Chapter 1

Introduction

1.1 What is a Plasma?

1.1.1 An ionized gas

A plasma is a gas in which an important fraction of the atoms is *ionized*, so that the electrons and ions are separately free.

When does this ionization occur? When the temperature is hot enough.

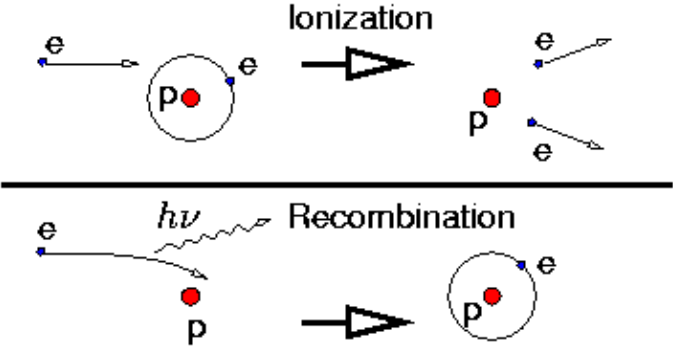Balance between collisional ionization and recombination:

Figure 1.1: Ionization and Recombination

Ionization has a threshold energy. Recombination has not but is much less probable.

Threshold is *ionization energy* (13.6eV, H). χ_i

Integral over Maxwellian distribution gives rate coefficients (reaction rates). Because of the tail of the Maxwellian distribution, the ionization rate extends below $T = \chi_i$. And in equilibrium, when

$$\frac{n_{\text{ions}}}{n_{\text{neutrals}}} = \frac{<\sigma_i v>}{<\sigma_r v>} \quad , \tag{1.1}$$
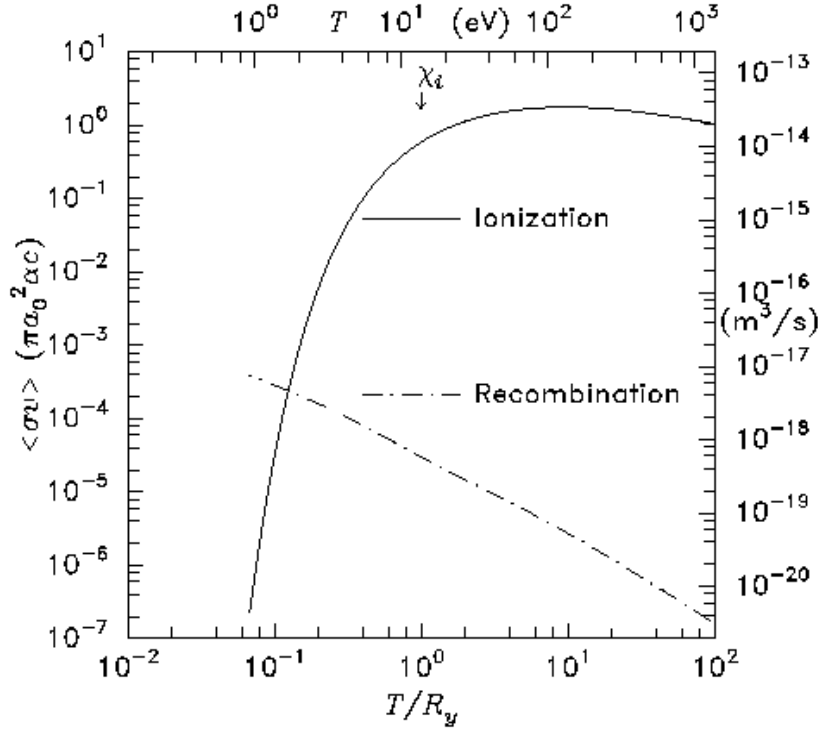
Figure 1.2: Ionization and radiative recombination rate coefficients for atomic hydrogen

the percentage of ions is large ($\sim 100\%$) if electron temperature: $T_e \gtrsim \chi_i/10$. e.g. Hydrogen is ionized for $T_e \gtrsim 1eV$ (11,600°k). At room temp **r** ionization is negligible.

