**22.101** Applied Nuclear Physics (Fall 2006)

**Lecture 14 (11/1/06)**

**Charged-Particle Interactions: Stopping Power, Collisions and Ionization**

**References**:

R. D. Evans, *The Atomic Nucleus* (McGraw-Hill, New York, 1955), Chaps 18-22.

W. E. Meyerhof, *Elements of Nuclear Physics* (McGraw-Hill, New York, 1967).

When a swift charged particle enters a materials medium it interacts with the electrons and nuclei in the medium and begins to lose energy as it penetrates into the material. The interactions can be thought of as individual *collisions* between the charged particle and the atomic electrons surrounding a nucleus or the nucleus itself (considered separately). The energy given off during these collisions will result in *ionization*, the production of ion-electron pairs, in the medium; also it can appear in the form of *electromagnetic radiation*, a process known as *bremsstrahlung* (braking radiation). We are interested in describing the energy loss per unit distance traveled by the charged particle, and the range of the particle in various materials. The range is defined as the distance traveled from the point of entry to the point of essentially coming to rest.

A charged particle is called 'heavy' if its rest mass is large compared to the rest mass of the electron. Thus, mesons, protons, $\alpha$-particles, and of course fission fragments, are all heavy charged particles. By the same token, electrons and positrons are 'light' particles.

If we ignore nuclear forces and consider only the interactions arising from Coulomb forces, then we can speak of four principal types of charged-particle interactions:

(i)     Inelastic Collision with Atomic Electrons. This is the principal process of energy transfer, particularly if the particle velocity is below the level where bremsstrahlung is significant. It leads to the excitation of the atomic electrons (still bound to the nucleus) and to ionization (electron stripped off the nucleus). Inelastic here refers to the excitation of electronic levels.

(ii)     Inelastic Collision with a Nucleus.  This process can leave the nucleus in an excited state or the particle can radiate (bremsstrahlung).

(iii)     Elastic Collision with a Nucleus.  This process is known as Rutherford scattering.  There is no excitation of the nucleus, nor emission of radiation. The particle loses energy only through the recoil of the nucleus.

(iv)     Elastic Collision with Atomic Electrons.  The process is elastic deflection which results in a small amount of energy transfer.  It is significant only for charged particles that are low-energy electrons.

In general, interaction of type (i), which is sometimes simply called collision, is the dominant process of energy loss, unless the charged particle has a kinetic energy exceeding its rest mass energy, in which case the radiation process, type (ii), becomes important.  For heavy particles, radiation occurs only at such kinetic energies, $\sim 10^3$ Mev, that it is of no practical interest to nuclear engineers.  The characteristic behavior of electron and proton energy loss in a high-Z medium like lead is shown in Fig. 13.1 [Meyerhof Fig. 3.7].  This is an important result to keep in mind for the discussions to follow.
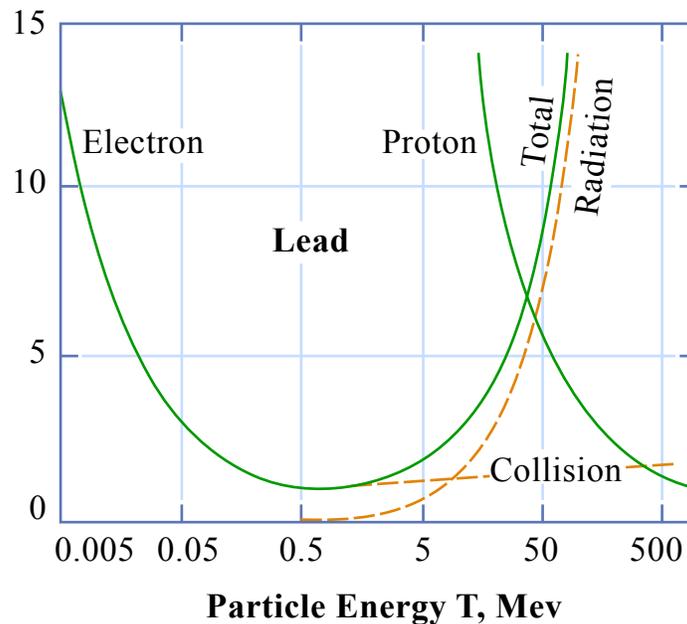


Figure by MIT OCW. Adapted from Burcham, 1963.

**Fig. 13.1.**  Stopping power (vertical axis in units of Mev/g-cm$^2$) showing the contributions from collisions and radiation.

