22.38 - PS#8 Solutions

4.5) For parallel systems: $X = \prod X_i \Rightarrow .01 = (1-.65)^n \Rightarrow n = 5$

4.6) Pump = .992, $\dagger$ = .968

a) System 1: $A_1 = [(.992)(.968)]^2 = .922$

System 2: $A_2 = [(.992)(.968)]^3 + 3(1-(.992)(.968)) = .995$

b) S1: $(\#15 + \#60)2 + \#10000(1-.922) = \#930$

S2: $(95+60)3 + 10000(1-.995) = \#275$

← clearly the 2-of-3 (system 2) system is better in this case where the cost of failure is so high.

4.7) $P_{system} = p^3 + 3(p^2(1-p)) = 3p^2 - 2p^3$

a) $\rightarrow R(s) = 3e^{-2\lambda t} - 2e^{-3\lambda t}$

$\Rightarrow f(t) = -R'(s) = 6\lambda(e^{-2\lambda t} - e^{-3\lambda t})$

b) MTTF: $= \int_0^\infty R(s)\,dt = \int_0^\infty 3e^{-2\lambda t} - 2e^{-3\lambda t}\,dt = \frac{3}{2\lambda} - \frac{2}{3\lambda} = 5/6\lambda$

mode: $\frac{df}{dt} = 0 \Rightarrow -2\lambda e^{-2\lambda t} + 3\lambda e^{-3\lambda t} = 0 \Rightarrow \tilde{t} = \ln\left(\frac{2}{3}\right)\frac{1}{\lambda} = \frac{.4}{\lambda}$

median:

$.5 = R(t) = 3e^{-2\lambda t} - 2e^{-3\lambda t}$

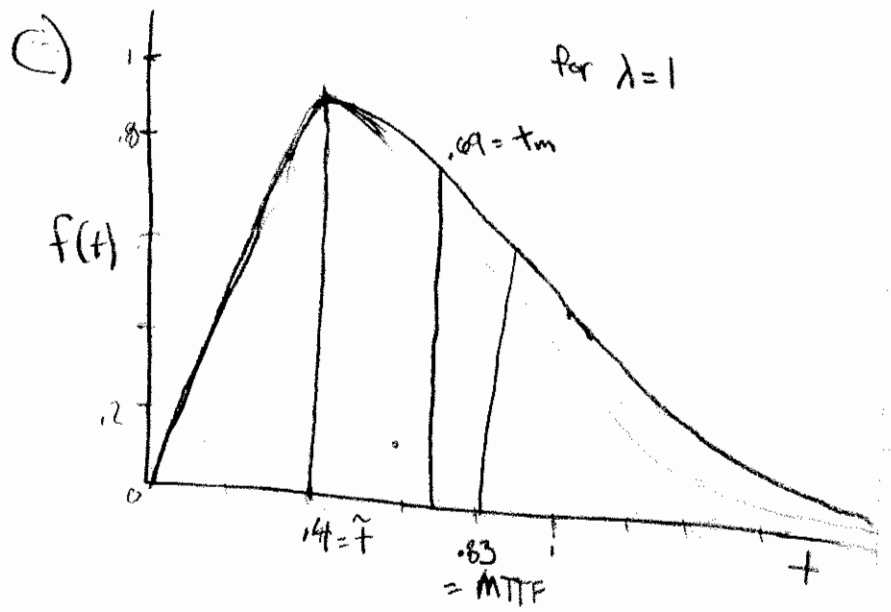
substituting $x = e^{-\lambda t}$,

$.5 = 3x^2 - 2x^3$

$\Rightarrow x = 1/2$ is the only root that makes sense

$\therefore t_m = \frac{\ln(2)}{\lambda}$

c)



4.12)

a) MTTF = $1/\lambda$ = 4950 hrs

b) P(4 months) = P(2880 hrs)