For dissociation and ionization balance figure see e.g. Delcroix *Plasma Physics* Wiley (1965) figure 1A.5, page 25.

1.1.2   Plasmas are Quasi-Neutral

If a gas of electrons and ions (singly charged) has unequal numbers, there will be a net charge density, ρ.

$$\rho = n_e(-e) + n_i(+e) = e(n_i - n_e) \tag{1.2}$$

This will give rise to an electric field via

$$\nabla.\mathbf{E} = \frac{\rho}{\epsilon_0} = \frac{e}{\epsilon_0}(n_i - n_e) \tag{1.3}$$

Example: Slab.

$$\frac{dE}{dx} = \frac{\rho}{\epsilon_0} \tag{1.4}$$

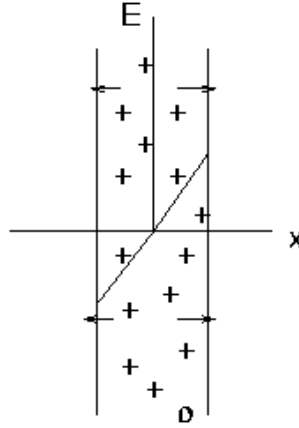$$\rightarrow E = \rho\frac{x}{\epsilon_0} \tag{1.5}$$

Figure 1.3: Charged slab

This results in a force on the charges tending to expel whichever species is in excess. That is, if $n_i > n_e$, the E field causes n_i to decrease, n_e to increase tending to reduce the charge.

This restoring *force is enormous*!

Example

Consider $T_e = 1eV$, $n_e = 10^{19} m^{-3}$ (a modest plasma; c.f. density of atmosphere $n_{\text{molecules}} \sim 3 \times 10^{25} m^{-3}$). Suppose there is a small difference in ion and electron densities $\Delta n = (n_i - n_e)$

$$\text{so} \quad \rho = \Delta n \, e \tag{1.6}$$

Then the force per unit volume at distance x is

$$F_e = \rho E = \rho^2 \frac{x}{\epsilon_0} = (\Delta n \, e)^2 \; \frac{x}{\epsilon_0} \tag{1.7}$$

Take $\Delta n / n_e = 1\%$, $x = 0.10m$.

$$F_e = (10^{17} \times 1.6 \times 10^{-19})^2 \; 0.1/8.8 \times 10^{-12} = 3 \times 10^6 N.m^{-3} \tag{1.8}$$

Compare with this the pressure force per unit volume $\sim p/x : p \sim n_e T_e (+n_i T_i)$

$$F_p \sim 10^{19} \times 1.6 \times 10^{-19}/0.1 = 16 N m^{-3} \tag{1.9}$$

Electrostatic force $\gg$ Kinetic Pressure Force.

This is one aspect of the fact that, because of being ionized, plasmas exhibit all sorts of collective behavior, different from neutral gases, mediated by the long distance electromagnetic forces E, B.

Another example (related) is that of longitudinal waves. In a normal gas, sound waves are propagated via the intermolecular action of collisions. In a plasma, waves can propagate when collisions are negligible because of the coulomb interaction of the particles.

1.2   Plasma Shielding

1.2.1   Elementary Derivation of the Boltzmann Distribution

Basic principle of Statistical Mechanics:

Thermal Equilibrium $\leftrightarrow$ *Most Probable State* i.e. State with large number of possible arrangements of micro-states.
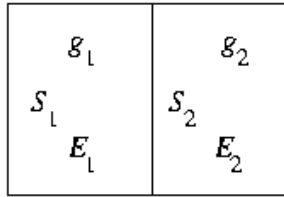

Figure 1.4: Statistical Systems in Thermal Contact

Consider two weakly coupled systems S_1, S_2 with energies E_1, E_2. Let g_1, g_2 be the number of microscopic states which give rise to these energies, for each system. Then the total number of micro-states of the *combined* system is (assuming states are independent)

$$g = g_1 g_2 \tag{1.10}$$

If the total energy of combined system is fixed $E_1 + E_2 = E_t$ then this can be written as a function of E_1:

$$g = g_1(E_1)g_2(E_t - E_1) \tag{1.11}$$

$$\text{and} \quad \frac{dg}{dE_1} = \frac{dg_1}{dE}g_2 - g_1\frac{dg_2}{dE} \quad . \tag{1.12}$$

