EPR paradox

Bell inequalities

Can quantum-mechanical description of physical reality be considered complete?

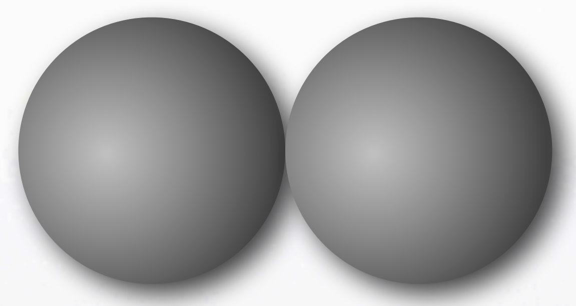
- A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 - 780 (1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

Entangled Pair

- Prepare the state $| \quad \rangle = (|01\rangle + |10\rangle)/\sqrt{2}$ of two identical particles (spins)
- Particles move to Alice and Bob, that measure their angular momentum S_z, obtaining either +1 or -1

A



B

- Experiment repeated many times: perfect anti-correlation

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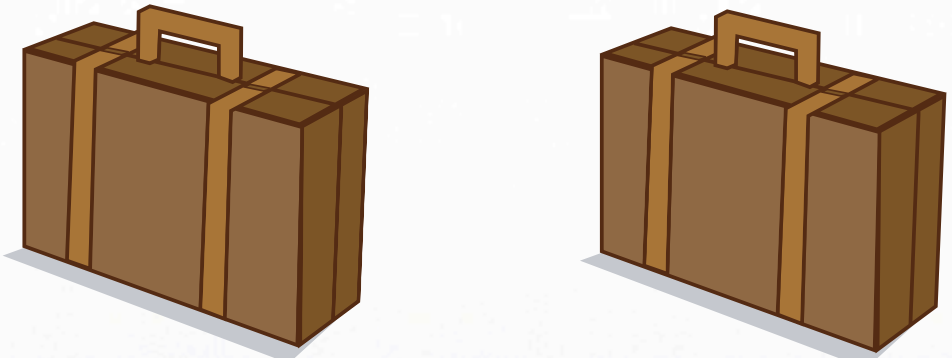
Traveler pair

- Anti-correlation also in “classical” experiment
- Two travelers with balls inside two luggages



Images by MIT OpenCourseWare.

- Alice and Bob check the luggages: perfect anti-correlation

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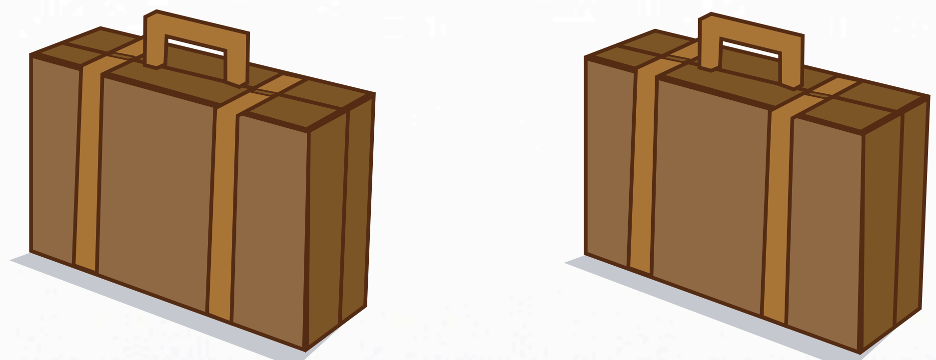

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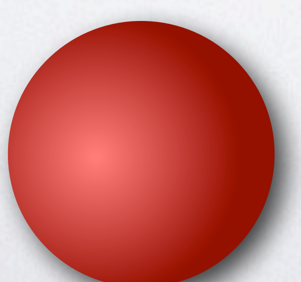
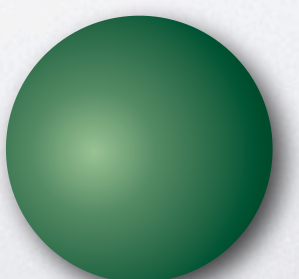




