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8.012 Physics I: Classical Mechanics
Fall 2008

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MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.012

Fall 2006

Exam 2
Thursday, November 2, 2006

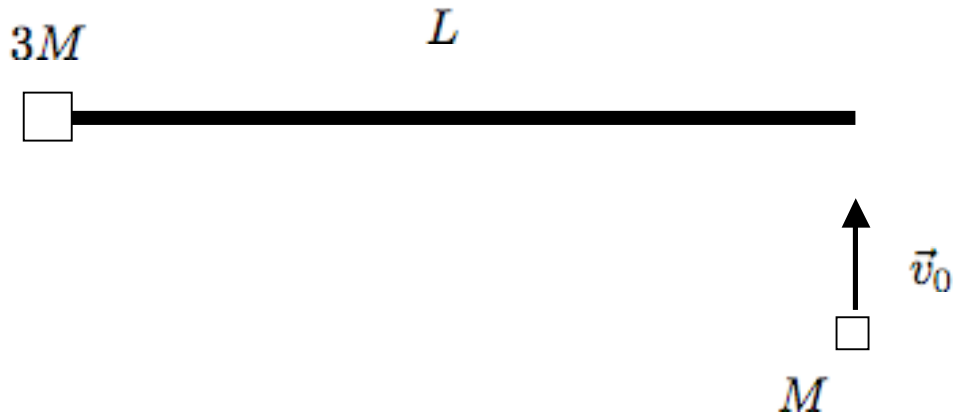
NAME: _____

Instructions:

1. Do all **FIVE (5)** problems. You have **90 minutes**.
2. **SHOW ALL WORK.** Be sure to **circle your final answer**.
3. Read the questions carefully
4. All work must be done in this booklet. Extra blank pages are provided.
5. **NO** books, notes, calculators or computers are permitted. **A sheet of useful equations is provided on the last page.**

Your Scores

Problem	Maximum	Score	Grader
1	15		
2	15		
3	30		
4	20		
5	20		
Total	100		

Problem 1: Masses and Rod [15 pts]

A block with mass $3M$ connected to a massless rod of length L lies at rest on a fixed frictionless table. A second block of mass M impinges on the system with speed v_0 , strikes the opposite end of the rod at a right angle, and sticks. For this problem, assume that the dimensions of the blocks are much smaller than the length of the rod.

- (a) [5 pts] What is the final velocity of the center of mass of the block-rod-block system?
- (b) [5 pts] What is the final angular velocity of the block-rod-block system about its center of mass?
- (d) [3 pts] What is the initial velocity of the more massive block after the collision? Be sure to indicate its direction and speed.

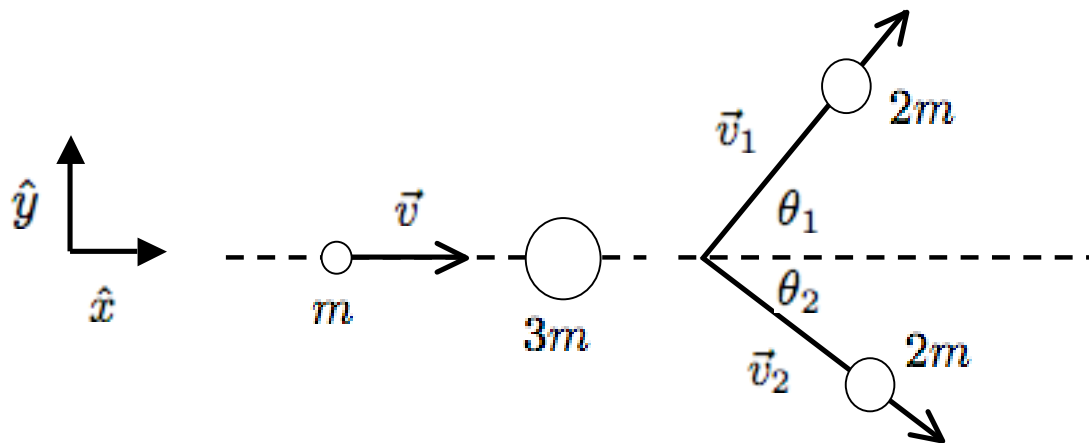
Problem 2: An Unusual Force Law [15 pts]

Consider the following force law:

$$F(x) = \frac{mc^2}{4} \left(\frac{4a^2}{x^3} - \frac{3a^3}{x^4} - \frac{a}{x^2} \right)$$

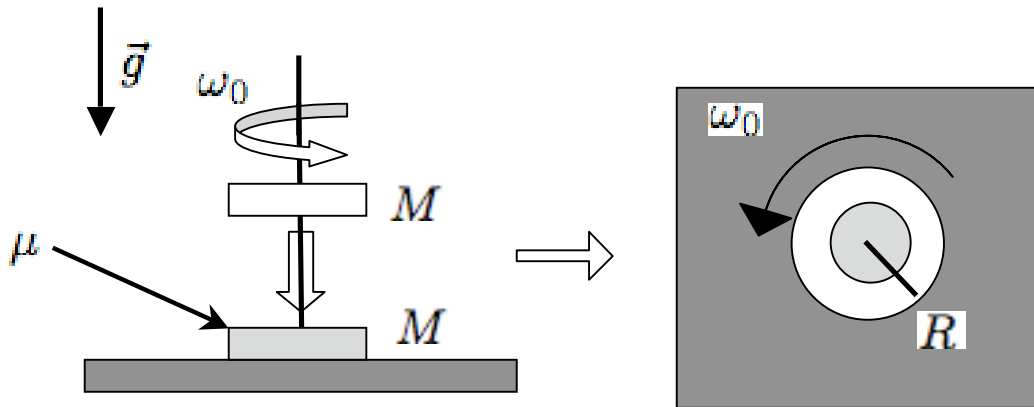
for $x > 0$, where m is the mass of the object, a is a constant length and c is the speed of light (Note: this force is related to the motion of a mass near a black hole).

