8.012 Physics I: Classical Mechanics
Fall 2008

Physics 8.012   Fall 2007

> **Midterm Exam 1**
> **Thursday, October 4, 2007**

NAME: _____

Instructions:
1. Do all **FIVE (5)** problems. You have **90 minutes**.
2. **SHOW ALL WORK**. Be sure to **circle your final answer**.
3. Read the questions carefully.
4. All work must be done in this booklet. Extra blank pages are provided.
5. NO books, notes, calculators or computers are permitted. **A sheet of useful equations is provided on the last page**.

Your Scores

Problem	Maximum	Score	Grader
1	10		
2	20		
3	25		
4	20		
5	25		
Total	100		

Problem 1: Quick Multiple Choice Questions [10 pts]

For each of the following questions circle the correct answer. You do not need to show any work.

(a) A block with mass M and contact area A slides down an inclined plane with friction, covering a distance L in time T. How much time does it take another block with the same mass and composition, but twice the surface area, to slide down the same length?

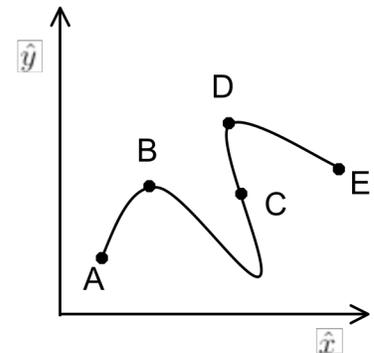0.5T   T   2T T^2

(b) A particle moves with constant speed from point A to point E on the path shown to the right. Among the points B, C and D, circle the point where the magnitude of acceleration is greatest.

(c) A tire rolls on a flat surface with constant angular velocity Ω and velocity $\vec{V}$ as shown in the diagram to the right. If $V = \Omega R$, then in which direction does friction from the road act on the tire?
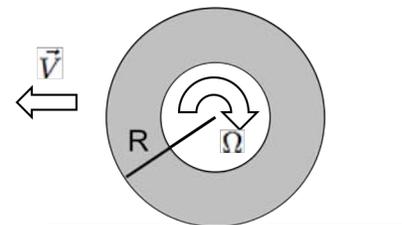
Toward the left   Toward the right   Friction does not act on the tire

(d) You pull a brick of mass M sitting on a flat table using a thick rope of mass m (see picture to right). Friction between the surface of the table and the brick is sufficient to prevent the brick from moving. Compared to the force you pull one end of the rope, the force that the other end of the rope exerts on the brick is:
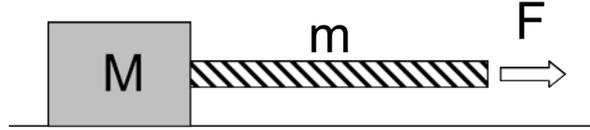
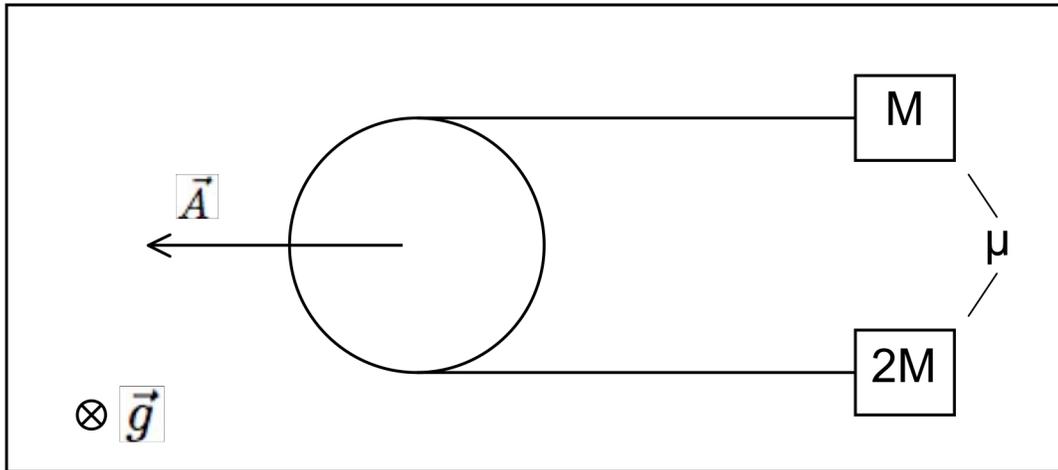   Less   Greater   The same   Zero

(e) As a swinging pendulum passes through its equilibrium point, in which direction does the total net force act?

