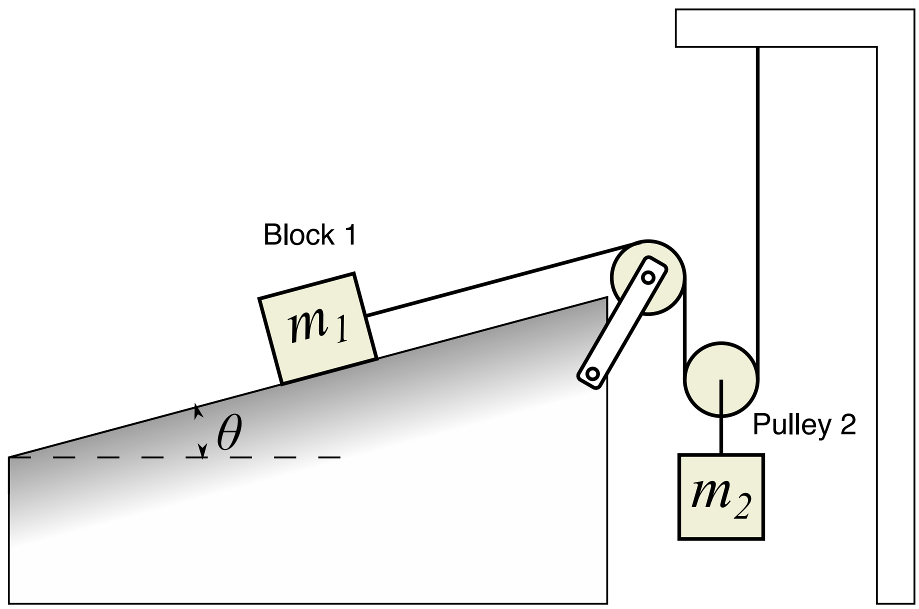
Problem Set 4

1. **Block and Pulley System**



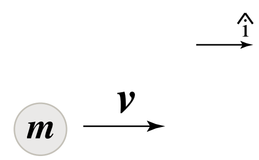
A block of mass m_1 is at rest on an inclined plane that makes an angle θ with the horizontal. The coefficient of static friction between the block and the incline surface is μ_s. A massless, inextensible string is attached to one end of the block, passes over a fixed pulley, pulley 1, around a second freely suspended pulley, pulley 2, and is finally attached to a fixed support. The left hand part of the string is parallel to the surface of the inclined plane. The sections of the string coming off the suspended pulley are vertical. The pulleys are massless, but a second block of mass m_2 is hung from the suspended pulley. Gravity acts downward.

(a) What is $m_{2,min}$, the minimum value of m_2 for which block 1 just barely slides **up** the incline? Express your answers in terms of some or all of the variables m_1, θ, μ_s and g.

(b) What is $m_{2,max}$, the maximum value of m_2 for which block 1 just barely slides **down** the incline? Express your answers in terms of some or all of the variables m_1, θ, μ_s and g.

(c) Now assume that the block on the incline plane is sliding upward. The coefficient of kinetic friction between the block and the incline surface is μ_k. Find the magnitude of the acceleration of the block on the inclined plane, a_{x1}. Express your answers in terms of some or all of the variables m_1, m_2, θ, μ_k and the acceleration of gravity g.

2. **Velocity Dependent Force**

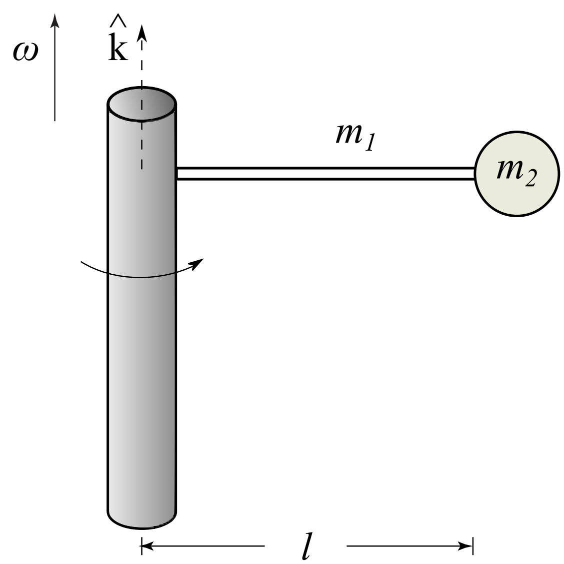


A particle of mass m moving parallel to the x-axis is acted on by a velocity dependent force directed against its motion. The force is given by:

$$\vec{F} = -be^{cv}\,\hat{i}$$

where b is a positive constant (units N), c ia also a positive constant (units s m^{-1}), and v is the speed, the magnitude of the particle's velocity (units m s^{-1}). If at $t = 0$, the particle is moving with speed v_0, find the speed $v(t)$ as a function of time t. Express your answer in terms of some or all of the following: b, m, t, c, and v_0.

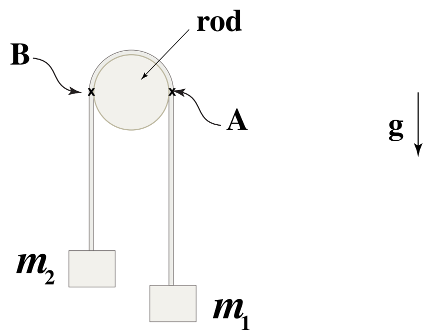
3. **Tension in Massive Rotating Rope with Object**



One end of a uniform rope of mass m_1 and length l is attached to a shaft that is rotating at constant angular velocity of magnitude ω. The other end is attached to a point-like object of mass m_2. Find $T(r)$, the tension in the rope as a function of r, the distance from the shaft.