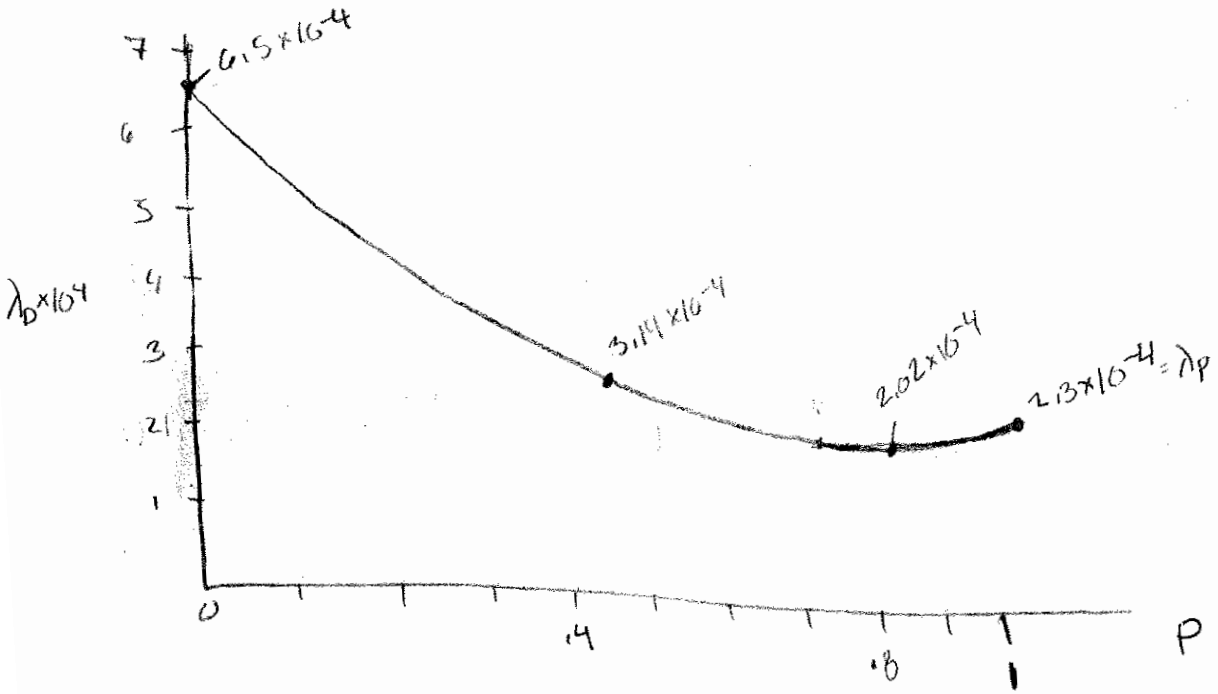
$R(s) = e^{-\lambda t} \Rightarrow R(2880) = e^{-2880\lambda} = .5589$ where $\lambda = 2.02 \times 10^{-4}$

c) $F(t) = 1 - e^{-\lambda_p t}$ where $\lambda_p = 6.5 \times 10^{-4} + 2p(p-1.6)\ 3.4 \times 10^{-4}$

$\lambda_p = 3.14 \times 10^{-4}\ hr^{-1}$ for 40% (p=.4)

$= 2.02 \times 10^{-4}$ for 80% (p=.8)

$= 2.3 \times 10^{-4}$ for 100% (p=1)



d) $R(t) = (e^{-\lambda_b t})^4 = .7488$ for $\lambda_b = 5 \times 10^{-5}$

$t = 1440$ hr

4.12 cont)

e) P(vessil fails to operated as needed) = P(failure of v) · P(burner works)
+ P(burner fails)

3-of-4 system: $R(t) = 4e^{-3\lambda t} - 3e^{-4\lambda t} \Rightarrow \lambda = 2.02\times10^{-4}, t = 1146$
$= .9736$

P(burners work) $= (.974)^4 (.9736) + 4(.97)^3(.03)(e^{-\lambda_b t})^3 (1-e^{-\lambda_b t}) = .888$

P(burners fail) $= 1-.888 = .1112$

P(vessil fail)@ 80% $= 1-e^{-\lambda_p t} = .252$ $\lambda_p = 2.02\times10^{-4}$

P(Fail) $= .252(.8887) + .1112 = .335$

$\Rightarrow$ P(success start + run 2 months) $= 1-P(\text{fail}) = 1-.335 = .665$

f) P(40 → 80%) = P(2 → 3 burners) + P(2 → 4 burners)
$= [.97(.03)]^2 + .97^2 = .9991$