   Radially outward   Radially inward   There is zero net force

Problem 2: The Accelerated Atwood Machine [20 pts]

An idealized Atwood machine (massless pulley and string) connected to two blocks of masses M and 2M sits initially at rest on a flat horizontal table. The coefficient of static and kinetic friction (assumed equal) between the block and table surfaces is μ. The pulley is accelerated to the left with magnitude of acceleration A. Assume that gravity acts with constant acceleration g down through the plane of the table.

(a) [15 pts] Find the distances each of the two blocks travel from their initial resting points as a function of time.

(b) [5 pts] What is the maximum acceleration A for which the block of mass 2M will remain stationary? Is there any case for $A > 0$ in which this block moves to the right?

Problem 3: Oscillation through a Massive Disk [25 pts]
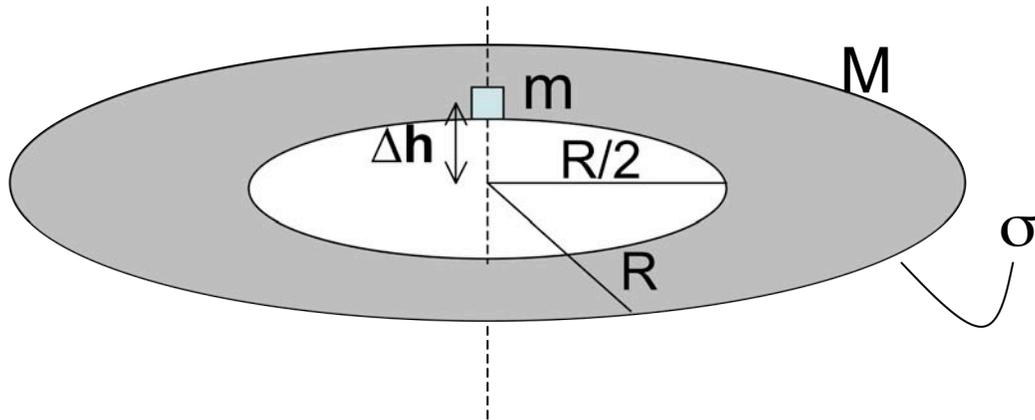
A small block of mass m lies above a thin disk of total mass M, constant surface density σ, and radius R. A hole of radius R/2 is cut into the center of the disk, allowing the mass to pass through. The mass starts at a position along the central axis of the disk but is displaced a height Δh from the plane of the disk, where Δh << R. There are no other external forces at work on this system; i.e., the disk and block are far out in deep space. Assume m << M.

(a) [15 pts] Show that to first order in Δh the block undergoes simple harmonic motion with respect to the center plane of the disk, and compute the period of oscillation of the block.

(b) [10 pts] What is the period of oscillation of the disk, and what is the ratio of the amplitude of the disk's oscillation to that of the block?

Problem 4: Rotating Spring [20 pts]
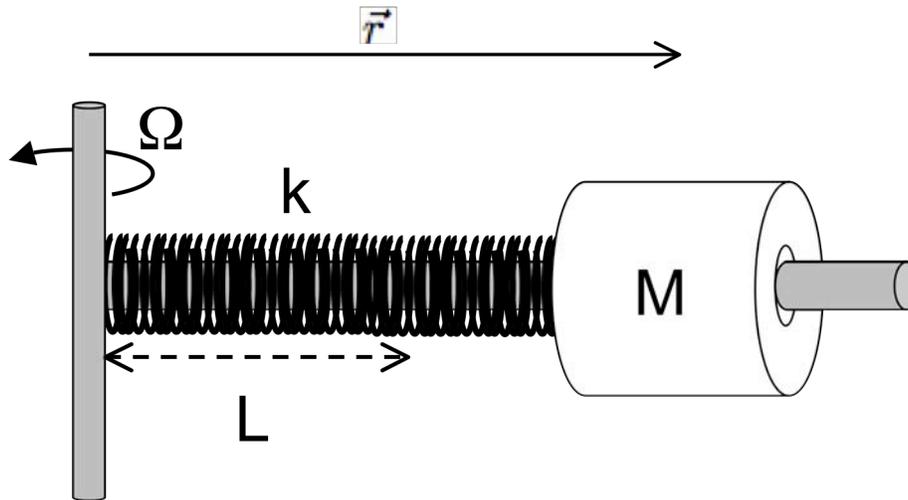
A cylindrical mass M is placed on a post connected to a rotating shaft. The post forces the mass to rotate with the shaft at constant angular velocity Ω. The mass is connected to the shaft by a spring with spring constant k and rest length L. Ignore friction and gravity.