Assume that the radius of the shaft is much smaller than l. You may also ignore the effect of gravitation - assume the shaft is rotating fast enough so that the mass and the rope are almost horizontal. Express your answer in terms of some or all of the following variables: m_1, m_2, ω, r and l.

4. **Tension in Rope Wrapped Around a Rod**



Two unequal blocks of masses m_1 and m_2, $m_1 > m_2$, are suspended by a rope over a fixed rod. The axis of the rod is perpendicular to the figure (only its cross section is shown). The coefficient of static friction between the rope and the rod is μ_s, and the coefficient of sliding friction is μ_k, $\mu_k < \mu_s$. The mass of the rope can be ignored.

For this problem , you can use the result from Chapter 8, Example 8.11 in the course textbook, titled "The Capstan", where it is shown that when the rope is about to slide the tension at point B in the rope, T_B, is related to the tension at point A in the rope, T_A, by:

$T_B = T_A e^{-\mu_s \theta}$

where θ is the angle subtended by the portion of the rope in contact with the rod. In this problem, the angle θ is $\theta = \pi$. (Note: points A and B are the points where the rope loses contact with the surface of the rod and we assume the cross section of the rod to be a perfect circle).

(a) Let T_1 be the magnitude of the force of tension exerted by the rope on block 1. Is T_A greater, less than, or qual to T_1?

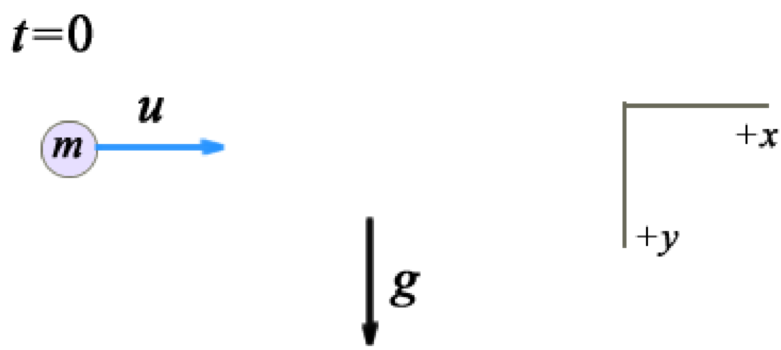
(b) What is the value of m_1 for which the rope starts sliding? Express your answer in terms of μ_s and m_2.

(c) Now assume that m_1 is large enough so that the rope starts to slip and the masses start to move. What is a, the magnitude of the acceleration of the masses after sliding has begun?

(Hint: Just when the masses start moving, the relationship between T_A and T_B becomes $T_B = T_A e^{-\mu_k \theta}$, where μ_s is replaced by μ_k. You can show this by following similar logic used in solving example 8.11 in the textbook.)

Express your answer in terms of some or all of the following: μ_k, m_1, m_2, and g.

5. **Drag Force at Low Speeds**



At low speeds (especially in liquids rather than gases), the drag force is proportional to the velocity, i.e., $\vec{F} = -C\,\vec{v}$, where C is a constant. At time $t = 0$, a small ball of mass m is projected into a liquid so that it initially has a horizontal velocity of magnitude u in the $+x$ direction as shown. (The vertical component of the velocity is zero). The gravitational acceleration is g. Consider the cartesian coordinate system shown in the figure (+x to the right and +y downwards).

(a) What is the component of the acceleration in the x direction for $t > 0$? Express your answer in terms of v_x (the component of the velocity in the x direction), C, g, m and u as needed.

(b) What is the component of the acceleration in the y direction for $t > 0$? Express your answer in terms of v_y (the component of the velocity in the y direction), C, g, m and u as needed.

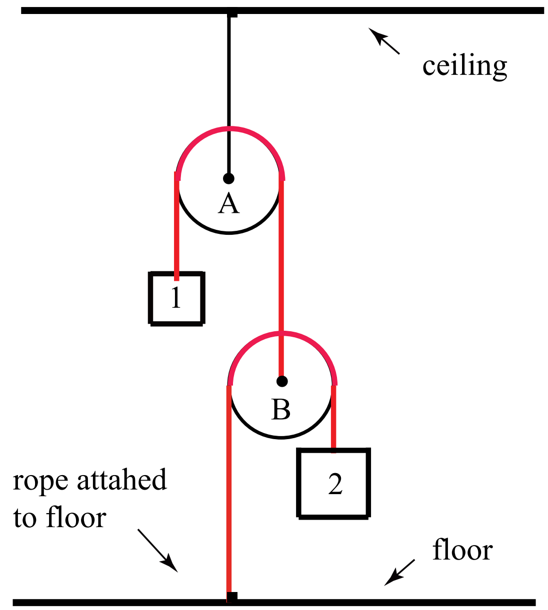
(c) Using your result from part (a), find $v_x(t)$, the horizontal component of the ball's velocity as a function of time t. Express your answer in terms of C, g, m, u and t as needed.

(d) Using your result from part (b), find $v_y(t)$, the vertical component of the ball's velocity as a function of time t. Express your answer in terms of C, g, m, u and t as needed.

(e) What is v_{xT}, the value of the horizontal component of the ball's terminal velocity? Express your answer in terms of C, g, m and u as needed.

(f) What is v_{yT}, the value of the vertical component of the ball's terminal velocity? Express your answer in terms of C, g, m and u as needed.

6. **Two Pulleys, Two Strings and Two Blocks**



Block 1 and block 2, with masses m_1 and m_2, are connected by a system of massless, inextensible ropes and massless pulleys as shown above.

Solve for the acceleration of block 2 in terms of m_1, m_2 and g. Assume that "down" is positive. Express your answer in terms of some or all of the following: g, m_1, and m_2.

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