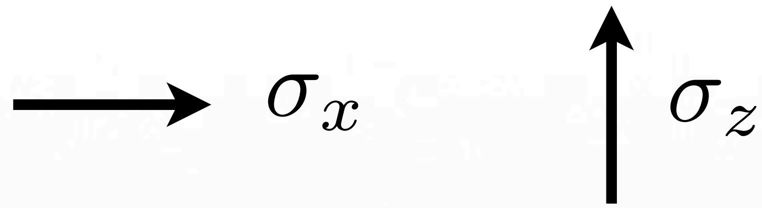
Two properties of balls

- Now assume that the balls can be red or green and matte or shiny.

- Anti-correlation also for the property of gloss

Two axes

- In QM, the 2 properties are the spin along 2 axes:



- We can rewrite the state in σ_x basis,

$$|\psi\rangle = (|01\rangle + |10\rangle)/\sqrt{2} = (|+-\rangle + |-+\rangle)/\sqrt{2}$$

- thus measuring σ_x Alice and Bob obtain same anti-correlation

Classical hypothesis

Realism & Locality

Realism

- At preparation, particles **a** and **b** possess both the properties

  (color and gloss for the classical balls
  σ_x, σ_z, with $\sigma_{x,z} = \pm 1$ for the quantum particles)

Locality

- When I measure particle **a**, I cannot modify instantaneously the result of measuring particle **b**.

  There is no action at distance (faster than light)

EPR Paradox

- Any complete description of the world must respect local realism
- Local realism is violated by quantum mechanics

➜ Quantum mechanism is not a complete description of the world

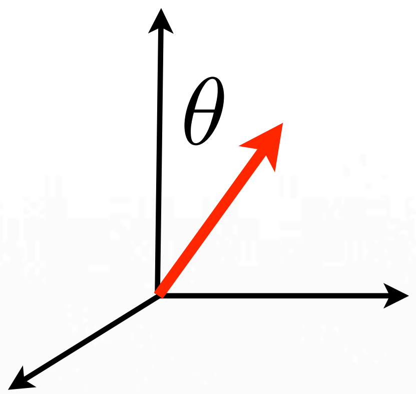
Bell Inequalities

Quantitative measure of violation of local realism

Correlation for any axes

- Assume two spins in Bell State

$$|\psi\rangle = (|01\rangle + |10\rangle)/\sqrt{2}$$



- Alice measure σ_z^A obtaining a, while Bob measure $\sigma_b^B = \cos\theta\sigma_z^B + \sin\theta\sigma_x^B$ getting $b \in \{+1,-1\}$.

- What is the correlation

$$\langle ab\rangle = \langle\sigma_z^A\sigma_b^B\rangle\ ?$$

Some calculations...

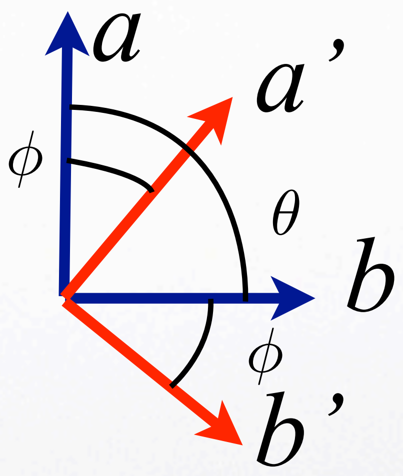
$$\langle \sigma_z^A \sigma_b^B \rangle = \frac{1}{2}(\langle 01|\sigma_z^A \sigma_b^B|01\rangle + \langle 01|\sigma_z^A \sigma_b^B|10\rangle +$$
$$+\langle 10|\sigma_z^A \sigma_b^B|01\rangle + \langle 10|\sigma_z^A \sigma_b^B|10\rangle)$$

$$= \frac{1}{2}\Big(\langle 0|\sigma_z^A|0\rangle\langle 1|\sigma_b^B|1\rangle + \cancel{\langle 0|\sigma_z^A|1\rangle}\langle 1|\sigma_b^B|0\rangle +$$
$$+\langle 1|\sigma_z^A|1\rangle\langle 0|\sigma_b^B|0\rangle + \cancel{\langle 1|\sigma_z^A|0\rangle}\langle 0|\sigma_b^B|1\rangle\Big)$$

$$= \frac{1}{2}\Big(\langle 1|\sigma_b^B|1\rangle - \langle 0|\sigma_b^B|0\rangle\Big) = -\cos\theta$$

