MHD

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What is MHD

- MHD stands for magnetohydrodynamics
- MHD is a simple, self-consistent fluid description of a fusion plasma
- Its main application involves the macroscopic equilibrium and stability of a plasma

Equilibrium and Stability

- Why separate the macroscopic behavior into two pieces?
- Even though MHD is simple, it is still involves nonlinear 3-D + time equations
- This is tough to solve
- Separation simplifies the problem
- Equilibrium requires 2-D non-linear time independent
- Stability requires 3-D+ time, but is linear
- This enormously simplifies the analysis

What is Equilibrium

- We must design a magnet system such that the plasma is in steady state force balance
- So far tokamaks are the best design
- The spherical torus is another option
- The stellarator is yet another option
- Each can provide force balance for a reasonably high plasma pressure

What is Stability?

- In general a plasma equilibrium may be stable or unstable

- Stability is good!

- Instability is bad!

Examples of Stability

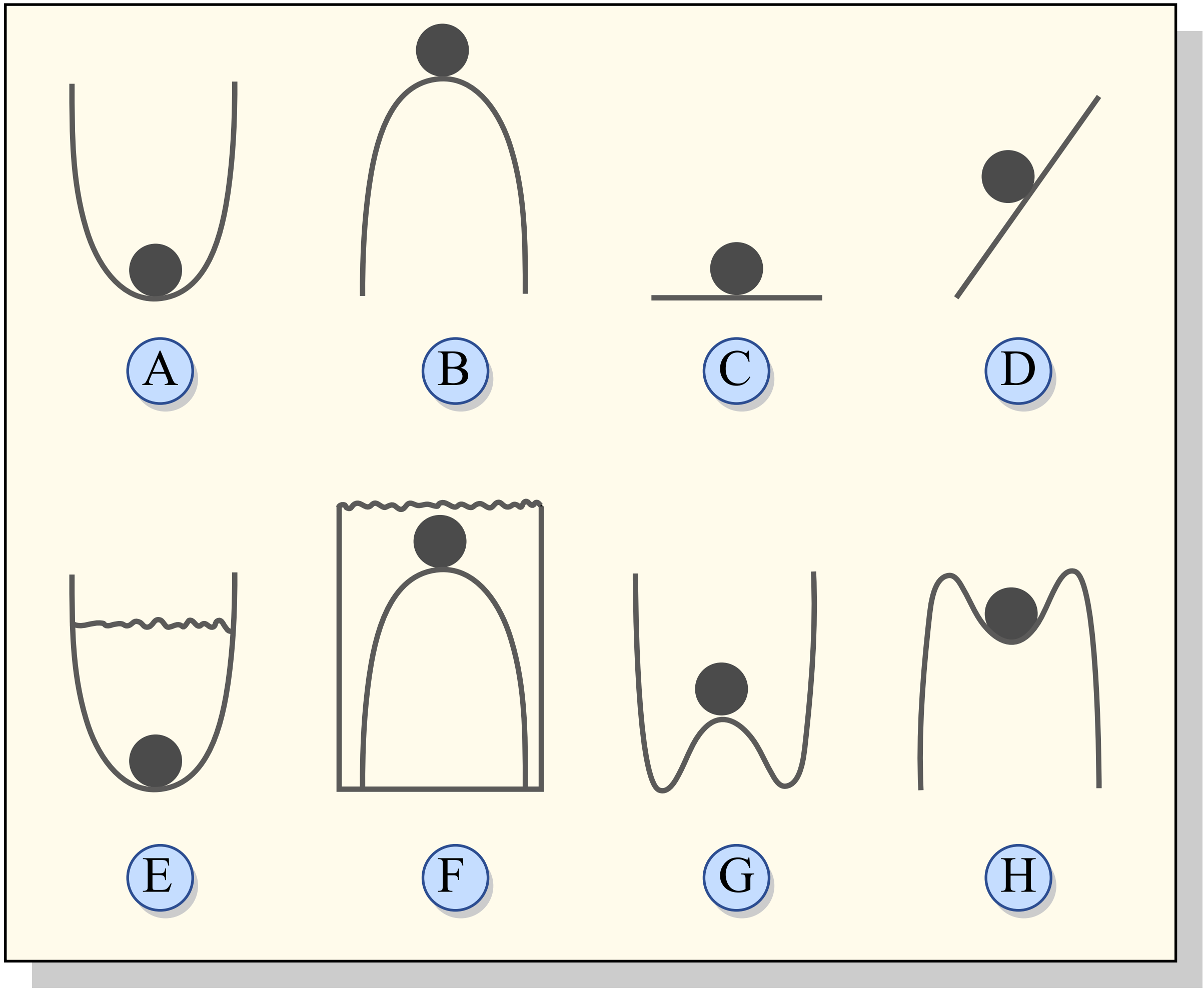


Figure by MIT OCW.