*Stopping Power:  Energy Loss of Charged Particles in Matter*

The kinetic energy loss per unit distance suffered by a charged particle, to be denoted as $-dT/dx$, is conventionally known as the *stopping power*. This is a positive quantity since $dT/dx$ is $<0$. There are quantum mechanical as well as classical theories for calculating this basic quantity. One wants to express $-dT/dx$ in terms of the properties specifying the incident charged particle, such as its velocity v and charge ze, and the properties pertaining to the atomic medium, the charge of the atomic nucleus Ze, the density of atoms n, and the average ionization potential $\bar{I}$ .

We consider only a crude, approximate derivation of the formula for $-dT/dx$. We begin with an estimate of the energy loss suffered by an incident charged particle when it interacts with a free and initially stationary electron. Referring to a collision cylinder whose radius is the impact parameter b and whose length is the small distance traveled dx shown in Fig. 13.2 [Meyerhof Fig. 3-1], we see that the net momentum transferred to the
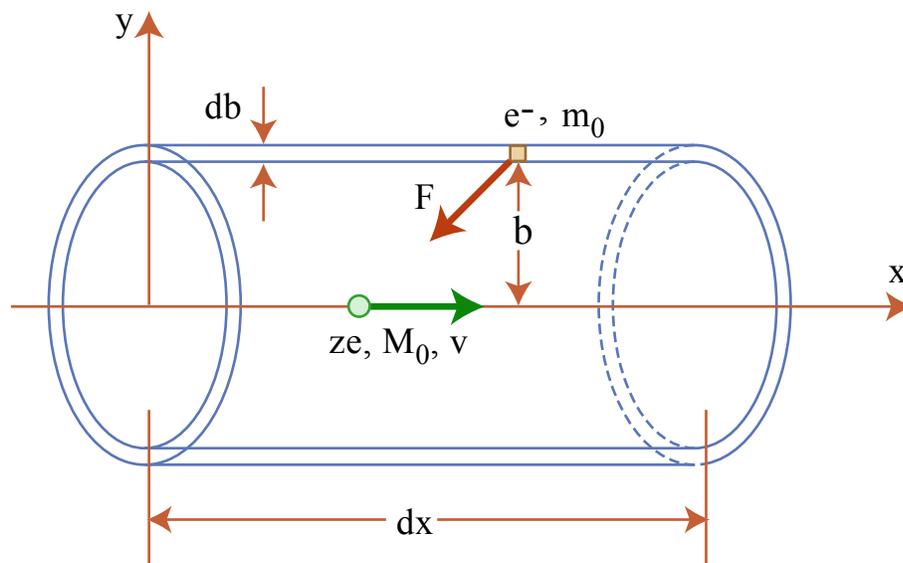


Figure by MIT OCW. Adapted from Meyerhof.

**Fig. 13.2.** Collision cylinder for deriving the energy loss to an atomic electron by an incident charged particle.

electron as the particle moves from one end of the cylinder to the other end is essentially entirely directed in the perpendicular direction (because $F_x$ changes sign, so the net momentum along the horizontal direction vanishes) along the negative y-axis. So we write

$$\int dt F_x(t) \approx 0 \tag{13.1}$$

$$p_e = \int dt F_y(t)$$

$$= \int \frac{ze^2}{x^2 + b^2} \frac{b}{\left(x^2 + b^2\right)^{1/2}} \frac{dx}{v}$$

$$\cong \frac{ze^2 b}{v} \int_{-\infty}^{\infty} \frac{dx}{\left(x^2 + b^2\right)^{3/2}} = \frac{2ze^2}{vb} \tag{13.2}$$

The kinetic energy transferred to the electron is therefore

$$\frac{p_e^2}{2m_e} = \frac{2(ze^2)^2}{m_e b^2 v^2} \tag{13.3}$$

If we assume this is equal to the energy loss of the charged particle, then multiplying (13.3) by $nZ(2\pi bdbdx)$, the number of electrons in the collision cylinder, we obtain

$$-\frac{dT}{dx} = \int_{b_{min}}^{b_{max}} nZ 2\pi bdb \frac{2}{m_e} \left(\frac{ze^2}{vb}\right)^2$$

$$= \frac{4\pi(ze^2)^2 nZ}{m_e v^2} \ell n\left(\frac{b_{max}}{b_{min}}\right) \tag{13.4}$$

where $b_{max}$ and $b_{min}$ are the maximum and minimum impact parameters which one should specify according to the physical description he wishes to treat.