Bell experiment

- Alice measures along either a or a'
- Bob measures along either b or b'



$$\widehat{ab} = \cos\theta \qquad \widehat{aa'} = \cos\phi$$

$$\widehat{bb'} = \cos\phi \qquad \widehat{a'b'} = \cos\theta$$

Correlations

- The correlations among the measurements are then:

$$\langle ab \rangle = \langle a'b' \rangle = -\cos\theta$$

$$\langle a'b \rangle = -\cos(\theta - \phi) \qquad \langle ab' \rangle = -\cos(\theta + \phi)$$

- We want to calculate $\langle S \rangle$

$$\langle S \rangle = \langle ab \rangle + \langle a'b' \rangle + \langle ab' \rangle - \langle a'b \rangle$$

Locality + Realism

- At each measurement, I should be able to calculate the value of the operator:

$$S_k = (\sigma_a^A \sigma_b^B)_k + (\sigma_{a'}^A \sigma_{b'}^B)_k + (\sigma_a^A \sigma_{b'}^B)_k - (\sigma_{a'}^A \sigma_b^B)_k$$

- The expectation value is then $\langle S \rangle = \lim_{N \to \infty} \frac{1}{N} \sum_k S_k$

Realism

- Even if I measure the spin along a, the property *spin along* a' is **real** (that is, it has a definite value)

Calculate outcome of S_k

- Rewrite as

$$S_k = \sigma_a^A(\sigma_b^B + \sigma_{b'}^B)_k - \sigma_{a'}^A(\sigma_b^B - \sigma_{b'}^B)_k$$

- Outcomes of $\sigma_b^B \pm \sigma_{b'}^B$ are $\{0, +2, -2\}$

- If $\sigma_b^B + \sigma_{b'}^B$ is ± 2, $\sigma_b^B - \sigma_{b'}^B$ is 0 and vice-versa
- Then $S_k = \pm 2\sigma_a$ or $S_k = \pm 2\sigma_a'$

Locality

- The fact of measuring b or b' does not change the value of a or a' (that have outcomes $+/-1$). Then:

$$S_k = \pm 2$$

- The expectation value of S is then

$$-2 < \langle S \rangle < +2$$

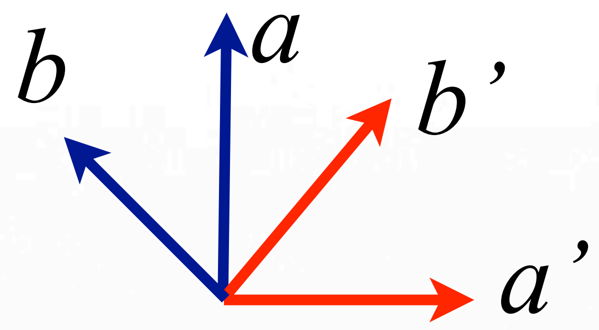
Bell inequality

- If $|\langle S \rangle| > 2$ at least one of the two hypothesis (locality or realism) is not true

Bell inequality

- Choose *a=z; a'=x;* *b=-x+z; b'=x+z;*

$$\langle ab\rangle = \langle a'b'\rangle = -\cos\theta_{ab} = -1/\sqrt{2}$$



$$\langle ab'\rangle = -\cos\theta_{ab'} = -1/\sqrt{2}$$

$$\langle a'b\rangle = -\cos\theta_{a'b} = +1/\sqrt{2}$$

- We obtain

$$\langle S\rangle = \langle ab\rangle + \langle a'b'\rangle + \langle ab'\rangle - \langle a'b\rangle = -\frac{4}{\sqrt{2}} = -2\sqrt{2} < -2$$