Effects of an MHD Instability

- Usually disastrous
- Plasma moves and crashes into the wall
- No more fusion
- No more wall (in a reactor)
- This is known as a major disruption

The Job of MHD

- Find magnetic geometries that stably confine high pressure plasmas
- Large amount of theoretical and computational work has been done
- Well tested in experiments

Current status of fusion MHD

- Some say there is nothing left to do in fusion MHD

    a. The theory is essentially complete
    b. Computational tools are readily available
    c. Used routinely in experiments

- There is some truth in this view

- But not really – there are major unsolved MHD problems

What do we know?

- MHD equilibrium in 2-D and 3-D
- MHD stability pressure limits (β)
- MHD stability current limits (q_*)
- MHD stability shaping limits (κ)
- Plasma engineering coil design

What don’t we know?

- Resistive wall mode
- Plasma rotation
- High bootstrap current
- Edge localized modes
- Neoclassical tearing modes

Goals of the Talk

- Tell the story of fusion
- Show how the unsolved MHD problems fit into the story
- Discuss the paths to a perhaps happy ending

Outline of the Talk

- Design a tokamak fusion reactor
- Describe the current status of the tokamak
- Describe one crucial unsolved problem
- Show how the unsolved MHD problems enter the picture
- Show how we might proceed into the future

