22.615, MHD Theory of Fusion Systems
Prof. Freidberg
Lecture #14: Real Tokamaks (with Bob Granetz)

1. Today's lecture presents a qualitative picture of various members of the tokamak family. These include:

    a. Ohmic tokamak - discussed in class

    b. High β tokamak - discussed in class (ITER)

    c. Advanced tokamak (AT operation – not so hot)

    d. Reversed shear tokamak (RS operation – good AT operation)

    e. Spherical tokamak (NSTX, MAST)

2. It will also include a discussion of the MHD behavior of Alcator C-Mod presented by Dr. Bob Granetz.

3. We begin with a brief discussion of elongation and why it is good as well as a short summary of the main instabilities of interest. These instabilities are discussed in more detail during the second part of the term.

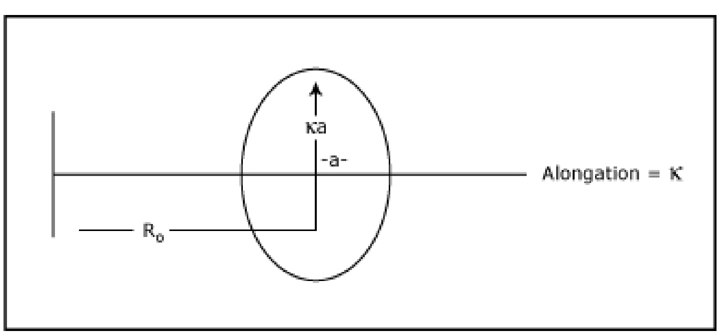
4. Elongation – from an MHD point of view, elongation is desirable because it allows higher current I and higher β without sacrificing stability. High I also improves transport: $\tau \propto I$.

5. Recall that $q_* \sim 1/I$ so that increasing I tends to decrease q_a, therby decreasing stability. Elongation can compensate this effect.

6. The idea is as follows. If we assume that stability depends upon the value of q_* or q_a, regardless of plasma shape, then we can show that elongation allows higher I without changing q.

7. The assumption that stability depends largely on q_* turns out to be approximately true for current driven kinks (that lead to disruption). Also, elongation uses the critical β for stability for a fixed q.

8. Let us determine an approximate relation between q, I, and elongation κ.



9. Definition of q

$$q \quad \oint \frac{B_\phi \cdot r d\theta}{R B_\theta}$$

10. Estimate terms

$B_\phi \quad B_o$

$R \quad R_o$

$\int r d\theta =$ lp (poloidal circumference)

$$\mu_0 I \quad \int B_\theta r d\theta \quad \bar{B}_\theta l_p \rightarrow \bar{B}_\theta \quad \frac{\mu_0 I}{l_p}$$

11. Therefore

$$q \approx \frac{B_0 l_p}{R_0 \left(\mu_0 I / l_p\right)} \quad \frac{l_p^2 B_0}{\mu_0 R_0 I}$$

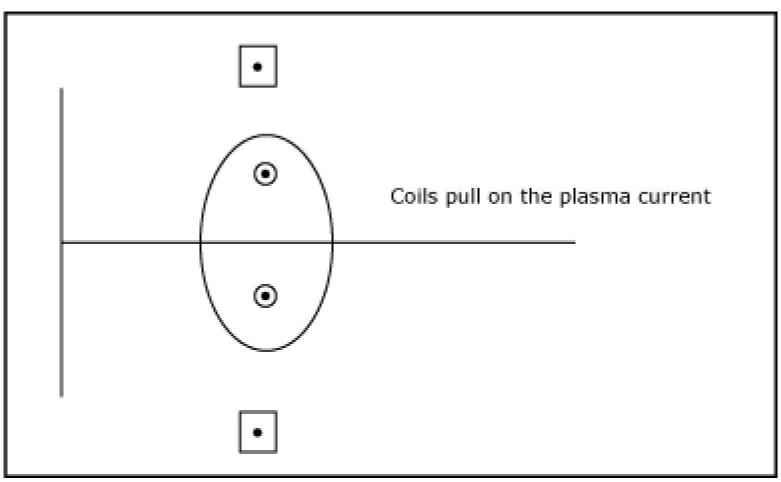
12. For a racetrack $l_p \quad 2\pi\, a\left[1 - \frac{2}{\pi}\kappa\right] \approx 2\pi a\left[.36 + .64\kappa\right]$