References

- J. S. Bell, *On the Einstein Podolsky Rosen Paradox*, Physics 1, 195-200 (1964)
- Alain Aspect, Philippe Grangier, and Gerard Roger, Phys. Rev. Lett. 47, 460 - 463 (1981) *Experimental Tests of Realistic Local Theories via Bell's Theorem*

State: informatio or object?

On the reality of the quantum state

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- No-go theorem: if the quantum state merely represents information about the real physical state of a system, then experimental predictions are obtained that contradict those of quantum theory.

Non-Locality

- If locality is lost, can it be used for action at distance?
- Teleportation?

Quantum Teleportation

- Alice has a qubit in a state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
  but does not know anything about it.
- As observing the state destroys it, Alice can't measure the qubit and tell the answer to Bob

Alice can't give the state to Bob by classical means.

- No-cloning theorem, no quantum channel.

Entangled pair

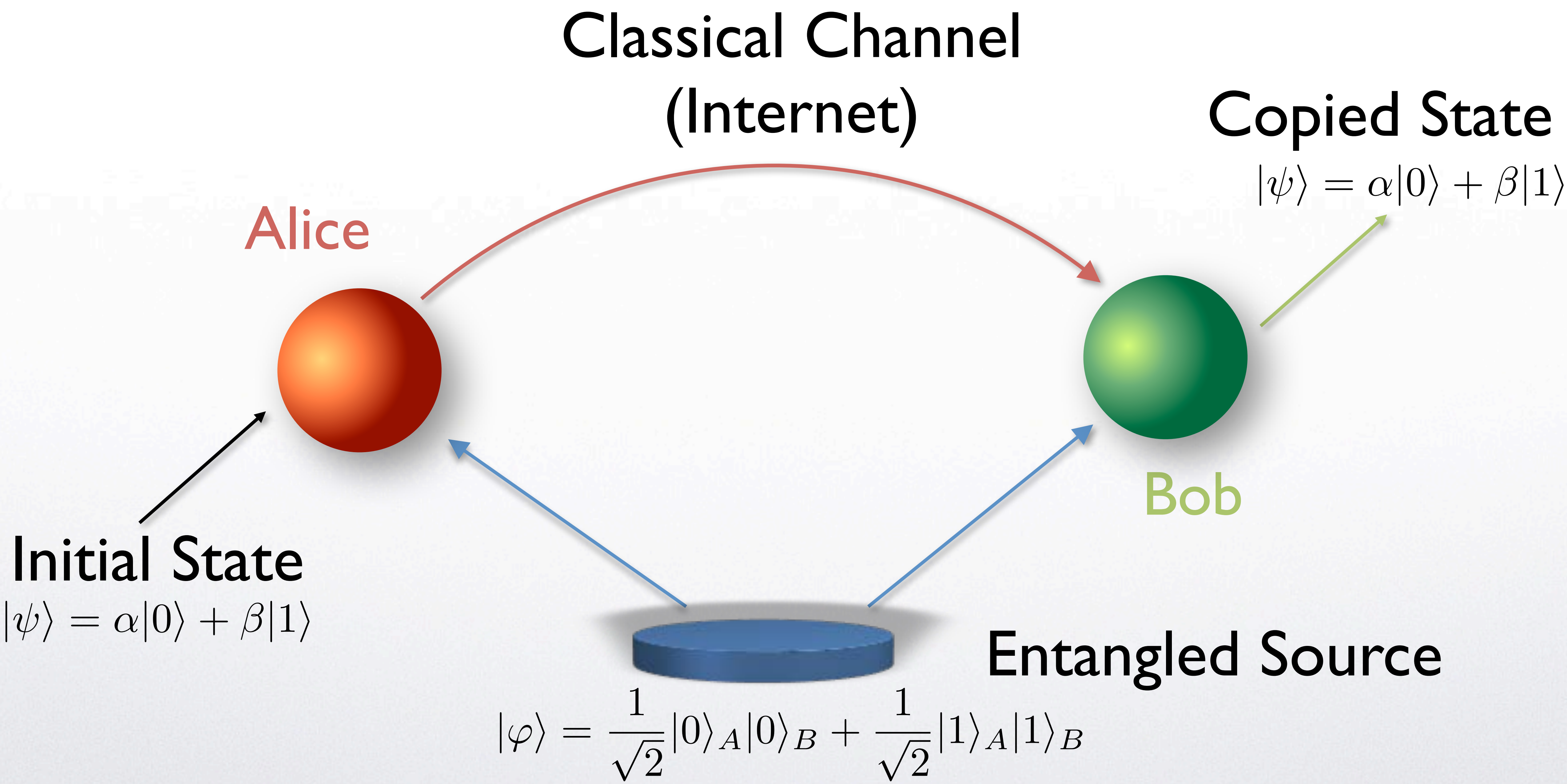
- Assume Alice and Bob share a pair of qubits that is prepared in an entangled state.
  - Alice and Bob each have access to one Qubit.