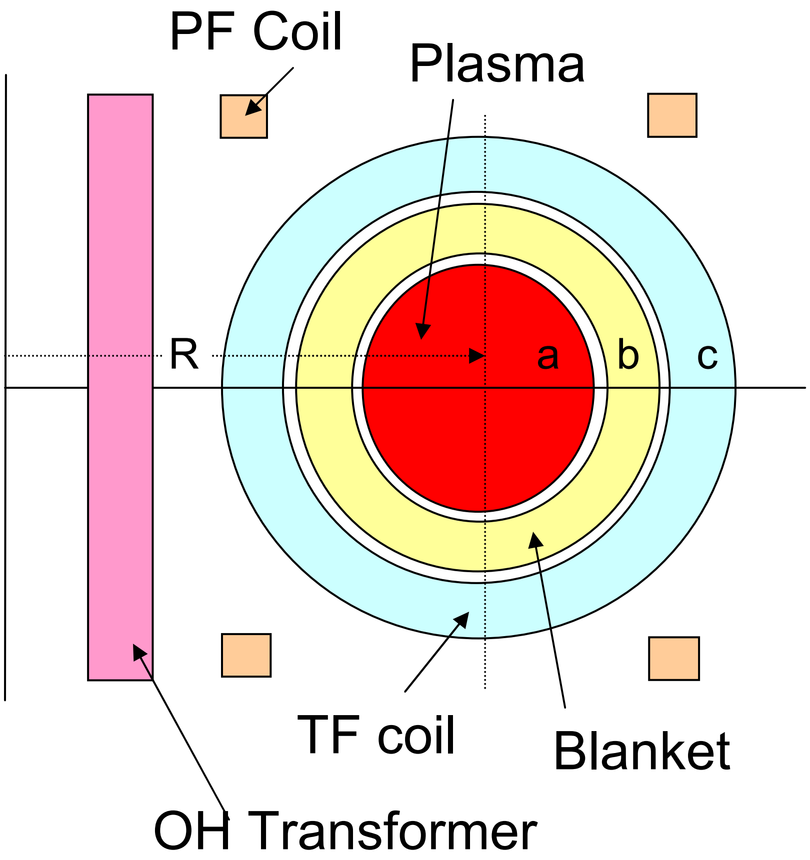
A Tokamak Fusion Reactor

- Based on the *D-T* reaction

  $D + T \rightarrow n + \alpha + 17.6\ MeV$

- *neutrons* escape and produce heat and electricity

- *alphas* stay confined and balance χ heat loss



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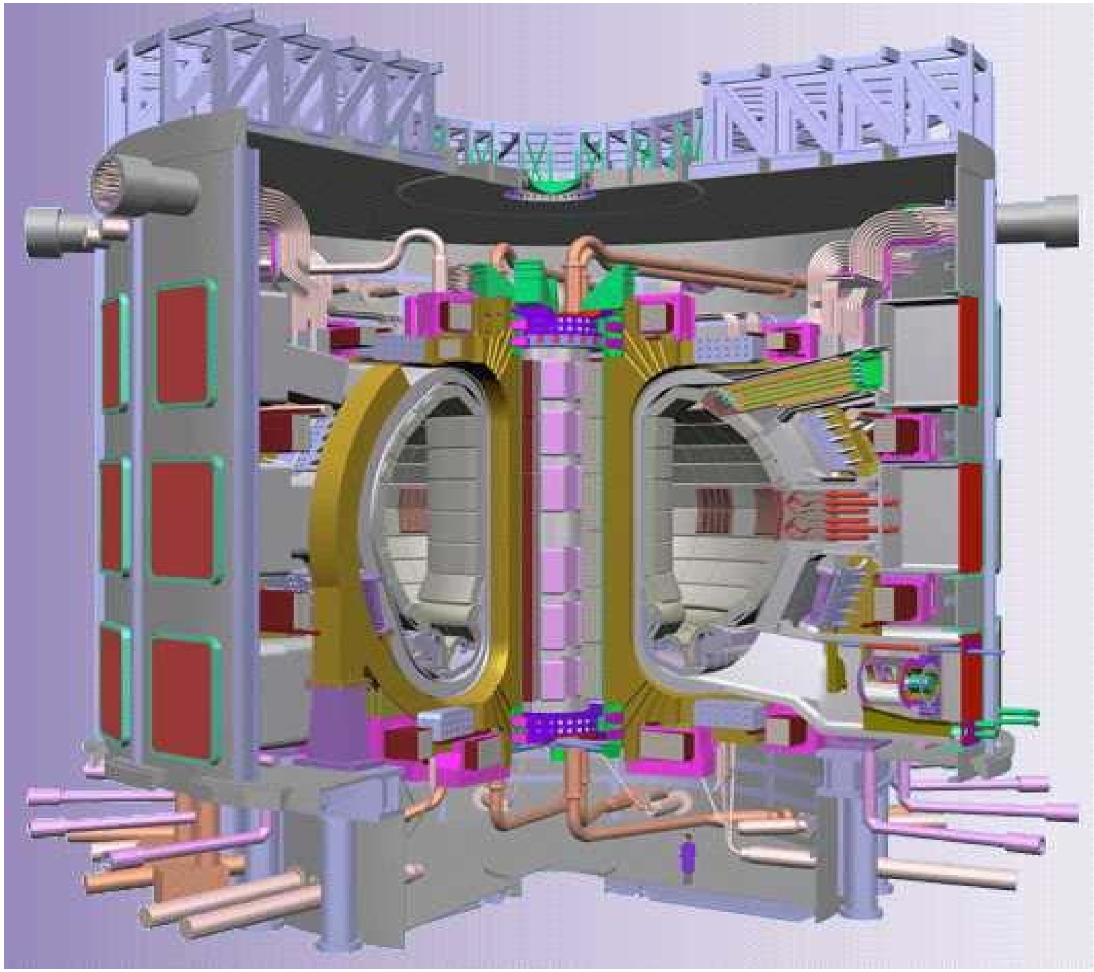