In reality the atomic electrons are of course not free electrons, so the charged particle must transfer at least an amount of energy equal to the first excited state of the atom. If we take the time interval of energy transfer to be $\Delta t \approx b/v$, then $(\Delta t)_{max} \sim 1/v$, where $h\nu \approx \bar{I}$ is the mean ionization potential. This gives an estimate of the maximum impact parameter,

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$$b_{max} \approx hv/\bar{I} \tag{13.5}$$

An empirical expression for $\bar{I}$ is $\bar{I} \approx kZ$, with k ~ 19 ev for H and ~ 10 ev for Pb.  Next we estimate $b_{min}$ by using the uncertainty principle to say that the electron position cannot be specified more precisely than its de Broglie wavelength (in the relative coordinate system of the electron and the charged particle).  Since electron momentum in the relative coordinate system is $m_e v$, we have

$$b_{min} \approx h/m_e v \tag{13.6}$$

Combining these two estimates we obtain

$$-\frac{dT}{dx} = \frac{4\pi z^2 e^4 nZ}{m_e v^2} \ell n\left(\frac{2m_e v^2}{\bar{I}}\right) \tag{13.7}$$

In (13.7) we have inserted a factor of 2 in the argument of the logarithm, this is to make our formula agree with the result of quantum mechanical calculation which was first carried out by H. Bethe using the Born approximation.

Eq.(13.7) describes the energy loss due to particle collisions in the nonrelativistic regime.  One can include relativistic effects by replacing the logarithm by

$$\ell n\left(\frac{2m_e v^2}{\bar{I}}\right) - \ell n\left(1 - \frac{v^2}{c^2}\right) - \frac{v^2}{c^2}$$

This correction can be important in the case of electrons and positrons.

Eq.(13.7) is a relatively simple expression, yet one can gain much insight into the factors that govern the energy loss of a charged particle by collisions with the atomic electrons.  We can see why the usual neglect of the contributions due to collisions with nuclei is justified.  In a collision with a nucleus the stopping power would increase by a

factor Z, because of the charge of the target with which the incident charged particle is colliding, and decrease by a factor of $m_e/M(Z)$, where $M(Z)$ is the mass of the atomic nucleus. The decrease is a result of the larger mass of the recoiling target. Since Z is always less than $10^2$ whereas $M(Z)$ is at least a factor $2 \times 10^3$ greater than $m_e$, the mass factor always dominates over the charge factor. Another useful observation is that (13.7) is independent of the mass of the incident charged particle. This means that nonrelativistic electrons and protons of the same velocity would lose energy at the same rate, or equivalently the stopper power of a proton at energy T is about the same as that of an electron at energy $\sim T/2000$. That this scaling is more or less correct can be seen in Fig. 13.1.

Fig. 13.3 shows an experimentally determined energy loss curve (stopping power) for a heavy charged particle (proton), on two energy scales, an expanded low-energy region where the stopping power decreases smoothly with increasing kinetic energy of the charged particle T below a certain peak centered about 0.1 Mev, and a more compressed high-energy region where the stopping power reaches a broad minimum around $10^3$ Mev. Notice also a slight upturn as one goes to higher energies past the broad minimum which we expect is associated with relativistic corrections. One should regard Fig. 13.3 as the extension, at both ends of the energy, of the curve for proton in Fig. 13.1.
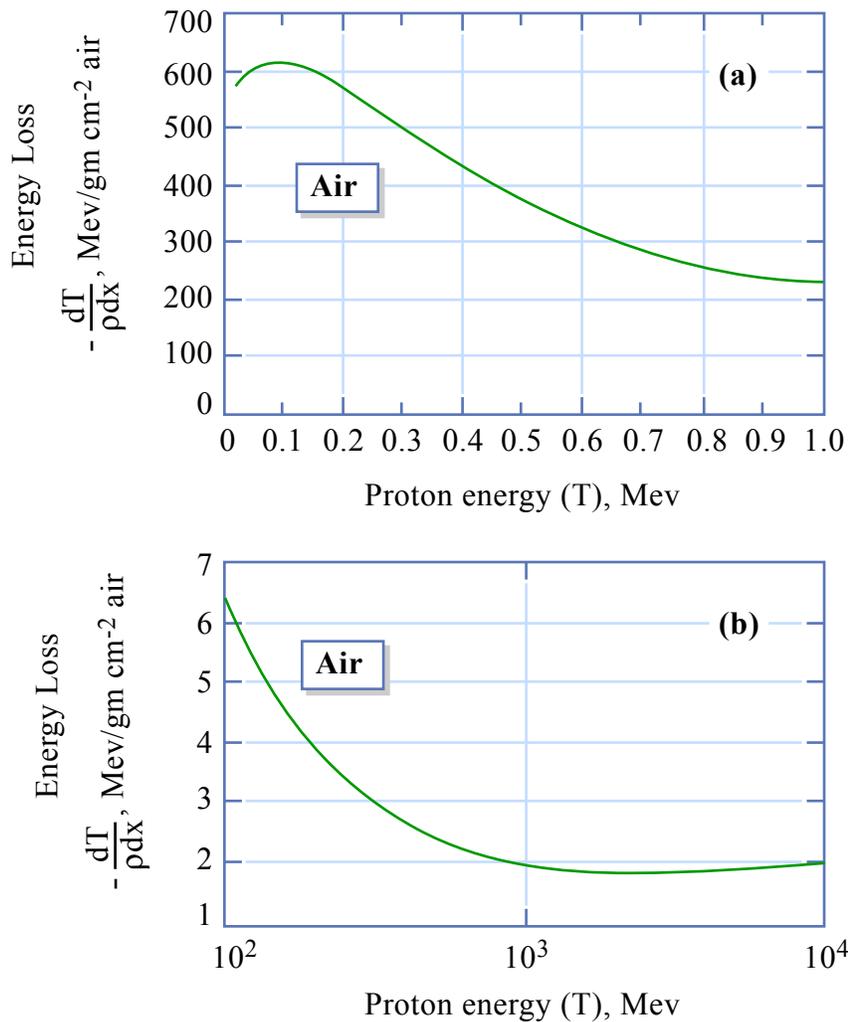
Figure by MIT OCW. Adapted from Meyerhof.

