^2}{kg^2})(.01\,kg)(2.6\times10^{15}\,kg)}{(5.0\times10^3\,m)(400\,N/m)} \right)^{1/2} = .042\,m = 4.2\,cm$$



$$\begin{array}{c|c} F_{rad} & m_s a_{rad} \\ \hline \dfrac{G m_s m_p}{r_s^{\,2}} & m_s r_s \left(\dfrac{2\pi}{T}\right)^2 \end{array}$$

solve for

$$r_s^3 = \frac{G m_p T^2}{4\pi^2}$$

$$r_s = \left(\frac{\left(6.67\times10^{-11}\frac{N-m^2}{kg^2}\right)\left(2.6\times10^{15}kg\right)\left((2)(3.6\times10^3 sec)\right)^2}{4\pi^2} \right)^{1/3}$$

$$r_s = 6.1\times10^3\ m$$

$$v_s = \frac{2\pi r_s}{T} = \frac{(2\pi)(6.1\times10^3 m)}{(7.2\times10^3 sec)} = 5.3\ m/s$$

d) At the radius r_s, the conservation of mechanical energy can be used to calculate the velocity of the pen

$$\begin{array}{c|c} \Delta K + \Delta P.E. = & W_{nc} \\ \hline \frac{1}{2} m_2 v'^2 - \frac{1}{2} k x_0^{\,2} - G m_1 m_2\left(\frac{1}{r_s} - \frac{1}{R_p}\right) = & 0 \end{array} \quad (1)$$

t_0, $v_0 = 0$
$x_0 = 4.2\ cm$
from part a)
$r_0 = R_p$

t'
$v' \neq 0$
$x_f = 0$
$r' = r_s$
from part c)

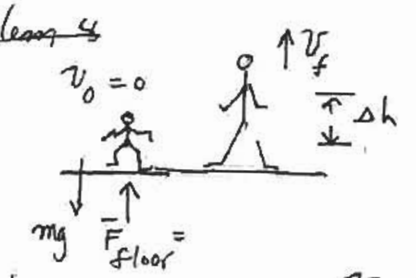
recall from part a) that $\frac{1}{2} k x_0^2 = \frac{G m_1 m_2}{R_p}$

So eq (1) becomes $\frac{1}{2} m_2 v'^2 - \frac{G m_1 m_2}{r_s} = 0$

$$v' = \left(\frac{2 G m_1}{r_s}\right)^{1/2} = \left(\frac{(2)\left(6.67\times10^{-11}\frac{N-m^2}{kg^2}\right)\left(2.6\times10^{15}kg\right)}{6.1\times10^3 m}\right)^{1/2}$$

$$v' = 7.54\ m/s$$

Problem 4



$$\Delta K + \Delta P.E. = W_{nc}$$

$$\frac{1}{2} m v_f^2 + mg(\Delta h) = \bar{F}_{floor}\, \Delta h$$

$$\bar{F}_{floor} = 3\,mg$$

solve for v_f: $\quad v_f = \left(2\left(\frac{\bar{F}_{floor} - mg}{m}\right)\Delta h\right)^{1/2} = \left((2)\frac{(2mg)}{m}\Delta h\right)^{1/2}$

$$v_f = (2)(g\Delta h)^{1/2} = (2)\left(\left(9.8\tfrac{m}{s^2}\right)(0.2\,m)\right)^{1/2} = 2.8\,\frac{m}{s}$$

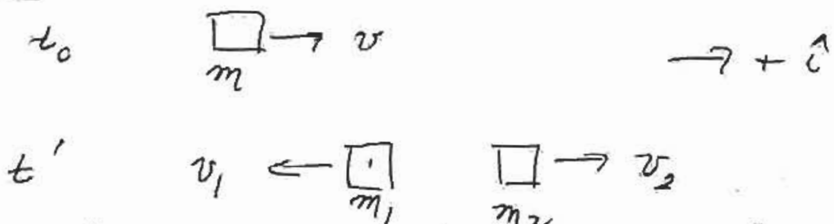
Problem 5

a) $\Delta Q = m\, C_w\, \Delta T$

$$= (.25\,kg)\left(4190\,\frac{J}{kg\text{-}K}\right)(28\,K) = 2.9 \times 10^4\,J$$

b) $P_{ave} = \dfrac{\Delta Q}{\Delta t} = \dfrac{2.9 \times 10^4\,J}{60\,s} = 4.9 \times 10^2\,W$