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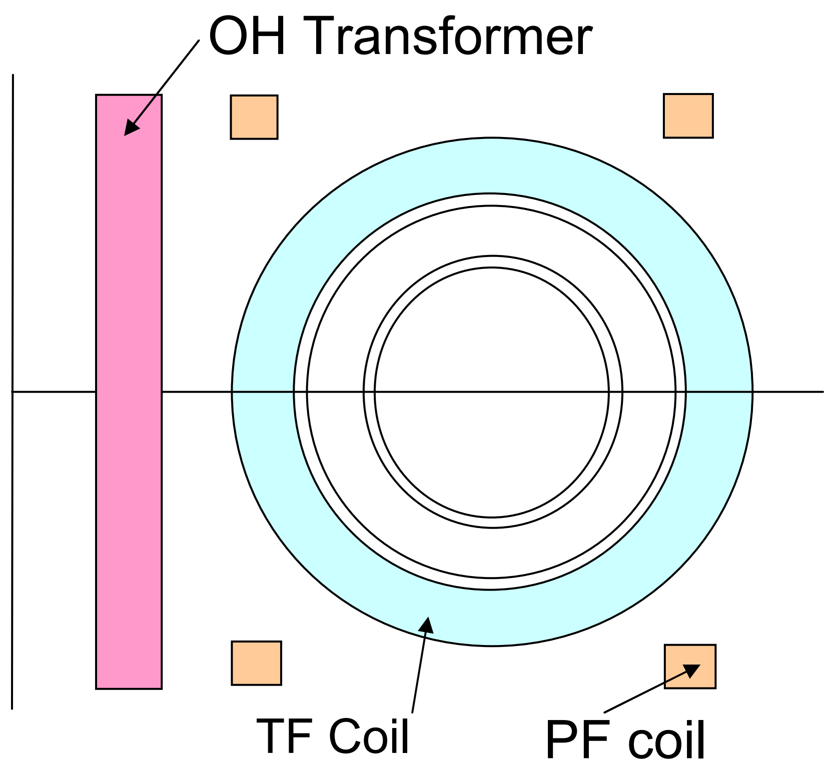
A Tokamak Fusion Reactor

- TF coils produce stabilizing toroidal magnetic field
- PF coils produce equilibrium poloidal magnetic field
- OH transformer induces toroidal plasma current



Design Strategy

- Minimize the Cost/Watt subject to
- Nuclear physics constraints: $\lambda_{mfp} \approx$ few cm
- Magnet Constraints: $B_{max} = 13\ T$, $\sigma_{max} = 300\ MPa$
- Wall loading constraint: $P_w = P_{neutron}/A = 4\ MW/m^2$
- Output power constraint: $P_E = \eta P_{fusion} = 1000\ MW$
- Self sustaining constraint: $P_\alpha = P_{loss}$
- Determine: a, R_0, T, p, n, τ_E
- Design is almost independent of plasma physics and MHD

Minimum Cost/Watt

- Minimum cost/watt is proportional to the volume of reactor material per watt of electricity
- Volume dominated by the blanket/shield and TF coils
- Minimize V/P_E

$$\frac{V}{P_E} = \frac{2\pi^2 R_0 \left[\left(a + b + c \right)^2 - a^2 \right]}{P_E}$$

Design

- Nuclear physics constraints: $b = 1.2\ m$
- Magnet Constraints: $c = 0.25(a + b)$
- Optimize $V/P_E(a)$: $a = 2\ m$, $c = 0.8\ m$
- Wall loading constraint: $R_0 = 0.04 P_E / a P_W = 5\ m$
- Output power constraint: $T = 15\ keV$, $p = 7\ atm$
- Self sustaining constraint: $\tau_E = 1.2\ sec$

Comparisons of Parameters

<u>Reactor</u>

- a = 2 m
- R_0 = 5 m
- T = 15 keV
- p = 7 atm
- n = 1.2 x 10^{20} m^{-3}
- τ_E = 1.2 sec

<u>Steady State ITER</u>

- a = 2.3 m
- R_0 = 8.7 m
- T = 13 keV
- p = 5 atm
- n = 1.0 x 10^{20} m^{-3}
- τ_E = 2.5 sec

Summary of Plasma Requirements

• $T = 15\ keV$	The RF community
• $p = 7\ atm$	The MHD community
• $\tau_E = 1.2$ sec	The transport community

Where do we stand now?

- Heating: Tokamaks have already achieved

$$T \approx 30 \ keV$$

- Should extrapolate to a fusion reactor

Where do we stand now?

- Pressure: Pressure is normally measured in terms of β

$$\beta = \frac{2\mu_0 p}{B_0^2} \qquad B_0 = B_{\max}\left(1 - \frac{a+b}{R_0}\right)$$

- Existing tokamaks have achieved $\beta \sim 10\%$ although at lower magnetic fields

- In a reactor $p = 7$ *atm* corresponds to $\beta \sim 8\%$

Where do we stand now?

- Energy confinement time: τ_E is usually determined empirically

$$\tau_E = 0.26 \frac{I_M^{1.06} R_0^{1.39} a^{0.58} n_{20}^{0.41} B_0^{0.41}}{P_M^{0.69}}$$

- Experiments have achieved $\tau_E \sim 0.3$ sec at lower magnetic fields

- In a reactor $\tau_E \sim 1$ sec should be achievable (requires I_M = 17 MA)

Doesn't this mean we have succeeded?

- No!!!

