Introduction to Plasma Physics I

Course 22.611j | I.H. Hutchinson
21 Oct 03 | Problem Set 6 | Due 30 Oct 03

---

1. A θ-pinch in MHD equilibrium has magnetic field that is

$$B(r) = B_o + (B_a - B_o)\, r/a, \text{ for } 0 \le r \le a \quad ,$$

where the plasma edge is $r = a$, at which point the plasma pressure, p, is zero. Calculate:

(a) The pressure profile, $p(r)$.

(b) The current density profile $j(r)$.

(c) The maximum possible value of the beta, $2\mu_o < p > /B_a^2$.
where $< p >$ is the volume averaged plasma pressure:

$$< p >= \int_o^a p \; 2\pi r \; dr \; / \; \pi a^2 \quad .$$

2. A pure z-pinch (no B_z) has current density

$$j = j_o\,(1 - r/a)$$

and pressure equal to zero at the plasma boundary, $r = a$.

(a) Calculate the magnetic field profile, $B(r)$.

(b) Calculate the pressure profile, $p(r)$.

(c) Hence show that the central pressure is $p(0) = \mu_o j_o^2 a^2/18$.

3. MHD power generators may possibly be a more efficient way of converting heat into electricity. Think of one as consisting of a simple rectangular channel of (x-) width a, (y-) height b, in which the plasma flows under pressure in the z-direction. Take the plasma density and velocity to be uniform. A uniform magnetic field, B, is applied in the y-direction and the walls at x = 0, a are electrodes where the electric current density (density j, assumed uniform) is picked off at a voltage difference ϕ. Use the MHD equations to answer the following questions.

(a) If the resistivity, η, of the plasma is negligible, what is the plasma velocity?

(b) If the pressure is p_o at z = 0, what is its value as a function of z?

(c) How much electric power is generated per unit length of the channel?

(d) What is the rate of doing work per unit channel length by the plasma pressure force?

(e) If η is not negligible but can be considered fixed, and the flow velocity and B-field are also fixed but the current density can be varied, what is the maximum electric power per unit length that can be generated?

4. The "osculating plane" at a certain point on a curve in 3-D differential geometry is the plane that contains both the tangent vector to the curve and the radius of curvature. Prove that for a *force-free* MHD equilibrium, ∇B lies in the osculating plane of the magnetic field line.

5. For a z-pinch equilibrium which has zero plasma pressure at the plasma edge, $r = a$, prove by integrating the MHD force balance equation a second time that the volume-averaged pressure is a function only of the total current, and find that function.

If a hydrogen plasma z-pinch has uniform density $n = 10^{20}$ m^{-3}, temperature $T_e = T_i = T_0(1 - r^2/a^2)$ with $T_0 = 10$ keV, and radius $a = 0.01$ m, what current is required?