MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01X Fall Term 2002

SOLUTIONS TO PROBLEM SET 4

1 Young & Friedman 5-26

A box of bananas weighing 40.0 N rests on a horizontal surface. The coefficient of static friction between the box and the surface is 0.40, and the coefficient of kinetic friction is 0.20.

(a) If no horizontal force is applied to the box and the box is at rest, how large is the friction force exerted on the box?

Static friction is the relevant type for an object at rest, and would oppose a horizontal force. With no applied force in the horizontal there's no friction force either.

(b) What is the magnitude of the friction force if a monkey applies a horizontal force of 6.0 N to the box and the box is initially at rest?

The maximum possible static friction force this box-surface interface could produce is $f_{\mathrm{S,max}} = \mu_{\mathrm{S}} N = (0.40)\,(40.0) = 16\,\mathrm{N}$. The applied 6.0 N is below that maximum, so static friction will negate it with an opposing force of 6.0 N.

(c) What minimum horizontal force must the monkey apply to start the box in motion?

The monkey would need to apply more than 16 N, as calculated in part (b).

(d) What minimum horizontal force must the monkey apply to keep the box moving at a constant velocity once it has been started?

In order that it move at a constant velocity, the net force on the box should be zero. The monkey will need to balance the friction force by carefully applying a force of equal magnitude and opposite direction. A moving box is in the realm of kinetic friction, so we use the kinetic friction coefficient to calculate the necessary applied force: $\left|F_{\mathrm{applied}}\right| = |f_{\mathrm{K}}| = \mu_{\mathrm{K}} N = (0.20)\,(40.0) = 8.0\,\mathrm{N}$.

(e) If the monkey applies a horizontal force of 18.0 N, what is the magnitude of the friction force?

18.0 N is more than either the static or kinetic friction are capable of opposing, so in this case the box will always experience a net horizontal force and hence accelerate. At any time when the box's velocity is nonzero, the friction force will be 8.0 N as in part (d).

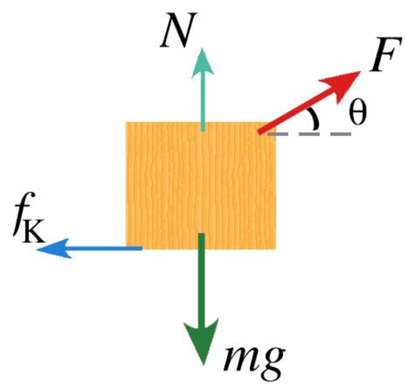
One could argue that even an accelerating object can have zero velocity for an instant, and in such an instant the maximum static friction force, 16 N, would exist instead. However it's unlikely that our empirical formulae for friction forces are so reliable as to pertain to such an extreme case.

2 Young & Friedman 5-38

A box with mass m is dragged across a level floor having a coefficient of kinetic friction μ_K by a rope that is pulled upward with an angle θ above the horizontal with a force of magnitude F.

(a) In terms of m, μ_K, θ, and g, obtain an expression for the magnitude of force required to move the box with constant speed.

A free-body diagram will be invaluable in determining the necessary force.



There's no acceleration in the vertical direction, so the sum of the vertical forces must be zero, i.e. $N - mg + F\sin\theta = 0$. This gives us N, which we don't know the value of, in terms of F, which we also don't know the value of (yet).

$$N = mg - F\sin\theta$$

The problem asks how to produce a constant horizontal velocity, which requires the sum of the forces in the horizontal to also be zero. $F\cos\theta - f_K = 0$. Since f_K can now be written in terms of F, we can solve for F.

$$\begin{aligned}
F\cos\theta &= f_K \\
&= \mu_K N \\
&= \mu_K (mg - F\sin\theta) \\
F(\cos\theta + \mu_K \sin\theta) &= \mu_K mg \\
F &= \frac{\mu_K mg}{(\cos\theta + \mu_K \sin\theta)}
\end{aligned}$$

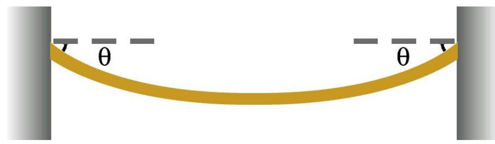
(b) Knowing that you are studying physics, a CPR instructor asks you how much force it would take to slide a 90-kg patient across a floor at constant speed by pulling on him at an angle of 25° above the horizontal. By dragging some weights wrapped in an old pair of pants down the hall with a spring balance, you find that $\mu_K = 0.35$. Use the result of part (a) to answer the instructor's question.

Despite the compelling plot about CPR and old pants, this question is simply asking you to plug $m = 90\,\text{kg}$, $\theta = 25^\circ$, $\mu_K = 0.35$, and $g = 9.81\,\frac{\text{m}}{\text{s}^2}$ into the last equation of part (a). Doing so yields $F = 293\,\text{N}$.

As a sanity check (to confirm we got a sensible number for F), note that $293\,\text{N}$ is the vertical force required to lift a 30-kg person up off the ground. Comparing to the 90-kg person in the problem, this agrees with our everyday intuition that even with friction present it's easier to drag a heavy object across the floor than to carry it.

3 Young & Friedman 5-62: A Rope with Mass.

In most problems in this book, the ropes, cords, or cables have so little mass compared to other objects in the problem that their masses can safely be neglected. But if the rope is the *only* object in the problem, then clearly its mass cannot be neglected. For example, suppose we have a clothesline attached to two poles. The clothesline has a mass M, and each end makes an angle θ with the horizontal.