Problem 6 :



momentum is conserved because there are no external forces. energy is not conserved due to the explosion.

$$\Delta P_x = 0$$

$$m_2 v_2 - m_1 v_1 - m v = 0$$

$$v_2 = \frac{m v + m_1 v_1}{m_2} = \frac{(2.0\,kg)(2.0\,\frac{m}{s}) + (0.5\,kg)(1.0 \times 10^1\,\frac{m}{s})}{(1.5\,kg)}$$

$$= 6\,m/s$$

$\Delta K_f = W_{n.c}$

$\frac{1}{2} m_2 v_2^2 + \frac{1}{2} m_1 v_1^2 - \frac{1}{2} m v^2 = W_{n.c.}$

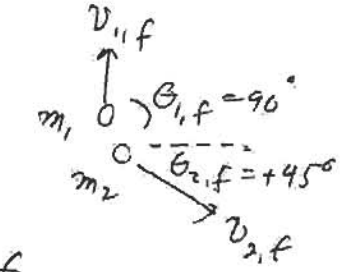
$\left(\frac{1}{2}\right)(1.5\,kg)\left(6\frac{m}{s}\right)^2 + \left(\frac{1}{2}\right)(.5\,kg)\left(10\frac{m}{s}\right)^2 - \frac{1}{2}(2.0\,kg)\left(2.0\frac{m}{s}\right)^2 = 48\,J$

is the increase in kinetic energy due to the explosion.

Problem 7:

$\longrightarrow +\hat{i}$

t_0 $\quad m_1 \; O \rightarrow v_{1,0} \quad v_{2,0} \leftarrow O \; m_2$



$$\Delta P_x = 0 \Rightarrow P_{x,0} = P_{x,f}$$

$$m_1 v_{1,0} - m_2 v_{2,0} = m_2 v_{2,f} \cos\theta_{2,f} \qquad (1)$$

$$\Delta P_y = 0 \Rightarrow P_{y,0} = P_{y,f}$$

$$0 = m_1 v_{1,f} - m_2 v_{2,f} \sin\theta_{2,f} \qquad (2)$$

$$\Delta K = 0 \Rightarrow K_0 = K_f$$

$$\frac{1}{2} m_1 v_{1,0}^2 + \frac{1}{2} m_2 v_{2,0}^2 = \frac{1}{2} m_1 v_{1,f}^2 + \frac{1}{2} m_2 v_{2,f}^2 \qquad (3)$$

Additionally we are told that $v_{1,f} = \frac{v_{1,0}}{2}$

eq (2) can be rewritten using this fact as

$$m_2 v_{2,f} \sin\theta_{2,f} = \frac{m_1 v_{1,0}}{2}$$

eq (1)

$$m_2 v_{2,f} \cos\theta_{2,f} = m_1 v_{1,0} - m_2 v_{2,0}$$

so dividing these equations yields

$$\tan\theta_{2,f} = \frac{m_1 v_{1,0}/2}{m_1 v_{1,0} - m_2 v_{2,0}}$$

since $\tan\theta_{2,f} = \tan 45° = 1$ we have

$$1 = \frac{m_1 v_{1,0}/2}{m_1 v_{1,0} - m_2 v_{2,0}} \quad \text{or} \quad m_1 v_{1,0} - m_2 v_{2,0} = \frac{1}{2} m_1 v_{1,0}$$

which we can solve for $v_{2,0}$

$$\boxed{v_{2,0} = \frac{1}{2} \frac{m_1}{m_2} v_{1,0}}$$

eq (2) can also be solved for $v_{2,f}$

$$\boxed{v_{2,f} = \frac{m_1}{m_2} \frac{v_{1,0}}{2 \sin\theta_{2,f}} = \frac{m_1}{m_2} \frac{v_{1,0}}{2 \frac{\sqrt{2}}{2}} = \frac{m_1}{m_2} \frac{v_{1,0}}{\sqrt{2}}}$$