(a) [5 pts] What is the equilibrium distance of the mass from the central shaft?

(b) [10 pts] At time t=0, the mass is struck, giving it an inward radial velocity V_R. Derive the period of oscillation of the cylindrical mass and an expression for its distance from the central shaft as a function of time t.

(c) [5 pts] What happens when $\Omega^2 > k/M$?

Problem 5: The Flyball Governor [25 pts]
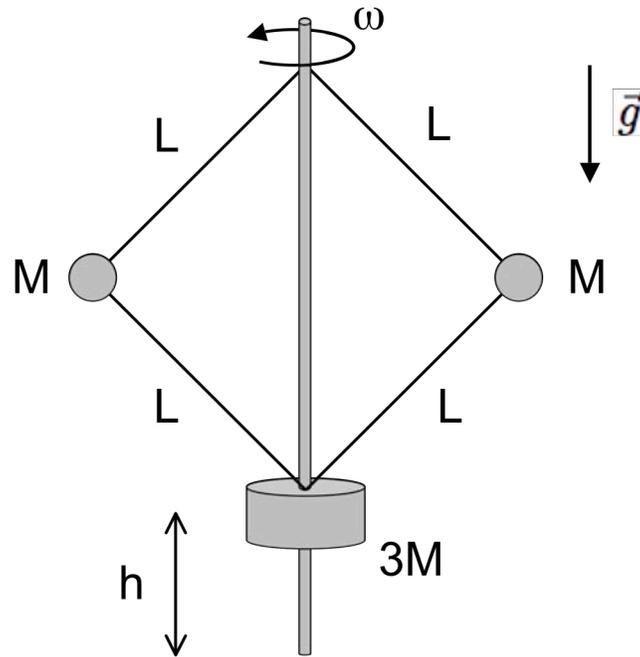
A flyball governor is a device commonly used in steam engines to control the flow of steam. In the simplified version shown above, a rotating shaft is connected to two hinges of mass M through rigid, massless rods of length L. The rods are also attached at the bottom of the device to a larger block of mass 3M which can slide freely up and down the shaft (in a practical flyball governor, this mass would be attached to a valve closer to reduce fuel flow). The shaft rotates at constant angular rate ω. Assume constant gravitational acceleration g acting downward and ignore friction and viscosity effects.

(a) [20 pts] Derive, as a function of ω, the height of the large block above its non-rotating position.

(b) [5 pts] Derive an expression relating the vertical speed of the block to the time rate change in angular rotation rate of the shaft, and confirm that the block moves upward as the shaft spins faster.

USEFUL EQUATIONS

Trajectory for constant acceleration $\vec{a}$

$$\vec{r}(t) = \frac{1}{2}\vec{a}t^2 + \vec{v}(0)t + \vec{r}(0)$$

Velocity in polar coordinates

$$\frac{d\vec{r}}{dt} = \dot{\vec{r}} = \dot{r}\hat{r} + r\dot{\theta}\hat{\theta}$$

Acceleration in polar coordinates

$$\frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}} = (\ddot{r} - r\dot{\theta}^2)\hat{r} + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\theta}$$

Taylor expansion of function f(x):

$$f(x) = f(a) + \frac{1}{1!}\frac{df}{dx}|_a(x-a) + \frac{1}{2!}\frac{d^2f}{dx^2}|_a(x-a)^2 + ...$$