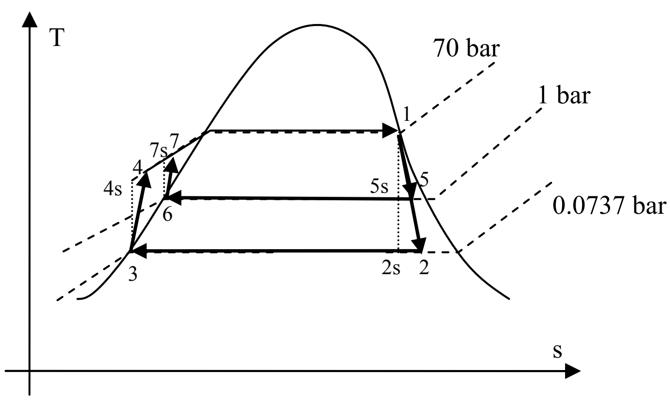
22.312 ENGINEERING OF NUCLEAR REACTORS

Tuesday, October 27th, 2009, 1:00 – 2:30 p.m.

OPEN BOOK QUIZ 1 (solution)

Problem 1 (55%) – Nuclear power plant for night-time desalination of seawater

i) T-s diagrams for daytime cycle ($1\rightarrow2\rightarrow3\rightarrow4$) and night-time cycle ($1\rightarrow5\rightarrow6\rightarrow7$):



ii) The total revenues, R (in \$), is the sum of the electricity and heat revenues over a period of 24 hours:

$$R = (\dot{Q}\cdot\eta_{cycle,day}\cdot 16\cdot C_{e,day} + \dot{Q}\cdot\eta_{cycle,night}\cdot 8\cdot C_{e,night} + \dot{Q}\cdot(1-\eta_{cycle,night})\cdot 8\cdot C_{heat})/100 \quad (1)$$

where $\dot{Q}$ is the reactor thermal power (in kW), $\eta_{cycle,day}$ is the daytime thermal efficiency of the Rankine cycle, $\eta_{cycle,night}$ is the thermal efficiency of the Rankine cycle at night, 16 and 8 are the daytime and night-time hours, respectively; $C_{e,day}$, $C_{e,night}$ and C_{heat} are the prices of electricity at day and night and the price of heat, respectively. The factor 100 at the denominator is to convert ¢ into \$.

The thermal efficiency of the cycle in daytime mode and night-time mode are:

$$\eta_{\text{cycle,day}} = \frac{(h_1-h_2)-(h_4-h_3)}{(h_1-h_4)} \quad (2)$$

$$\eta_{\text{cycle,night}} = \frac{(h_1-h_5)-(h_7-h_6)}{(h_1-h_7)} \quad (3)$$

So, one has to find the enthalpies h_1, h_2, h_3, h_4, h_5, h_6 and h_7.

Daytime Cycle
Turbine inlet (**Point 1**): T_1=285.7°C, P_1=70 bar, h_1=2772 kJ/kg, s_1=5.815 kJ/kg·K
Turbine outlet (**Point 2**): T_2=40°C, P_2=0.0737 bar, $s_{2s}=s_1$=5.815 kJ/kg·K, $x_{2s}=(s_{2s}-s_f)/(s_g-s_f)$=0.6822, $h_{2s}=h_f+x_{2s}(h_g-h_f)$=1809 kJ/kg, $h_2=h_1-(h_1-h_{2s})\eta_T$=1905 kJ/kg
Condenser outlet (**Point 3**): T_3=40°C, P_3=0.0737 bar, h_3= 167 kJ/kg
Pump outlet (**Point 4**): P_4=70 bar, $h_{4s}=h_3+(P_4-P_3)v_f \approx$ 174 kJ/kg, $h_4=h_3+(h_{4s}-h_3)/\eta_P$=175 kJ/kg

Night-time Cycle
Turbine inlet (**Point 1**): T_1=285.7°C, P_1=70 bar, h_1=2772 kJ/kg, s_1=5.815 kJ/kg·K
Turbine outlet (**Point 5**): T_5=100°C, P_5=1 bar, $s_{5s}=s_1$=5.815 kJ/kg·K, $x_{5s}=(s_{5s}-s_f)/(s_g-s_f)$= 0.7455, $h_{5s}=h_f+x_{5s}(h_g-h_f)$= 2102 kJ/kg, $h_5=h_1-(h_1-h_{5s})\eta_T$= 2169 kJ/kg
Condenser outlet (**Point 6**): T_6=100°C, P_6=1 bar, h_6= 419 kJ/kg
Pump outlet (**Point 7**): P_7=70 bar, $h_{7s}=h_6+(P_7-P_6)v_f \approx$ 426 kJ/kg, $h_7=h_6+(h_{7s}-h_6)/\eta_P$= 427 kJ/kg

Therefore, from Eqs. (2) and (3) we get $\eta_{cycle,day}$ ~0.330 and $\eta_{cycle,night}$ ~0.254, and from Eq. (1) R~\$152,000.