The most probable state is that for which $\frac{dg}{dE_1} = 0$ i.e.

$$\frac{1}{g_1}\frac{dg_1}{dE} = \frac{1}{g_2}\frac{dg_2}{dE} \quad \text{or} \quad \frac{d}{dE}\ln g_1 = \frac{d}{dE}\ln g_2 \tag{1.13}$$

Thus, in equilibrium, states in thermal contact have equal values of $\frac{d}{dE}\ln g$.

One defines $\sigma \equiv \ln g$ as the *Entropy*.

And $[\frac{d}{dE}\ln g]^{-1} = T$ the *Temperature*.

Now suppose that we want to know the relative probability of 2 *micro*-states of system 1 in equilibrium. There are, in all, g_1 of these states, for each specific E_1 but we want to know how many states of the *combined* system correspond to a *single* microstate of S_1.

Obviously that is just equal to the number of states of system 2. So, denoting the two values of the energies of S_1 for the two microstates we are comparing by E_A, E_B the ratio of the number of combined system states for S_{1A} and S_{1B} is

$$\frac{g_2(E_t - E_A)}{g_2(E_t - E_B)} = \exp[\sigma(E_t - E_A) - \sigma(E_t - E_B)] \tag{1.14}$$

Now we suppose that system S_2 is large compared with S_1 so that E_A and E_B represent very small changes in S_2's energy, and we can Taylor expand

$$\frac{g_2(E_t - E_A)}{g_2(E_t - E_A)} \simeq \exp\left[-E_A \frac{d\sigma}{dE} + E_B \frac{d\sigma}{dE}\right] \tag{1.15}$$

Thus we have shown that the ratio of the probability of a system (S_1) being in any two micro-states A, B is simply

$$\exp\left[\frac{-(E_A - E_B)}{T}\right] , \tag{1.16}$$

when in equilibrium with a (large) thermal "reservoir". This is the well-known "Boltzmann factor".

You may notice that Boltzmann's constant is absent from this formula. That is because of using natural thermodynamic units for entropy (dimensionless) and temperature (energy).

Boltzmann's constant is simply a conversion factor between the *natural units of temperature* (energy, e.g. Joules) and (e.g.) degrees Kelvin. Kelvins are based on °C which arbitrarily choose melting and boiling points of water and divide into 100.

Plasma physics is done almost always using energy units for temperature. Because Joules are very large, usually electron-volts (eV) are used.

$$1eV = 11600K = 1.6 \times 10^{-19} \text{Joules.} \tag{1.17}$$

One consequence of our Botzmann factor is that a gas of moving particles whose energy is $\frac{1}{2}mv^2$ adopts the Maxwell-Boltzmann (Maxwellian) distribution of velocities $\propto \exp[-\frac{mv^2}{2T}]$.

1.2.2   Plasma Density in Electrostatic Potential

When there is a varying potential, ϕ, the densities of electrons (and ions) is affected by it.

If electrons are in thermal equilibrium, they will adopt a Boltzmann distribution of density

$$n_e \propto \exp(\frac{e\phi}{T_e}) . \tag{1.18}$$

This is because each electron, regardless of velocity possesses a potential energy $-e\phi$.

Consequence is that (fig 1.5) a self-consistent loop of dependencies occurs.

This is one elementary example of the general principle of plasmas requiring a self-consistent solution of Maxwell's equations of electrodynamics *plus* the particle dynamics of the plasma.

1.2.3   Debye Shielding

A slightly different approach to discussing quasi-neutrality leads to the important quantity called the Debye Length.