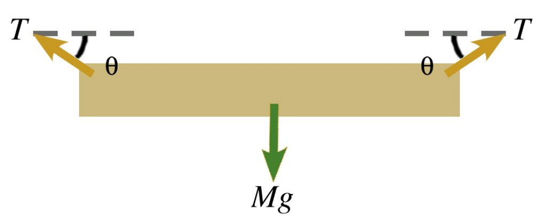


The curve of the clothesline, or of any flexible cable hanging under its own weight, is called a *catenary*.

(a) What is the tension at the ends of the clothesline?

To find this we need to treat the rope as if it were made of three separate pieces: two very short (infinitesimal) ropes of mass $\ll M$ and one large object of mass M.

Now we can draw a free-body diagram for the large object.



A symmetry argument can be used to demonstrate that both short ropes must be under the same tension T. That is to say the physical situation is identical if one looks instead at the left-right mirror image of the problem, so there's no way the two ends' tensions can be unequal.

From here we note that there is no vertical acceleration, which means the sum of the vertical forces must be zero.

$$\begin{aligned} T\sin\theta + T\sin\theta - Mg &= 0 \\ T &= \frac{Mg}{2\sin\theta} \end{aligned}$$

(b) What is the tension at the lowest point?

At its lowest point, the center point, the rope is horizontal. Only horizontal force can contribute to the tension there. Also, since no part of the rope is accelerating horizontally, any segment of the rope feels a force to left and an identical force to the right. In other words, the horizontal force is constant throughout the rope. From part (a) we know at each end the horizontal force on the rope is $T\cos\theta = \frac{Mg}{2}\cot\theta$, so that's also the value of the tension at the center.

(c) Why can't we have $\theta = 0$?

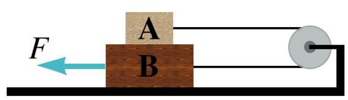
This can be approached from two equally valid perspectives.

We can argue that because force due to tension exists only in the direction of the rope's length, if θ were 0 the rope would be horizontal and tension could in no way counteract the force of gravity. The rope would inevitably accelerate downward.

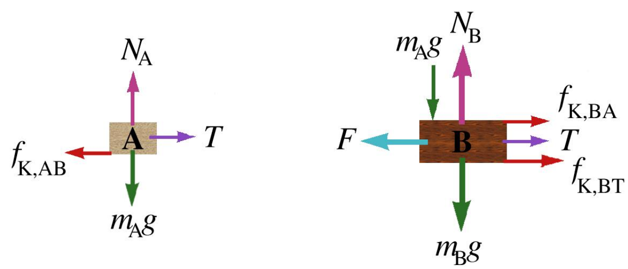
We can also show that the specific results of parts (a) and (b) become unphysical in the limit that $\theta \to 0$. Since $\sin 0 = 0$, T would be infinite, which not even the best-crafted rope could withstand.

4 Young & Friedman 5-77

Block A weighs 1.40 N, and block B weighs 4.20 N. The coefficient of kinetic friction between all surfaces is 0.30. Find the magnitude of the horizontal force $\vec{F}$ necessary to drag block B to the left at a constant speed if A and B are connected by a light, flexible cord passing around a fixed, frictionless pulley.



Free-body diagrams are profoundly helpful in visualizing the many forces at play here.



If block B is to move with no acceleration, then due to the cord block A must undergo no acceleration. Thus the total force on each block will be zero. Let's write down the equation for the horizontal motion of block B and see what we need to figure out.

$$\begin{aligned} F - T - f_{K,BT} - f_{K,BA} &= 0 \\ F &= T + f_{K,BT} + f_{K,BA} \end{aligned}$$

Finding $f_{\mathrm{K,BA}}$, the magnitude of the friction force that A exerts on B, will be the simplest, since it has the same magnitude as $f_{\mathrm{K,AB}}$ by virtue of the two being Third-Law spouses. The free-body diagram for block A looks like a simple friction problem we've encountered several times, so we can easily calculate the kinetic friction on A due to its contact with B.

$$f_{\mathrm{K,BA}} = f_{\mathrm{K,AB}} = \mu_{\mathrm{K}} m_{\mathrm{A}} g$$

We've already deduced that block A feels no net force, so the cord's tension must balance the friction force on A.

$$T = f_{\mathrm{K,AB}} = \mu_{\mathrm{K}} m_{\mathrm{A}} g$$

That leaves us with the friction between the table and block B. This is a little tricky because the weight of A is an externally applied force on B, so B transmits this force down on the table along with its own weight. So in response, the normal force on B is $(m_A + m_B)\,g$. This makes the friction force between the table and block B

$$f_{\mathrm{K,BT}} = \mu_{\mathrm{K}} (m_A + m_B)\, g$$

Now we can solve for F.

$$\begin{aligned}
F &= \mu_{\mathrm{K}} m_{\mathrm{A}} g + \mu_{\mathrm{K}} (m_A + m_B)\, g + \mu_{\mathrm{K}} m_{\mathrm{A}} g \\
&= \mu_{\mathrm{K}} (3m_A + m_B)\, g \\
&= (0.30)(3)(1.40) + (0.3)(4.20) \\
&= 2.52\,\mathrm{N}
\end{aligned}$$