iii) If the plant sold only electricity, the total revenues per day would be:

$$\mathrm{R} = (\dot{Q}\cdot\eta_{cycle,day}\cdot 16\cdot C_{e,day} + \dot{Q}\cdot\eta_{cycle,day}\cdot 8\cdot C_{e,night})/100 \sim \$143{,}000$$

Therefore, the utility makes more money selling part of its night-time output as heat to the desalination plant.

iv)

$$\mathrm{EUF} = \frac{\dot{Q}\cdot\eta_{cycle,day}\cdot 16 + \dot{Q}\cdot\eta_{cycle,night}\cdot 8 + \dot{Q}\cdot(1-\eta_{cycle,night})\cdot 8}{\dot{Q}\cdot 24} \sim 0.554$$

v) If the plant operated in the electricity-production mode all the time, the EUF would be simply equal to the daytime thermal efficiency (~0.330), and thus lower than the EUF for the bimodal operation.

Problem 2 (40%) – Nuclear energy storage in molten-salt pool

i)
Conservation of mass:

$$\frac{\partial M_{CV}}{\partial t} = -\dot{m}_0$$

Integrating and recognizing that the mass of nitrogen in the system does not change:

$$M_{s2} - M_{s1} = -\dot{m}_o t_2 \qquad (4)$$

Where M_{s2} and M_{s1} are the final and initial mass of molten salt in the pool, respectively, and $\dot{m}_o$=500 kg/s is the mass flow rate of the outgoing molten salt, and t_2=180 s is the duration of the transient. Note that M_{s1} is easily found, $M_{s1} = V_{s1}\rho_s$=2.56×10^6 kg, where V_{s1}= 1600 m^3 is the initial volume of molten salt and ρ_s=1600 kg/m^3 its density. Then Eq. (4) gives M_{2s}= 2.47×10^6 kg.

Conservation of energy (with negligible gravitational and kinetic terms):

$$\frac{\partial E_{CV}}{\partial t} = -\dot{m}_0 h_{s1} + \dot{Q}$$

Integrating and recognizing that h_{s1} is constant, as per the problem statement:

$$E_2 - E_1 = -h_{s1}\dot{m}_o t_2 + \dot{Q}t_2$$

Where $\dot{Q}$=500 MW. Expanding the left-hand side, one gets:

$$M_N c_{v,N}(T_2 - T_1) + M_{s2}u_{s2} - M_{s1}u_{s1} = -h_{s1}\dot{m}_o t_2 + \dot{Q}t_2 \qquad (5)$$

Where $c_{v,N}$=742 J/kg-K is the nitrogen specific heat at constant volume, T_1=873 K (600°C) is the initial temperature, $M_N=(P_1V_{N1})/(RT_1)$=123.4 kg is the mass of nitrogen in the system, P_1=200 kPa and V_{N1}=160 m^3 are the nitrogen initial pressure and volume, respectively; R=297 J/kg-K is the gas constant for nitrogen. Eliminating $\dot{m}_o t_2$ from Eqs. (4) and (5) and re-arranging the terms, one gets:

$$M_N c_{v,N}(T_2 - T_1) + M_{s2}(u_{s2} - h_{s1}) + M_{s1}(h_{s1} - u_{s1}) = \dot{Q}t_2$$

Using the constitutive relations for enthalpy and internal energy of an incompressible fluid, the second and third terms on the left-hand side of the above equation can be found as $u_{s2} - h_{s1} = c_{p,s}(T_2 - T_1) - \dfrac{P_1}{\rho_s}$ and $h_{s1} - u_{s1} = \dfrac{P_1}{\rho_s}$, where $c_{p,s}$ = 1200 J/kg-K is the specific heat of the molten salt. Therefore:

$$M_N c_{v,N}(T_2 - T_1) + M_{s2}[c_{p,s}(T_2 - T_1) - \frac{P_1}{\rho_s}] + M_{s1}\frac{P_1}{\rho_s} = \dot{Q}t_2 \qquad (6)$$

Eq. (6) can be solved for its only unknown T_2:

$$T_2 = T_1 + \frac{\dot{Q}t_2 + (M_{s2} - M_{s1})\dfrac{P_1}{\rho_s}}{M_N c_{v,N} + M_{s2}c_{p,s}} \approx 903.4 \text{ K } (630°\text{C})$$

The final nitrogen pressure is $P_2 = M_N R T_2 / V_{N2} \approx$ 153 kPa, where $V_{N2} = V_{N1} + (M_{1s} - M_{2s})/\rho_s \approx 216.25$ m^3, as the total volume of the pool is fixed.

ii)
The equation of state for the gas yields the following relation between pressure before and after the transient:

$$\frac{P_2}{P_1} = \frac{T_2}{T_1}\frac{V_{N1}}{V_{N2}} \qquad (7)$$

The thermal capacity of the gas is negligible with respect to the thermal capacity of the molten salt, so the temperature change during the transient would not be affected by the type of cover gas, i.e., T_2/T_1 is independent of the gas. Also, the change in gas volume during the transient depends only on the amount of salt extracted, which is not affected by the type of gas. Then Eq. (7) suggests that also the change in pressure P_2/P_1 is independent of the type of gas. Bottom line: final pressure and temperature are expected to be the same whether one uses nitrogen or helium.