**Fig. 13.3.** The experimentally determined stopping power, (-dT/dx), for protons in air, (a) low-energy region where the Bethe formula applies down to T ~ 0.3 Mev with $\bar{I}$ ~ 80 ev. Below this range charge loss due to electron capture causes the stopper power to reach a peak and start to decrease (see also Fig. 13.4 below), (b) high-energy region where a broad minimum occurs at T ~ 1500 Mev. [from Meyerhof]

Experimentally, collisional energy loss is measured through the number of ion pairs formed along the trajectory path of the charged particle. Suppose a heavy charged particle loses on the average an amount of energy W in producing an ion pair, an electron and an ion. Then the number of ion pairs produced per unit path is

$$i = \frac{1}{W}\left(-\frac{dT}{dx}\right) \tag{13.8}$$

We will say more about the energy W in the next lecture.

Eq.(13.7) is valid only in a certain energy range because of the assumptions we have made in its derivation. We have seen from Figs. 13.1 and 13.3 that the atomic stopping power varies with energy in the manner sketched below. In the intermediate
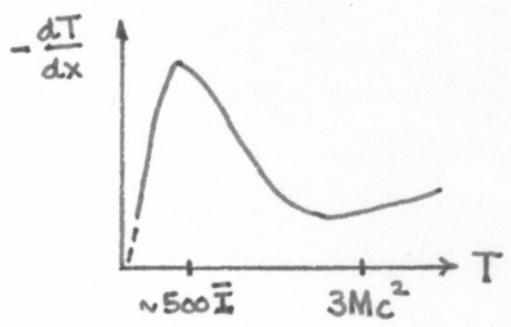


