# 22.312 ENGINEERING OF NUCLEAR REACTORS

Tuesday, December 16th, 2014, 9:00 a.m. – 12:00 p.m.

OPEN BOOK FINAL EXAM (SOLUTIONS)

**Problem 1 (20%) – Power Uprate in a PWR Core**

i)
The heat flux is axially uniform, thus the location of MDNBR is the channel exit. Recall the definition of the MDNBR:

$$MDNBR \equiv \frac{q''_{DNB}}{q''}$$

In both Approaches A and B the operating heat flux $q''$ is increased by 30% with respect to the reference case. The DNB heat flux $q''_{DNB}$ depends on the local equilibrium quality, mass flux, pressure and equivalent diameter, i.e. $q''_{DNB} = q''_{DNB}(x_e, G, P, D_e)$.

In Approach A the exit temperature (and thus equilibrium quality), pressure and mass flux are unchanged; therefore the $q''_{DNB}$ is also unchanged. As such, the MDNBR is simply reduced by 30%, i.e. MDNBR=1.6/1.3≈1.23.

In Approach B the mass flow rate (and thus the mass flux) has to increase by 30% to maintain the inlet temperature constant, according to the conservation of energy $\dot{Q} = \dot{m}c(T_{out} - T_{in})$. Using the Tong-68 correlation, we can see that an increase by 30% of the mass flux results in an increase of $1.3^{0.4}$ in the DNB heat flux ($q''_{DNB} \propto G^{0.4}$). Thus, for approach B we get the MDNBR=$1.6\times1.3^{0.4}/1.3\approx1.37$.

ii)
The results in Part 'i' suggest that Approach B has a better margin to DNB than Approach A, and in fact the MDNBR for Approach A is below the typical limit mandated by the NRC for U.S. PWRs. Also, Approach A will lower the inlet core temperature, which reduces the thermal efficiency of the plant, clearly undesirable. However, Approach B has its drawbacks: for example, it requires higher pumping power; also, the higher mass flux could cause excessive vibrations of the fuel rods. Likely the best approach is a combination of Approaches A and B that would preserve an acceptable MDNBR without inducing excessive vibrations in the fuel assemblies.

**Problem 2 (15%) – Use of a Spring to Reduce the Cladding Stresses**

The axial force balance for a section of the cladding is as follows:

$$P_{out}\frac{\pi}{4}D^2 - P_{in}\frac{\pi}{4}D^2 + \sigma_z \pi D t - kx = 0 \qquad (1)$$

where $P_{in}$ = 3 MPa, $P_{out}$ = 7 MPa, $D$ = 11.2 mm, $t$ = 0.5 mm, $x$ = 1 cm and $k$ is the (unknown) spring rigidity constant. Setting the axial stress $\sigma_z$ equal to zero and solving for $k$, we get:

$$k = \frac{(P_{out} - P_{in})\frac{\pi}{4}D^2}{x} \approx 39.4 \text{ kN/m}$$

Note that in writing Eq. 1 we assumed the cladding is a thin shell ($R/t$>10). If we do account for the thickness of the cladding in calculating the force due to the pressure forces, then the expression for the rigidity constant becomes:

$$k = \frac{[P_{out} - P_{in}(1-2t/D)^2]\frac{\pi}{4}D^2}{x} \approx 44.4 \text{ kN/m}$$

Thus in this case the thin-shell approximation results in an error of about 11%.

**Problem 3 (65%) – Debris Transport following a LOCA in a PWR Containment**

i)
The minimum mass flow rate is obtained when the coolant is allowed to evaporate completely in the core. The conservation of energy yields:

$$\dot{Q} = \dot{m}(h_g - h_f) = \dot{m}h_{fg} \qquad (2)$$

where $h_{fg}$ = 2257 kJ/kg, $\dot{Q} = 0.066\dot{Q}_0 t^{-0.2}$ ≈ 51.3 MW, $\dot{Q}_0$ =4000 MW and $t$ = 3600 s. Solving Eq. 2 for the mass flow rate, we get $\dot{m} = \dot{Q}/h_{fg}$ ≈ 22.7 kg/s.

ii)
The *terminal* velocity corresponds to zero acceleration, thus is obtained from the force balance (in the vertical direction) for the debris:

$$F_{weight} + F_{bouyancy} + F_{drag} = 0 \Rightarrow \rho_d g \frac{\pi}{6} D^3 - \rho_f g \frac{\pi}{6} D^3 - C_d A_d \rho_f V_d^2 / 2 = 0 \qquad (3)$$

where $\rho_d$ = 3500 kg/m$^3$, $\rho_f$ = 958 kg/m$^3$, $C_d$ =3, $A_d = \pi D^2/4$, $D$=100 μm. Solving Eq. 3 for $V_d$, we get:

$$V_d = \sqrt{\frac{4\left(\frac{\rho_d}{\rho_f} - 1\right) gD}{3C_d}} \approx 3.4 \text{ cm/s}$$

iii)
The debris settling time is equal to the ratio of the sump depth (4 m) to the settling velocity, thus 4/0.034 ≈ 118 s. The flow-through time is equal to the ratio of the sump volume (100 m$^3$) to the volumetric flow rate ($\dot{m}/\rho_f$ ≈0.0237 m$^3$/s), thus 100/0.0237 ≈ 4212 s. Since the settling time is much shorter than the flow-through time, the debris actually settle at the bottom of the sump, which is desirable.

