Now we'd like to discuss angular acceleration for circular motion.

So suppose we have our angle theta, radius r, and r hat and theta hat.

Recall that we described the angular velocity as the derivative of d theta dt, and we made this perpendicular to our right-handed coordinate system, direction k hat.

Now let's differentiate that to get our concept of angular acceleration.

So alpha is the second derivative d theta dt squared k hat.

And this quantity is what we call angular acceleration.

Now we'll describe the component alpha z as d squared theta dt squared.

So it's the second derivative of the angle.

And also if we wrote this as omega z k hat, we can write that as the derivative of d omega z dt, as well.

So this is the component.

And now in circular motion, the quantities of omega and alpha z are very much like the linear quantities of the x component of the velocity and the x component of the acceleration.

And again, when we've chosen a reference frame, let's look at what various components mean.

Let's begin with the case 1 where omega z is positive.

So when omega z is positive, that tells us that the angle d theta dt is increasing.

And that corresponds to counterclockwise motion.

Now given that case, let's look at what happens when alpha z is positive.

Remember, that's the statement that d omega dt is positive, that omega z is increasing.

So if an object is moving with a positive component of omega z and the angular acceleration component is positive, that corresponds to increasing.

The linear example, if you had one dimensional motion, i hat, you had vx positive and a x positive, corresponds to an object increasing in its speed in the x direction.

That's our first case.

Now let's look at the second example when alpha z is less than 0.

So now the derivative of d omega z dt is negative.

What that corresponds to-- remember, omega z is the z component of the angular speed.

And if that's slowing down, then, with alpha z less than 0, the object is slowing down.

So in our linear case, if we had a x less than 0, this is the classic example of breaking.

The object is moving in the x direction and slowing down.

Now let's look at case 2.

This is always a little bit complicated for circular motion where omega z is less than 0.

In that case, the object is moving in the clockwise direction because the angle theta is decreasing, corresponding to clockwise motion.

So in that case, once again, let's consider the two examples.

Well, the first example is a positive component of angular acceleration.

Now this is the one that can be a little bit confusing.

The object is moving clockwise but it has a positive alpha z, which will correspond to slowing the object down.

And if the alpha z remains positive, it will actually come to rest and then reverse its motion and start to speed up.

So this is the case where d omega dz is increasing.

And that's our first case.

So something like that could correspond to, if we plotted omega z and we had an object that starts off with a negative omega z and increases.

Notice that the slope here, which is alpha z positive, corresponds to a positive angular acceleration component.

And the object slows down as omega gets closer to 0, stops, and now has a positive omega z, corresponding to motion in a counterclockwise direction.

For our linear case, this corresponds to, again, with i hat, our object moving to the left, vx negative, and if a x is positive, it breaks in this direction, which means it's slowing down.

And then eventually if alpha x, ax, stays positive, it continues in that direction.

Now our final case, and I'll put it down here, b, this is again where omega z negative and alpha z negative.

It's always helpful to see this immediately with the graph.

Omega z is negative.

Here, alpha z, which is the slope, is also negative.

This corresponds to an object moving in the clockwise direction.

And actually its speed is increasing because alpha z is negative.

So it's going faster and faster in the clockwise direction, even though alpha z is negative.

And for our linear case, again, this corresponds to an object moving in the negative x direction.

And a x is negative, it's moving faster in the negative x direction.

And so these are the cases of how we analyze the various cases for angular acceleration and angular velocity.