$$|\varphi\rangle = \frac{1}{\sqrt{2}}|0\rangle_A|0\rangle_B + \frac{1}{\sqrt{2}}|1\rangle_A|1\rangle_B$$

- The full-state then is the product of Alice's Qubit and the shared register:

$$|\psi\rangle|\varphi\rangle = \alpha\frac{1}{\sqrt{2}}\,|000\rangle + \frac{1}{\sqrt{2}}\beta\,|100\rangle + \frac{1}{\sqrt{2}}\alpha\,|011\rangle + \frac{1}{\sqrt{2}}\beta\,|111\rangle$$

Communication Scheme



Algorithm

- Alice then performs a CNOT on her half of the register, using her mystery bit as the control.

$$|\varphi\rangle|\psi\rangle = \alpha\frac{1}{\sqrt{2}}\left|000\right\rangle + \frac{1}{\sqrt{2}}\beta\left|110\right\rangle + \frac{1}{\sqrt{2}}\alpha\left|011\right\rangle + \frac{1}{\sqrt{2}}\beta\left|101\right\rangle$$

- She then applies the Hadamard gate to $|\psi\rangle_A$

$$\begin{aligned}|\varphi\rangle|\psi\rangle = &\frac{1}{2}\left|00\right\rangle(\alpha|0\rangle + \beta|1\rangle) + \frac{1}{2}\left|01\right\rangle(\alpha|1\rangle + \beta|0\rangle)\\ &+\frac{1}{2}\left|10\right\rangle(\alpha|0\rangle - \beta|1\rangle) + \frac{1}{2}\left|11\right\rangle(\alpha|1\rangle - \beta|0\rangle)\end{aligned}$$

Measurement

- Alice then measures her 2 qubits and tells Bob
  - Note that this destroys her original state.
  - The outcome of this observation is unpredictable.
- If Alice measures 00, then Bob has the original state. Otherwise, Bob has some other state.
- The state is **known**, so Bob can perform a known operation to retrieve the original state

$$\mathbb{1} \rightarrow \frac{1}{2}|00\rangle(\alpha|0\rangle + \beta|1\rangle) + \frac{1}{2}|01\rangle(\alpha|1\rangle + \beta|0\rangle) \leftarrow \sigma_x$$

$$\sigma_z \rightarrow +\frac{1}{2}|10\rangle(\alpha|0\rangle - \beta|1\rangle) + \frac{1}{2}|11\rangle(\alpha|1\rangle - \beta|0\rangle) \leftarrow \sigma_y$$

Quantum Teleportation

- This procedure relied on superposition and entanglement.
  - It was necessary to account for the probabilistic nature of QM by giving Bob particular actions to take, depending on the (unpredictable) outcome of the observation.
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