- There is one crucial unsolved reactor problem

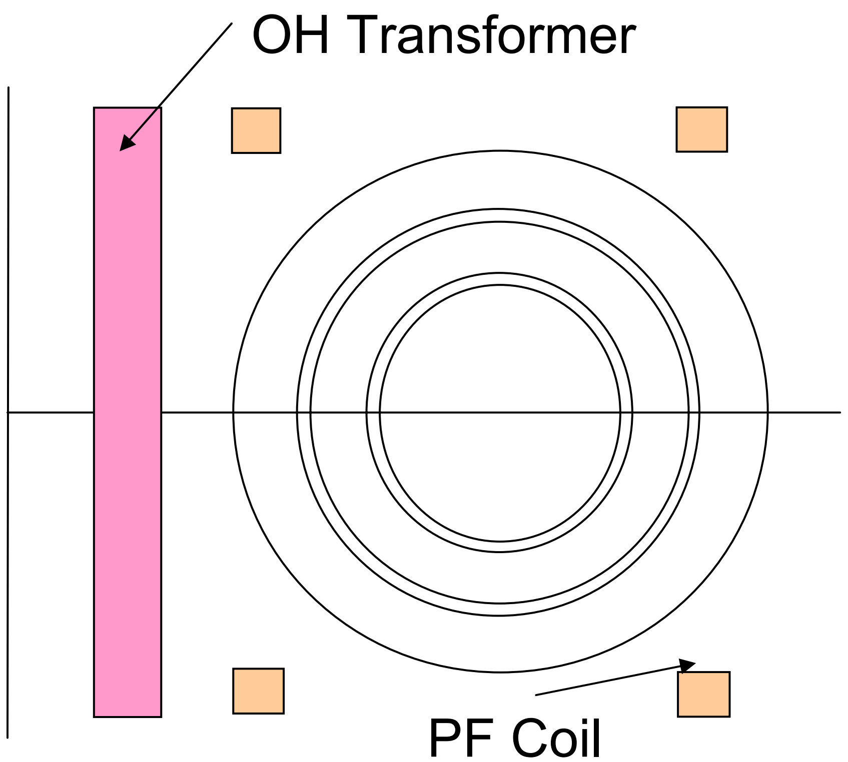
- A power reactor should be a steady state device, not a pulsed device

- Simple tokamaks are inherently pulsed devices because of the OH transformer

Why are tokamaks pulsed devices?

- To hold the plasma in equilibrium an OH-PF field system is needed

- The toroidal current is normally driven by a transformer which is inherently pulsed



How can we resolve this problem?

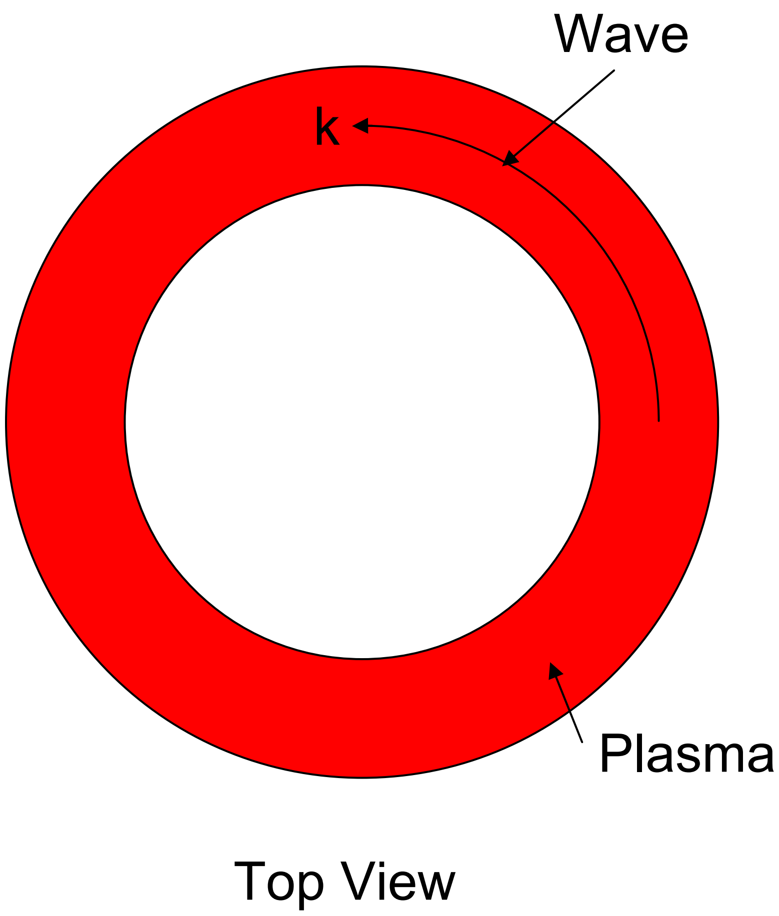
- Approach #1 Advanced tokamak operation