$$q \approx \frac{2\pi a^2 B_0 \left(.36 + .64\kappa\right)^2}{\mu_0 R_0 I}$$

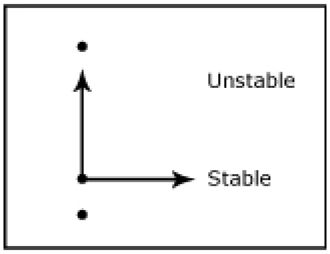
13. Note that as κ increases, I can also increase without changing q. As κ increases from 1 to 1.6, I can increase by a factor of 1.9 without changing q.

14. Why not make the plasma very long? Elongated cross sections are very susceptible to vertical instabilities. Usually $\kappa \leq 1.8$.

15. How do we make an elongated equilibrium? Add shaping coils.



16. Why in this unstable?



17. Other tokamak instabilities to worry about.

    a. Too much current – kink instability $\rightarrow$ disruptions.

    b. Too much pressure – ballooning, ballooning–kink modes-disruptions.

    c. Close conducting shale can rise the β limit. But, a real resistive shale leads to the resistive wall mode – may need feedback.

    d. New classical tearing modes – seed island leads to disruption. Need low enough β and I.

Advanced tokamak (not so bad) and RS good

1. AT operation is characterized by a substantial amount of profile control – both J_θ and p.

2. This is accomplished by various localized heating and CD sources, as well as different fueling techniques (e.g. pellets, edge puffing).

3. What problem is AT operation supposed to solve?

4. Tokamaks are supposed to operate in a steady state manner.

5. Ohmic drive is finite in duration because of the transformer.

6. RF current drive can in principle drive current in a steady state fashion – directional waves drag electrons with them causing a flow of current.

7. But, current drive is not very efficient – lots of watts to make 1 ampere.

8. Driving all the current in a tokamak requires high priced RF power comparable to the entire output of the plant – bad power balance.

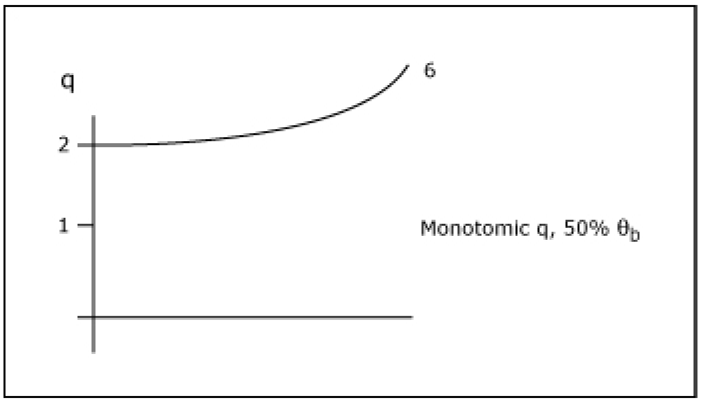
9. Here is where the bootstrap current comes in. This is a transport driven current similar to the magnetization current $(\sim \nabla n)$ in a plasma except involving trapped and nearly trapped particles.