- (a) [5 pts] Sketch the potential energy of this force law as a function of x . What are the two finite equilibrium points for this force law (i.e., excluding $x = \infty$)? Which of these is stable?
- (b) [5 pts] Compute the period of small oscillations about the stable equilibrium point.
- (c) [5 pts] An object of mass m , initially at rest at the unstable equilibrium point, moves toward the stable equilibrium point. What is its velocity at the stable equilibrium point?

Problem 3: Particle Collision [30 pts]

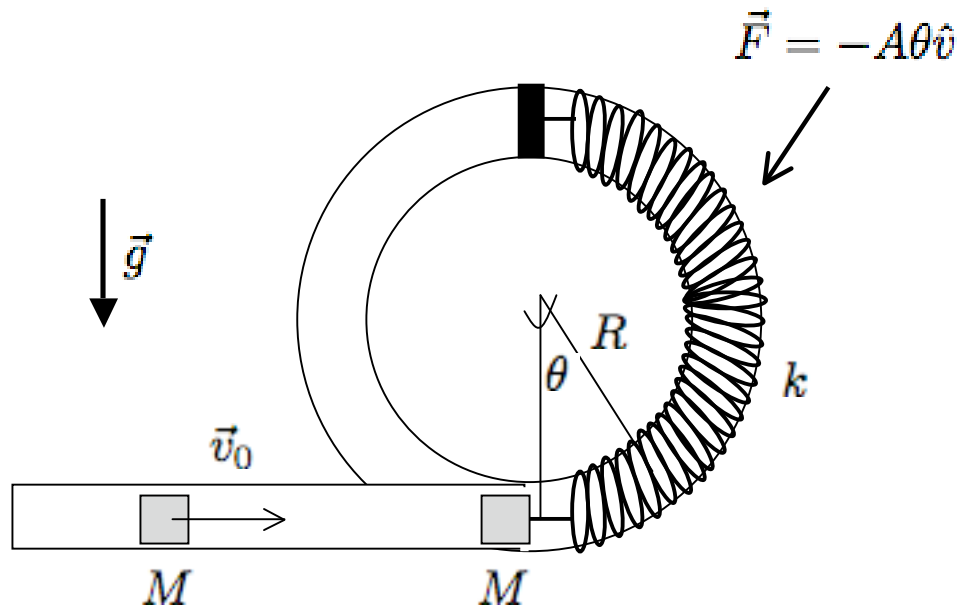
A particle of mass m and velocity $\vec{v} = v\hat{x}$ collides with a particle of mass $3m$ at rest in the LAB frame. The ensuing interaction creates two particles with identical masses $2m$ that scatter off with velocities \vec{v}_1 and \vec{v}_2 at angles θ_1 and θ_2 from the \hat{x} axis, respectively, in the LAB frame. A mechanical energy Q is lost as a result of the interaction. Assume that there are no external forces at work.

- (a) [10 pts] What is the minimum kinetic energy (K_0) of the incident particle required for this interaction to take place?
- (b) [10 pts] Assume that the incident particle has initial kinetic energy $K = \alpha^2 K_0$ where $\alpha^2 > 1$. Write an expression relating the scattering angle θ_1 in the LAB frame to the center of mass scattering angle Θ (the deflection angle of the particles in the center of mass frame) that depends only on α .
- (c) [10 pts] One of the scattered particles is observed to emerge at an angle of 90° with respect to the \hat{x} axis in the LAB frame. In the limit that the kinetic energy of the incident particle is much larger than the minimum energy (i.e., $\alpha^2 \gg 1$), what is the maximum interaction scattering angle Θ for which this could occur? Note: this can be solved without doing part (b).

Problem 4: Spinning Disk and Hoop [20 pts]

A uniform hoop with mass M , outer radius R and inner radius $R/2$ is initially spinning with angular velocity ω_0 . The hoop is placed onto a uniform disk, also of mass M and outer radius R , that is initially at rest on a fixed frictionless table. The hoop and disk are aligned along their centers of mass. There is a coefficient of friction μ between the hoop and disk where their surfaces make contact. Gravitational acceleration is assumed to be constant and points downward.

- (a) [10 pts] What is the initial torque (about its center of mass) acting on the disk? What is the initial torque (about its center of mass) acting on the hoop?
- (b) [5 pts] The friction force between the hoop and disk cause them to eventually spin at the same angular velocity. What is this angular velocity?
- (c) [5 pts] How much time does it take for the disk and hoop to reach this common angular velocity?

Problem 5: Spring-loaded Tube [20 pts]

A block of mass M and velocity v_0 is moving through a fixed tube with frictionless walls. At the bottom of a loop of radius R , the block collides perfectly inelastically (i.e., it sticks) with a plunger of mass M connected to a long fixed spring with spring constant k and equilibrium length πR . As the plunger and block travel around the loop, they are also acted on by a drag force $\vec{F} = -A\theta\hat{v}$; i.e., a force that acts opposite to the direction of motion. Gravitational acceleration acts downward. Assume that the dimensions of the block and plunger are much smaller than R , so that their rotation about their centers of mass are negligible.

- (a) [10 pts] What is the incoming block's velocity if the plunger and block come to rest at $\theta = \pi/2$?
- (b) [10 pts] Continuing from (a), after the block/plunger pair stop at $\theta = \pi/2$, they swing back around the loop and out into the straight, frictionless linear tube. If there are no adhesive forces that keep the block and plunger bound to each other, at what point does the block move away from the plunger and with what speed?