Suppose we put a plane grid into a plasma, held at a certain potential, ϕ_g.
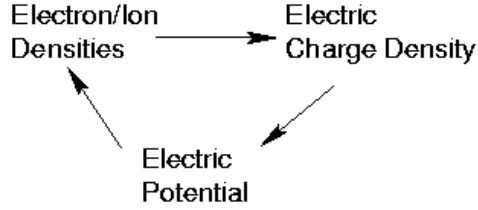
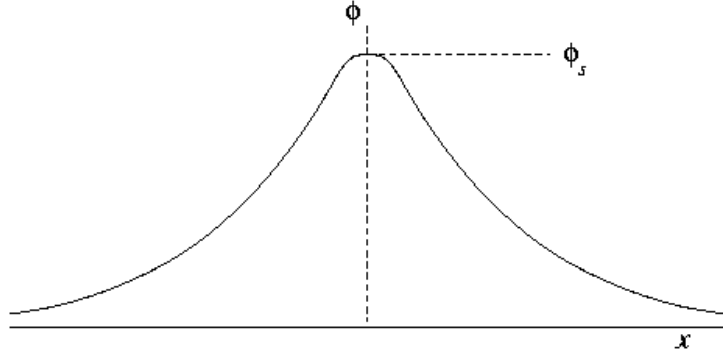Figure 1.5: Self-consistent loop of dependencies

Figure 1.6: Shielding of fields from a 1-D grid.

Then, unlike the vacuum case, the perturbation to the potential falls off rather rapidly into the plasma. We can show this as follows. The important equations are:

$$\text{Poisson's Equation} \qquad \nabla^2\phi = \frac{d^2\phi}{dx^2} = -\frac{e}{\epsilon_0}\left(n_i - n_e\right) \qquad (1.19)$$

$$\text{Electron Density} \qquad n_e = n_\infty \exp(e\phi/T_e). \qquad (1.20)$$

[This is a Boltzmann factor; it assumes that electrons are in thermal equilibrium. n_∞ is density far from the grid (where we take $\phi = 0$).]

$$\text{Ion Density} \qquad n_i = n_\infty . \qquad (1.21)$$

[Applies far from grid by quasineutrality; we just *assume*, for the sake of this illustrative calculation that ion density is not perturbed by ϕ-perturbation.]

Substitute:

$$\frac{d^2\phi}{dx^2} = \frac{en_\infty}{\epsilon_0}\left[\exp\left(\frac{e\phi}{T_e}\right) - 1\right] . \qquad (1.22)$$

This is a nasty nonlinear equation, but *far from the grid* $|e\phi/T_e| << 1$ so we can use a Taylor expression: $\exp\frac{e\phi}{T_e} \simeq 1 + \frac{e\phi}{T_e}$. So

$$\frac{d^2\phi}{dx^2} = \frac{en_\infty}{\epsilon_0}\frac{e}{T_e}\phi = \frac{e^2 n_\infty}{\epsilon_0 T_e}\phi \qquad (1.23)$$

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Solutions: $\phi = \phi_0 \exp(-|x|/\lambda_D)$ where

$$\lambda_D \equiv \left(\frac{\epsilon_0 T_e}{e^2 n_\infty} \right)^{\frac{1}{2}} \tag{1.24}$$

This is called the *Debye Length*

Perturbations to the charge density and potential in a plasma tend to fall off with characteristic length λ_D.

In Fusion plasmas λ_D is typically small. [e.g. $n_e = 10^{20}m^{-3} T_e = 1keV \quad \lambda_D = 2 \times 10^{-5}m = 20\mu m$]

Usually we include as part of the *definition* of a plasma that $\lambda_D <<$ the size of plasma. This ensures that collective effects, quasi-neutrality etc. are important. Otherwise they probably aren't.

1.2.4   Plasma-Solid Boundaries (Elementary)

When a plasma is in contact with a solid, the solid acts as a "sink" draining away the plasma. *Recombination* of electrons and ions occur at surface. Then:

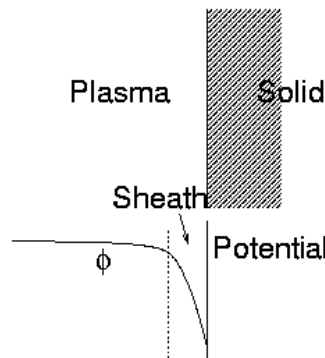1. Plasma is normally charged positively with respect to the solid.

