Fall Term 2003
Plasma Transport Theory, 22.616
Problem Set #4
Prof. Molvig

Passed Out: Oct. 2, 2003 DUE: Oct. 9, 2003

1. **Collisional Guiding Center Scattering:** Carry out the calculation of the guiding center radial scattering operator for Lorentz scattering resulting from the transformation of the original "velocity scattering at fixed" postion operator. Recall from lecture that the spatial diffusion operator is (taking the spatial gradients, ∇_R, parallel to $\mathbf{e}_x$,

$$\begin{aligned}\mathcal{C}_{ei}^{R}(f_e) &= \nabla_R \cdot D\left(v_\parallel, v_\perp\right)\nabla_R f_e \\ D\left(v_\parallel, v_\perp\right) &= \left\langle \nu_{ei}\frac{v_{Te}^3}{2\Omega_e^2}\mathbf{e}_y\cdot\mathbf{U}\cdot\mathbf{e}_y\right\rangle_\phi\end{aligned}$$

with,

$$\mathbf{U} = \frac{1}{v}\left(\mathbf{I} - \frac{\mathbf{vv}}{v^2}\right)$$

Show that the result is,

$$D\left(v_\parallel, v_\perp\right) = \nu_{ei}\frac{v_{Te}^3}{2\Omega_e^2}\frac{v_\parallel^2 + \frac{1}{2}v_\perp^2}{v^3}$$

Show also that the gyrophase average velocity scattering operator,

$$\mathcal{C}_{ei}^{v}(f_e) = \left\langle \frac{\partial}{\partial\mathbf{v}}\cdot\nu_{ei}\frac{v_{Te}^3}{2}\mathbf{U}\cdot\frac{\partial}{\partial\mathbf{v}}\right\rangle_\phi f_e$$

is,

$$\mathcal{C}_{ei}^{v}(f_e) = \nu_{ei}\frac{v_{Te}^3}{v^3}\frac{1}{2}\frac{\partial}{\partial\mu}\left(1-\mu^2\right)\frac{\partial}{\partial\mu}f_e$$

2. **Diamagnetic Flow:** Show that the diamagnetic flow computed from expanding the stationary Maxwellian distribution of *guiding centers* about *fixed particle position* gives,

$$n\mathbf{u}^* = \frac{1}{m_e\Omega_e}\mathbf{b}\times\nabla p_e$$

In other words show that the separate temperature gradient term integrates to zero.

3. **Electron-Ion Temperature Equilibration:** Use the guiding center transport theory developed in class and include the electron-ion energy exchange operator from class notes (modified slightly!),

$$\mathcal{C}_{ei}^{E}(f_e) = \frac{m_e}{2m_i}v_{Te}^3\frac{1}{v^2}\frac{\partial}{\partial v}\left[\nu_{ei}(v)\left(f_e + \frac{T_i}{m_e}\frac{1}{v}\frac{\partial}{\partial v}f_e\right)\right]$$

to show that the ordering, $m_e/m_i \sim \rho_e/L_\perp \sim E/E_R$ leads to an ill-posed transport theory. A detailed note is important here: the particle and energy moments of this operator need to be zero for any distribution, f_e. It is necessary that the collision frequency, ν_{ei}, go to zero at small velocities, actually, $v < v_{Ti}$. This detail is critical for these moments, *but for very little else in transport theory*, so you can use the simpler form above for the rest of this calculation. Explain the physical basis of this result.

4. **Flux-Friction Calculation of Radial Flux:** Consider the momentum moment equation for electrons using the Lorentz collision operator,

$$0 \simeq -\nabla p_e + m_e \Omega_e n \mathbf{V} \times \mathbf{b} + \mathbf{F}_{ei}$$

where $\mathbf{F}_{ei}$ is the friction moment of the collision operator,

$$\mathbf{F}_{ei} = \int d^3 v m_e \mathbf{v} \mathcal{C}_{ei}^L (f_e)$$

and we have dropped the slow time and space derivatives of the fluid velocity, $\mathbf{V}$. Now consider a confinement type geometry where $\mathbf{b} = \mathbf{e}_z$,the spatial gradients vary in the $\mathbf{e}_x$ direction and the $\mathbf{e}_y$ direction is homogeneous. For strong magnetic fields, one can compute the diamagnetic flows in the $\mathbf{e}_y$ direction from radial, $\mathbf{e}_x$, momentum balance, $0 \simeq -\mathbf{e}_x \cdot \nabla p_e + m_e \Omega_e n \mathbf{e}_x \cdot \mathbf{V} \times \mathbf{b} = -\partial p_e/\partial x + m_e \Omega_e n V_y$. You can now evaluate the y-component of the momentum balance (use a shifted Maxwellian for f_e with fluid velocity, V_y, as computed here) to give a Flux-Friction relation,

$$nV_x = \frac{1}{m_e \Omega_e} \mathbf{e}_y \cdot \mathbf{F}_{ei}$$

Compute $\mathbf{e}_y \cdot \mathbf{F}_{ei}$ by including the diamagnetic flow in $\mathcal{C}_{ei}^L (f_e)$, and show the the perpendicular flux that results is identical to that derived in class (and text) for the pressure driven flow. This is the macroscopic version of the guiding center diffusion flux we talked about in class.