

Let's determine the moment of inertia of a big wheel.

Here we have a big wheel and on the side it's attached here at the center .

And there's a string going around here, and on that string is a little mass hanging.

And our disk here has a radius  $R$ .

And we now want to in a little experiment what the moment of inertia is of this disk.

And what we have to do is we have to drop this mass to the ground, and we need to measure what height is here, and we need to measure how long it takes to drop.

So we need to measure  $t$ .

So how are we going to go about that?

Well the moment of inertia, that probably has something to do with the torque of this wheel.

So we have to start doing a torque analysis.

So torque about the center of mass equals moment of inertia about the center of mass times the angular acceleration.

And we need to recall that the torque is the product of the radius times the perpendicular force, so we're going to have  $R$  here, and then the force that's acting on this wheel here is actually this tension force here.

So we are going to have our  $RT$  here.

And now we need to look at how things actually moving with respect to coordinate system.

So if I let this mass drop, to the disk it's going to spin clockwise.

So we're going to have  $\theta$  hat going this way.

And if  $\theta$  hat is going clockwise, my  $\mathbf{k}$  hat vector is going to go into the board.

And if  $\mathbf{k}$  hat is going to go there, and it's rotating clockwise, the angular acceleration is going to go in the  $\mathbf{k}$  hat direction.

And if that is the case, then the torque is going to follow suit.

So torque also goes into the board.

And so that means here we're going to deal with  $\hat{k}$  direction.

And we're going to have  $l\alpha$  and then  $\alpha Z$  in the  $\hat{k}$  direction.

OK, so we can solve this  $lRT$  over  $\alpha Z$ . OK, well that's pretty good, but we have two unknowns-- the  $T$  and the  $\alpha Z$ . And we can use some other concepts to actually get information on those.

The  $T$ , as you can guess already, plays a role here in this massless string, so we can do a quick free body diagram, an  $F = ma$  analysis to get to that tension force.

So we have a little mass,  $m$ , here gravity is acting on it.

And we have this tension force here.

And we're going to put  $\hat{j}$  down.

So we're going to get  $mg - t = ma$ .

It's only going to go in  $y$  direction, so we can just leave it here.

And we can solve this for  $t$ .

And then we have  $mg - a$ .

OK, good, so we have that.

Now about the angular acceleration.

And whenever there was a string going around a disk then, we of course, have a constrained condition, because the linear acceleration of this little mass going down is related to the radius of the disk, times the angular acceleration.

So we can solve this for  $\alpha$ .

And then we have  $a$  over  $R$ . So let's put that in here.

$R$ , and then we have  $mg - a$  over  $a$ , and then we get another  $R$  here.

And we can write that a little bit more compact,  $mR^2 g$  over  $a$  minus 1.

Good, so now we have one last hurdle, namely that a here.

That a we can't measure.

I said in the beginning, we want to make an experiment.

Actually we need experiment, because we can't otherwise get to this a, so what we need is a relation that connects what we can measure, which is the time.

It falls down the height here to the acceleration of this block.

And of course, that comes from one dimensional kinematics.

And we know that  $h$  equals  $\frac{1}{2} a t^2$ , so we can solve for  $a$ .

$2h$  over  $t$  squared.

And now we can stick that in here.

And we have  $mR$  squared,  $g t$  squared over  $2h$  minus 1.

And let's just write this here again.

And that is our final solution.

So now we have only measurable quantities here.

The  $t$  we can measure.

We just need a stopwatch.

And the  $h$  we can measure, as well.

And this actually already resembles, if you know the theoretical, this already resembles the theoretical solution, which of course is for a disk.

$\frac{1}{2} mR$  squared, so that's what one expects for a disk.

And you see that we're very close, so this term here is probably something like 1.5, or should better come out to be 1.5, because if we subtract 1, we get to that  $\frac{1}{2}$  here.

And you can use it to in return-- in return, you can also use it to predict the time it takes to fall down if you know

what the height is or vice versa.