Figure 1.7: Plasma-Solid interface: Sheath

2. There is a relatively thin region called the "sheath", at the boundary of the plasma, where the main potential variation occurs.

Reason for potential drop:
   Different velocities of electrons and ions.

If there were no potential variation ($\mathbf{E}= 0$) the electrons and ions would hit the surface at the random rate

$$\frac{1}{4}n\bar{v} \quad \text{per unit area} \tag{1.25}$$

[This equation comes from elementary gas-kinetic theory. See problems if not familiar.]

The mean speed $\bar{v} = \sqrt{\frac{8T}{\pi m}} \sim \sqrt{\frac{T}{m}}$.

Because of mass difference electrons move $\sim \sqrt{\frac{m_i}{m_e}}$ faster and hence would drain out of plasma faster. Hence, plasma charges up enough that an electric field opposes electron escape and reduces total electric current to zero.

Estimate of potential:

Ion escape flux $\qquad \frac{1}{4} n_i' \bar{v}_i$

Electron escape flux $\qquad \frac{1}{4} n_e' \bar{v}_i$

Prime denotes values at solid surface.

Boltzmann factor applied to electrons:

$$n_e' = n_\infty \exp[e\phi_s / T_e] \tag{1.26}$$

where ϕ_s is solid potential relative to distant (∞) plasma.

Since ions are being dragged out by potential assume $n_i' \sim n_\infty$ ($Z_i = 1$). [This is only approximately correct.]

Hence total current density out of plasma is

$$j \;=\; q_i \frac{1}{4} n_i' \bar{v}_i + q_e \frac{1}{4} n_e' \bar{v}_e \tag{1.27}$$

$$\;=\; \frac{en_\infty}{4} \{ \bar{v}_i - \exp\left[\frac{e\phi_s}{T_e} \right] \bar{v}_e \} \tag{1.28}$$

This must be *zero* so

$$\phi_s \;=\; \frac{T_e}{e} \ln |\frac{\bar{v}_i}{\bar{v}_e}| = \frac{T_e}{e} \frac{1}{2} \ln \left(\frac{T_i}{T_e} \frac{m_e}{m_i} \right) \tag{1.29}$$

$$\;=\; \frac{T_e}{e} \frac{1}{2} \ln \left(\frac{m_e}{m_i} \right) \quad \text{[if } T_e = T_i.\text{]} \tag{1.30}$$

For hydrogen $\frac{m_i}{m_e} = 1800$ so $\frac{1}{2} \ln \frac{m_e}{m_i} = -3.75$.

The potential of the surface relative to plasma is approximately $-4 \frac{T_e}{e}$.

[Note $\frac{T_e}{e}$ is just the electron temp **r** in electron-volts expressed as a voltage.]

1.2.5 Thickness of the sheath

Crude estimates of sheath thickness can be obtained by assuming that ion density is uniform. Then equation of potential is, as before,

$$\frac{d^2\phi}{dx^2} = \frac{en_\infty}{\epsilon_0} \left[\exp\left(\frac{e\phi}{T_e} \right) - 1 \right] \tag{1.31}$$

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We know the rough scale-length of solutions of this equation is

$$\lambda_D = \left(\frac{\epsilon_0 T_e}{e^2 n_\infty} \right)^{\frac{1}{2}} \qquad \text{the Debye Length.} \qquad (1.32)$$

Actually our previous solution was valid only for $|e\phi/T_e| << 1$ which is no longer valid.

When $-e\phi/T_e > 1$ (as will be the case in the sheath). We can practically ignore the electron density, in which case the solution will continue only quadratically. One might expect, therefore, that the sheath thickness is roughly given by an electric potential gradient

$$-\frac{T}{e}\frac{1}{\lambda_D} \qquad (1.33)$$

extending sufficient distance to reach $\phi_S = -4\frac{T_e}{e}$ i.e.

$$\text{distance } x \sim 4\lambda_D$$

This is correct for the typical sheath thickness but not at all rigorous.