So eq (3), can be rewritten as

$$\frac{1}{2} m_1 v_{1,0}^2 + \frac{1}{2} m_2 \left(\frac{1}{2} \frac{m_1}{m_2} v_{1,0}\right)^2 = \frac{1}{2} m_1 \left(\frac{v_{1,0}}{2}\right)^2 + \frac{1}{2} m_2 \left(\frac{m_1}{m_2} \frac{v_{1,0}}{\sqrt{2}}\right)^2$$

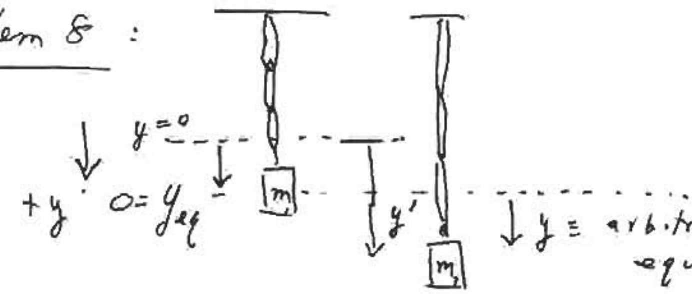
or

$$\frac{1}{2} m_1 v_{1,0}^2 + \frac{1}{2} m_2 \frac{1}{4} \frac{m_1^2}{m_2^2} v_{1,0}^2 = \frac{1}{2} m_1 \frac{v_{1,0}^2}{4} + \frac{1}{2} m_2 \frac{m_1^2}{m_2^2} \frac{v_{1,0}^2}{2}$$

$$\frac{3}{4} m_1 v_{1,0}^2 = \frac{1}{4} \frac{m_1^2 v_{1,0}^2}{m_2}$$

$$\Rightarrow \quad \boxed{3 = \frac{m_1}{m_2}}$$

Problem 8 :



$\downarrow y \equiv$ arbitrary stretch from equilibrium positon

The equilibrium positon is already a slightly stretched positon since at equilibrium

F_y	$m a_y$
$m_1 g - k y_{eq}$	0

$\Rightarrow y_{eq} = \dfrac{m_1 g}{k}$

Then when the system is stretched an additional distance y_0 at $t=0$

$F_y =$	$m_1 a_y$
$m_1 g - k(y + y_{eq})$	$= m_1 \dfrac{d^2y}{dt^2}$

here y is an arbitrary stretch from eq. pos we get

$$\underbrace{m_1 g - k y_{eq}}_{= 0} \quad - k y = m_1 \frac{d^2 y}{dt^2}$$

$$m_1 \frac{d^2 y}{dt^2} + k y = 0$$

simple harmonic motion about y_{eq} positon

$$y = A \cos\sqrt{\frac{k}{m_1}}\, t + B \sin\sqrt{\frac{k}{m_1}}\, t$$

$$A = y_0 \ , \quad B = \frac{v_0}{\sqrt{k/m_1}} = 0$$

released from rest

$$v = \frac{dy}{dt} = -\sqrt{\frac{k}{m_1}}\, A \sin\sqrt{\frac{k}{m_1}}\, t + \sqrt{\frac{k}{m_1}}\, B \cos\sqrt{\frac{k}{m_1}}\, t$$

period T : $\sqrt{\dfrac{k}{m_1}}\, T = 2\pi \Rightarrow \boxed{T = 2\pi\sqrt{\dfrac{m_1}{k}}}$

we can find the velocity using

$$y = y_0 \cos\sqrt{\frac{k}{m_1}}\, t,$$

$$v_y = -\sqrt{\frac{k}{m_1}}\, y_0 \sin\sqrt{\frac{k}{m_1}}\, t$$

noting that when $\sqrt{\frac{k}{m_1}}\, t = \pi/2$, $\cos\left(\sqrt{\frac{k}{m_1}}\, t\right) = 0$

so $y = 0$, mass is back at eq. pos.