**Fig. 13.4.** Schematic of overall behavior of stopping power of a charged particle with rest mass M.

energy region, $500\bar{I} < T \le Mc^2$, where M is the mass of the charged particle, the stopping power behaves like 1/T, which is roughly what is predicted by (13.7). In this region the relativistic correction is small and the logarithm factor varies slowly. At higher energies the logarithm factor along with the relativistic correction terms give rise to a gradual increase so that a broad minimum is set up in the neighborhood of $\sim 3\,Mc^2$. At energies below the maximum in the stopping power, $T < 500\bar{I}$, Eq.(13.7) is not valid because the charged particle is moving slow enough to capture electrons and begin to lose its charge. Fig. 13.5 [Meyerhof Fig. 3-3] shows the correlation between the mean charge
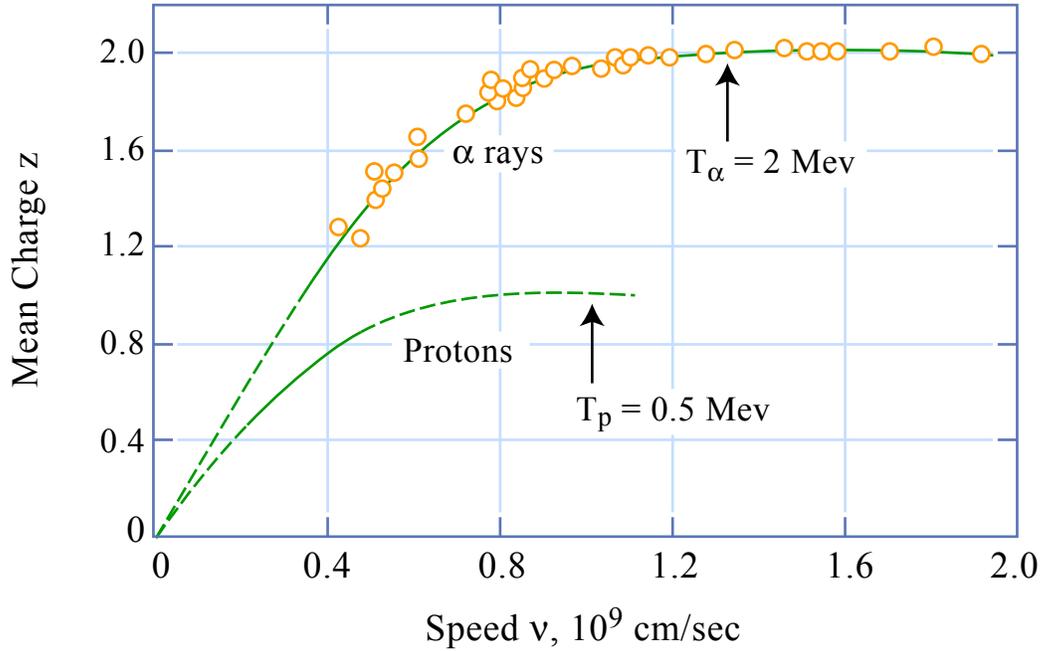
Figure by MIT OCW. Adapted from Evans, 1955.

**Fig. 13.5.** Loss of charge with speed as a charged particle slows down.

of a charged particle and its velocity. This is a difficult region to analyze theoretically. For $\alpha$-particles and protons the range begins at $\sim 1$ Mev and 0.1 Mev respectively [H. A. Bethe and J. Ashkin, "Passage of Radiation Through Matter", in *Experimental Nuclear Physics*, E. Segrè, ed (Wiley, New York, 1953), Vol. I, p.166].

Eq.(13.7) is generally known as the Bethe formula. It is a quantum mechanical result derived on the basis of the Born approximation which is essentially an assumption of weak scattering [E. J. Williams, Rev. Mod. Phys. **17**, 217 (1945)]. The result is valid provided

$$\frac{ze^2}{\hbar v} = \left(\frac{e^2}{\hbar c}\right)\frac{z}{v/c} = \frac{z}{137(v/c)} << 1 \tag{13.9}$$

On the other hand, Bohr has used classical theory to derive a somewhat similar expression for the stopping power,

$$-\left(\frac{dT}{dx}\right)_{class} = \frac{4\pi z^2 e^4 nZ}{m_e v^2} \ell n\left[\frac{M\hbar v}{2ze^2(m_e + M)}\frac{2m_e v^2}{\bar{I}}\right] \tag{13.10}$$

which holds if

$$\frac{ze^2}{\hbar v} \gg 1 \tag{13.11}$$

Thus (13.7) and the classical formula apply under opposite conditions. Notice that the two expressions agree when the arguments of the logarithms are equal, that is, $2ze^2/\hbar v = 1$, which is another way of saying that their regions of validity do not overlap. According to Evans (p. 584), the error of either approximation tends to be an overestimate, so the expression that gives the smaller energy loss is likely to be the more correct. This turns out to be the classical expression when z > 137(v/c), and (13.7) when 2z < 137(v/c). Knowing the charge of the incident particle and its velocity, one can use this criterion to choose the appropriate stopper power formula. In the case of fission fragments (high Z nuclides) the classical result should be used. Also, it should be noted that a quantum mechanical theory has been developed by Bloch that gives the Bohr and Bethe results as limiting cases.

The Bethe formula,(13.7), is appropriate for heavy charged particles. For fast electrons (relativistic) one should use

$$-\frac{dT}{dx} = \frac{2\pi e^4 nZ}{m_e v^2}\left[\ell n\left(\frac{m_e v^2 T}{\bar{I}^2(1-\beta^2)}\right) - \beta^2\right] \tag{13.12}$$

where $\beta = v/c$. For further discussions see Evans.