1.3 The 'Plasma Parameter'

Notice that in our development of Debye shielding we used $n_e e$ as the charge density and supposed that it could be taken as smooth and continuous. However if the density were so low that there were less than approximately one electron in the Debye shielding region this approach would not be valid. Actually we have to address this problem in 3-d by defining the 'Plasma Parameter', N_D, as

$$N_D = \text{Number of particles in the 'Debye Sphere'.}$$

$$= n.\frac{4}{3}\pi\lambda_D^3 \qquad \left(\propto \frac{T^{\frac{3}{2}}}{n^{\frac{1}{2}}} \right) \qquad . \qquad (1.34)$$

If $N_D \lesssim 1$ then the individual particles cannot be treated as a smooth continuum. It will be seen later that this means that collisions dominate the behaviour: i.e. short range correlation is just as important as the long range collective effects.

Often, therefore we add a further qualification of plasma:

$$N_D >> 1 \quad \text{(Collective effects dominate over collisions)} \qquad (1.35)$$

1.4 Summary

Plasma is an *ionized* gas in which collective effects dominate over collisions.

$$[\lambda_D << \text{size} \quad , \quad N_D >> 1 \quad .] \qquad (1.36)$$

1.5 Occurrence of Plasmas

Gas Discharges: Fluorescent Lights, Spark gaps, arcs, welding, lighting

Controlled Fusion

Ionosphere: Ionized belt surrounding earth

Interplanetary Medium: Magnetospheres of planets and starts. Solar Wind.

Stellar Astrophysics: Stars. Pulsars. Radiation-processes.

Ion Propulsion: Advanced space drives, etc.

& Space Technology   Interaction of Spacecraft with environment

Gas Lasers: Plasma discharge pumped lasers: CO_2, He, Ne, HCN.

Materials Processing: Surface treatment for hardening. Crystal Growing.

Semiconductor Processing: Ion beam doping, plasma etching & sputtering.

Solid State Plasmas: Behavior of semiconductors.

For a figure locating different types of plasma in the plane of density versus temperature see for example Goldston and Rutherford *Introduction to Plasma Physics* IOP Publishing, 1995, figure 1.3 page 9. Another is at `http://www.plasmas.org/basics.htm`

1.6 Different Descriptions of Plasma

1. Single Particle Approach. (Incomplete in itself). Eq. of Motion.

2. Kinetic Theory. Boltzmann Equation.

$$\left[\frac{\partial}{\partial t} + \mathbf{v}.\frac{\partial}{\partial \mathbf{x}} + \mathbf{a}.\frac{\partial}{\partial \mathbf{v}}\right] f = \frac{\partial f}{\partial t}\bigg)_{col.} \tag{1.37}$$

3. Fluid Description. Moments, Velocity, Pressure, Currents, etc.

Uses of these.

    Single Particle Solutions $\rightarrow$ Orbits
    $\rightarrow$ Kinetic Theory Solutions $\rightarrow$ Transport Coefs.
    $\rightarrow$ Fluid Theory $\rightarrow$ Macroscopic Description

All descriptions should be consistent. Sometimes they are different ways of looking at the same thing.

1.6.1 Equations of Plasma Physics

$$\nabla.\mathbf{E} \;=\; \frac{\rho}{\epsilon_0} \qquad\qquad\qquad \nabla.\mathbf{B} \;=\; 0$$

$$\nabla \wedge \mathbf{E} \;=\; -\frac{\partial \mathbf{B}}{\partial t} \qquad\qquad \nabla \wedge \mathbf{B} \;=\; \mu_0 \mathbf{j} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} \qquad (1.38)$$

$$F = q(\mathbf{E} + \mathbf{v} \wedge \mathbf{B})$$

1.6.2 Self Consistency

In solving plasma problems one usually has a 'circular' system:

The problem is solved only when we have a model in which all parts are self consistent. We need a 'bootstrap' procedure.

Generally we have to do it in stages:

- Calculate Plasma Response (to given E,B)

- Get currents & charge densities

- Calculate **E** & **B** for **j**, p.

Then put it all together.