Also, $\sin\left(\frac{\pi}{2}\right) = 1$ so

$v_y = -\sqrt{\frac{k}{m_1}}\, y_0$ at eq. pos. $t_{eq} = \frac{\pi}{2}\sqrt{\frac{m_1}{k}}$

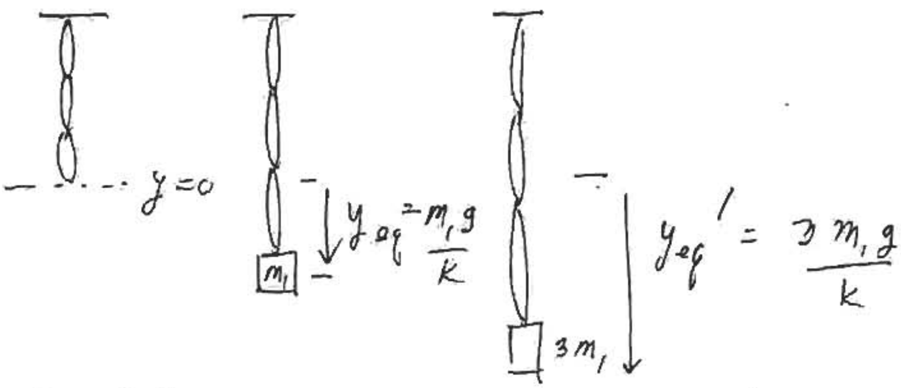
c) since a new mass $m_2 = 2m_1$

$m_{total} = 3m_1$ and $T = 2\pi\sqrt{\frac{3m_1}{k}}$

d) since the collision occured when the mass was completely compressed, the velocity was zero, hence for the collision $\Delta K = 0$, no energy was lost. Therefore, the new system of mass $3m_1$ will satisfy a new equilibrium condition

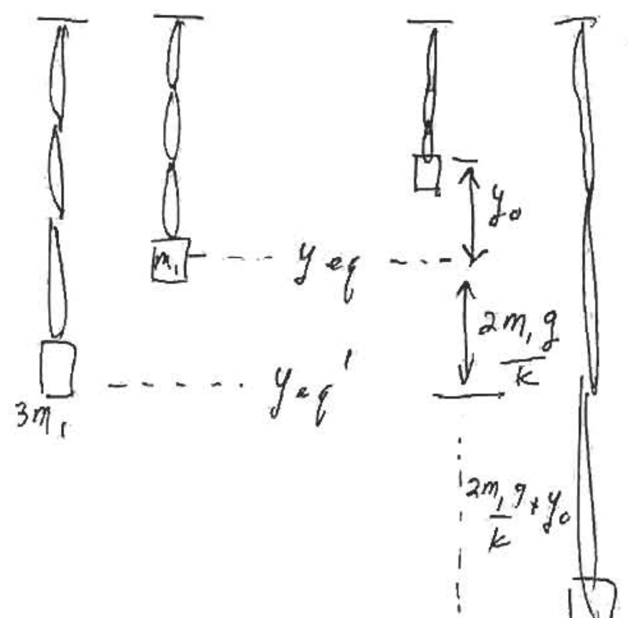
$3m_1 g = k y_{eq}' \Rightarrow y_{eq}' = 3m_1 g/k$

and the oscillations are about this position,



So the new equilibrium position is lowered by $\frac{2 m_1 g}{k}$

When the rubber bands were fully compressed by y_0, the collision occured. The



So with respect to the new equilibrium position the stretch is

$$y_0 + \frac{2 m_1 g}{k}$$

Hence when the system is fully stretched, the mass $3m_1$ is at a position

$$y_0 + \frac{4 m_1 g}{k}$$ from the original equilibrium position