10. General scaling

$$J_b \sim -\frac{\varepsilon^{1/2}}{B_p}\frac{dp}{dr}$$

$$\frac{I_b}{I_p} \sim \varepsilon^{1/2}\beta_p$$

11. Early AT attempts

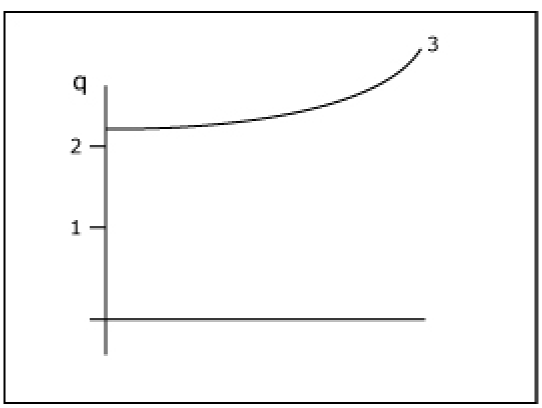


a. low β stability limits.

b. low current $\rightarrow$ poor transport.

c. bootstrap fraction OK but not great – need 80%.

12. This led to experimental discovery of reversed shear equilibria.



a. reversed shear $\rightarrow$ hollow current, profile control.

b. lower q_a $\rightarrow$ higher current, improved transport.

c. $J_b \sim \in^{1/2} p'$ naturally makes hollow current profile.

d. bootstrap fractions on the order of 75% - 93% can be readily achieved (theoretically).

e. For $F_b \le 75\%$, no conducting wall is needed.

f. For $F_b \ge 75\%$, resistive wall mode appears and some type of feedback is required.

13. There are strong RS-AT experimental programs at GA and MIT.

14. Big issues

a. Can "natural" bootstrap profiles with hollow current be maintained in steady state with profile control.

b. Can $F_b \ge 75\%$ be maintained in steady state with profile control and resistive wall stability.

c. What happens in a reactor dominated by α heating? Will external profile control still be effective? $P_{cd} \ll P_\alpha$

Spherical Tokamak = Spherical Torus

1. An ST is a way to make very high β in a tokamak.

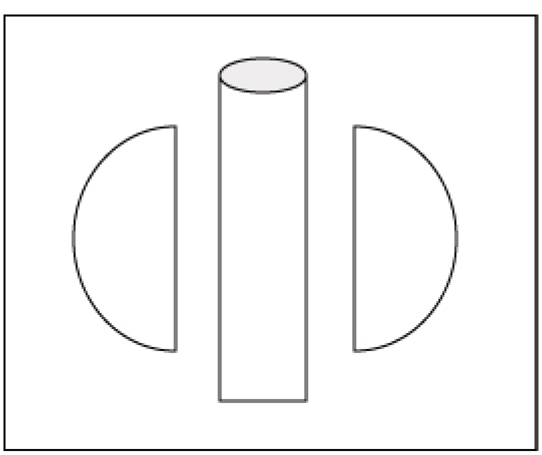
2. High β is good in a reactor.

3. Idea is based on the physics result that in a tokamak, both the equilibrium and stability β limits scale is

$$\beta \le \kappa\varepsilon$$

4. Thus, making $\varepsilon \to 1$ increases the β limit.

5. An ST is then a very tight aspect ratio tokamak: $R/a \sim 1.2$.
Typical tokamak has $R/a \sim 3$.



6. The idea seems to work in terms of β : MAST, NSTX.
   β values of 20-40% have been observed.

7. Issues – not quite as good as it seems.

8. With a blanket/shield and magnets on the inboard side, there is an optimum aspect ratio (homework problem).

$$\rho_F \,\alpha\, p^2 \,\alpha \beta^2 B^4 \,\alpha \left(\frac{\beta}{\varepsilon}\right)^2 B_c^4 \varepsilon^4 \left(1-\varepsilon\right)^4 \to \text{optimum}\, \varepsilon \sim 1/3$$

9. This is not a major issue in existing experiments without a blanket and with copper coils.

10. Small central hole does not leave much room for a transformer. Inherently short pulse unless there is significant current drive and bootstrap current.

11. Rector scale-ups require a copper TF core – tough for power balance.

12. Needs very high $f_b \sim 95\%$ to keep CD power low.

13. With $f_b \sim 95\%$ resistive wall mode becomes important.

14. Overall, ST may have some advantages over regular RS tokamak, but gains are not that clear.

15. Does not help significantly in solving the main tokamak problems – steady state – high f_b operation.