10.3 External and Internal Forces and the Change in Momentum of a System

So far we have restricted ourselves to considering how the momentum of an object changes under the action of a force. For example, if we analyze in detail the forces acting on the cart rolling down the inclined plane (Figure 10.4), we determine that there are three forces acting on the cart: the force $\vec{\mathbf{F}}_{\text{spring, cart}}$ the spring applies to the cart; the gravitational interaction $\vec{\mathbf{F}}_{\text{earth, cart}}$ between the cart and the earth; and the contact force $\vec{\mathbf{F}}_{\text{plane, cart}}$ between the inclined plane and the cart. If we define the cart as our *system*, then everything else acts as the *surroundings*. We illustrate this division of system and surroundings in Figure 10.4.

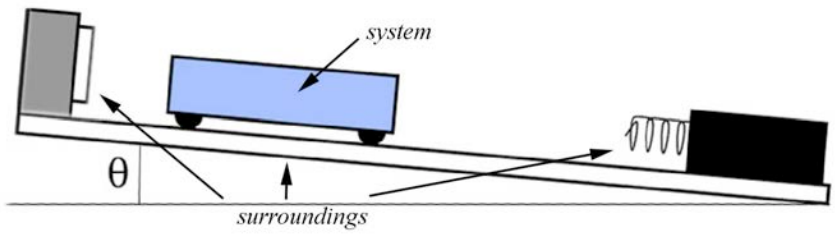

Figure 10.4 A diagram of a cart as a system and its surroundings

The forces acting on the cart are *external* forces. We refer to the vector sum of these external forces that are applied to the system (the cart) as the external force,

$$\vec{\mathbf{F}}^{\text{ext}} = \vec{\mathbf{F}}_{\text{spring, cart}} + \vec{\mathbf{F}}_{\text{earth, cart}} + \vec{\mathbf{F}}_{\text{plane, cart}} . \tag{10.3.1}$$

Then Newton's Second Law applied to the cart, in terms of impulse, is

$$\Delta\vec{\mathbf{p}}_{\text{sys}} = \int_{t_0}^{t_f} \vec{\mathbf{F}}^{\text{ext}}\, dt \equiv \vec{\mathbf{I}}_{\text{sys}} . \tag{10.3.2}$$

Let's extend our system to two interacting objects, for example the cart and the spring. The forces between the spring and cart are now *internal* forces. Both objects, the cart and the spring, experience these internal forces, which by Newton's Third Law are equal in magnitude and applied in opposite directions. So when we sum up the internal forces for the whole system, they cancel. Thus the sum of all the internal forces is always zero,

$$\vec{\mathbf{F}}^{\text{int}} = \vec{\mathbf{0}}. \tag{10.3.3}$$

External forces are still acting on our system; the gravitational force, the contact force between the inclined plane and the cart, and also a new external force, the force between the spring and the force sensor. The force acting on the system is the sum of the internal and the external forces. However, as we have shown, the internal forces cancel, so we have that

$$\vec{\mathbf{F}} = \vec{\mathbf{F}}^{\text{ext}} + \vec{\mathbf{F}}^{\text{int}} = \vec{\mathbf{F}}^{\text{ext}}. \tag{10.3.4}$$

8.01 Classical Mechanics
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