iv)
The pumping power, $\dot{W}_p$, is:

$$\dot{W}_p = \dot{m}\frac{\Delta P_{pump}}{\rho_f} \cdot \frac{1}{\eta_p} \qquad (4)$$

where $\dot{m}$ =23 kg/s, $\eta_p$=0.85 and $\Delta P_{pump}$ is the pressure head provided by the pump. To find $\Delta P_{pump}$, we have to add up the pressure changes in the loop:

$$0 = -\Delta P_{pump} + \rho_f g(H_{bc} - H_{sump}) + \bar{\rho}_{m,core} g H_{core} + K_{core}\frac{(\dot{m}/A_{core})^2}{2\rho_g} - \rho_g g(H_{bc} - H_{sump} + H_{core})$$

$$\Delta P_{pump} = K_{core}\frac{(\dot{m}/A_{core})^2}{2\rho_g} + (\rho_f - \rho_g)g(H_{bc} - H_{sump}) + (\bar{\rho}_{m,core} - \rho_g)gH_{core} \qquad (5)$$

where $K_{core}$ = 150, $A_{core}$ = 6 m$^2$, $H_{bc}$ = 6 m is the bottom core elevation relative to the pump, $H_{sump}$ = 4 m is the depth of the sump, $H_{core}$ = 4 m is the core height; also, we have neglected acceleration pressure changes, and all friction and form losses in the loop except the form loss in the core, per the problem statement. In Eq. 5, the average density in the core is calculated as follows:

$$\bar{\rho}_{m,core} = \frac{1}{H_{core}}\int_0^{H_{core}} \rho_m dz = \frac{1}{H_{core}}\int_0^{H_{core}} [\rho_g \alpha + \rho_f(1-\alpha)]dz = \frac{1}{H_{core}}\int_0^{H_{core}} \frac{1}{x/\rho_g + (1-x)/\rho_f} dz =$$

$$= \therefore \frac{\ln(\rho_f/\rho_g)}{\frac{1}{\rho_g} - \frac{1}{\rho_f}} \approx 4.43 \text{ kg/m}^3$$

where we used HEM with a linear variation of the steam quality in the core ($x=z/H_{core}$), as suggested by the problem statement.

Substituting all numerical values in Eq. 5, we get $\Delta P_{pump} \approx$ 20.7 kPa. Finally, Eq. 4 gives $\dot{W}_p \approx$ 580 W.

v)
Heat is transferred through three thermal resistances in series: steam condensation + conduction in the containment wall + air convection. Thus the heat flux can be calculated as follows:

$$q'' = \frac{T_{steam} - T_{air}}{\frac{1}{h_{steam}} + \frac{\delta_{cont}}{k_{cont}} + \frac{1}{h_{air}}} \approx 2{,}200 \text{ W/m}^2$$

where $T_{steam}$=100°C, $T_{air}$=40°C, $h_{steam}$=800 W/m$^2$°C, $\delta_{cont}$=3 cm, $k_{cont}$=35 W/m°C and $h_{air}$=40 W/m$^2$°C. The total heat rate to be transferred to condense $\dot{m}$ =23 kg/s of steam is $\dot{Q} = \dot{m}h_{fg}$ ≈51.9 MW. Therefore, the minimum containment area required is $A_{cont} = \dot{Q}/q''$≈23,400 m$^2$, which is very large, and would result in an enormous (un-economical) containment.

vi)
Since the dominant thermal resistance is on the air side, any design modification should aim at reducing that thermal resistance. For example, one could implant ribs on the outer surface to increase the heat transfer area exposed to air, or enhance the heat transfer coefficient by actively blowing the air on the containment shell, or increase the heat transfer coefficient by dripping water on the outer containment shell, to take advantage of evaporative cooling. The latter approach is actually implemented in the AP1000 passive containment cooling system.

vii)
At the conditions of interest, water occupies the whole containment volume:

$$V_c = M_w[v_f(T_{sat})(1-x) + v_g(T_{sat})x]$$

where $V_c$ = 60,000 m$^3$ , $T_{sat}$=100°C, $v_f$ = 0.001 m$^3$/kg and $v_g$ =1.67 m$^3$/kg are the specific volumes of saturated liquid water and steam, respectively, and $x$ is the static quality in the containment. Solving for $x$, we get $x \approx 0.119$. Therefore, the volume occupied by liquid water is $V_f = M_w v_f (1-x) \approx 276$ m$^3$ and that of steam is $V_g = V_c - M_w v_f (1-x) \approx 59{,}724$ m$^3$.

viii)
The total pressure in the containment is the sum of the partial pressures of air and water:

$$P_c = P_w + P_{air} = P_w(T_{sat}) + \frac{M_a R T_{sat}}{V_g} \approx 225 \text{ kPa}$$

where $P_w(T_{sat})$ =101 kPa, $M_a$ = 7×10$^4$ kg and $R$=286 J/kg-K.

22.312 Engineering of Nuclear Reactors
Fall 2015