- Approach #2 The stellarator

The Advanced Tokamak

- The advanced tokamak achieves steady state by non-inductive current drive

- Directed RF waves trap and drag electrons with the wave generating a current



This works but…

- Current drive efficiency is low

$$P_{RF}(watts) \approx 10\ I_{CD}(amps)$$

- For our reactor I = 17 MA

- With efficiencies this implies $P_{RF} \approx 340\ MW$

- This is unacceptable from an economic point of view

Is there any way out?

- Possibly
- In a torus there is a naturally driven transport current
- It is known as the bootstrap current J_B
- No current drive is required
- If enough J_B current flows (75%), the current drive requirements can be dramatically reduced

How much bootstrap current flows?

- The formula for the bootstrap fraction is

$$f_B \equiv \frac{I_B}{I} \approx \frac{1}{3}\frac{\beta q_*^2}{\varepsilon^{3/2}} \propto \frac{a^{5/2}}{R_0^{1/2}}\frac{p}{I^2}$$

$$q_* = \frac{5a^2 B_0}{R_0 I_M} \sim \frac{1}{I_M}$$

$$\varepsilon = \frac{a}{R_0}$$

- To make $f_B = 75\%$ requires a combination of high pressure and low current

Are there limits on *p* and *I*?

- Yes!!
- If they are violated a major disruption can occur
- A catastrophic collapse of the *p* and *I*
- Major disruptions must be avoided in a reactor or ITER

Specific Plasma limitations

- I_{max} is limited by kink stability condition

$$q_* \equiv \frac{5a^2 \kappa B_0}{I_M} \geq 2$$

- The elongation κ is limited by vertical instabilities

$$\kappa < 2$$

- But I_{min} is limited by transport: $\tau_E \sim I_M$
- Beta is limited by the no wall Troyon limit

$$\beta < \beta_N \frac{I_M}{aB_0} \qquad \beta_N \approx 0.03$$

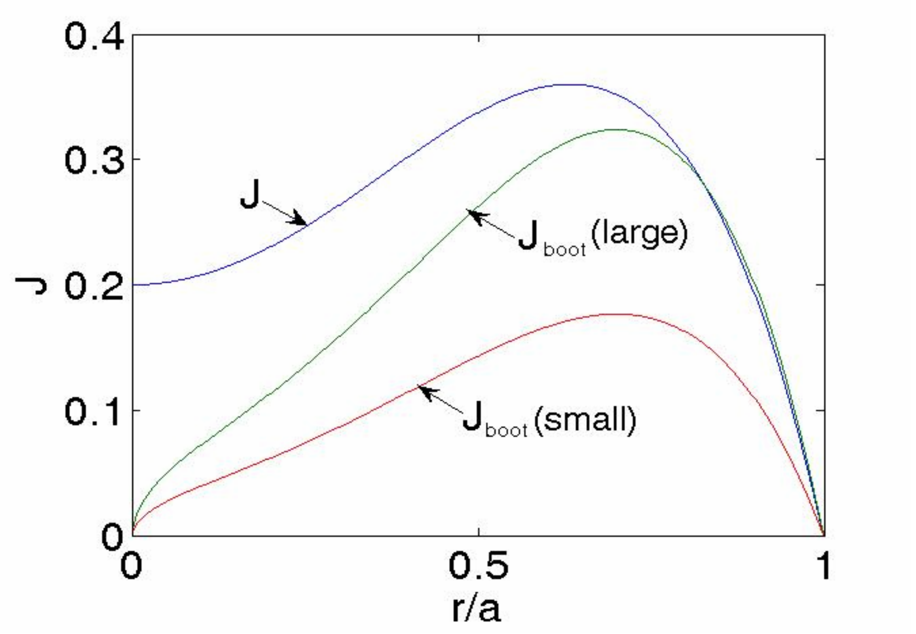
- This generates too low a bootstrap fraction

What do we do now?

- The best approach:
  Hollow J profiles
  Perfectly conducting wall

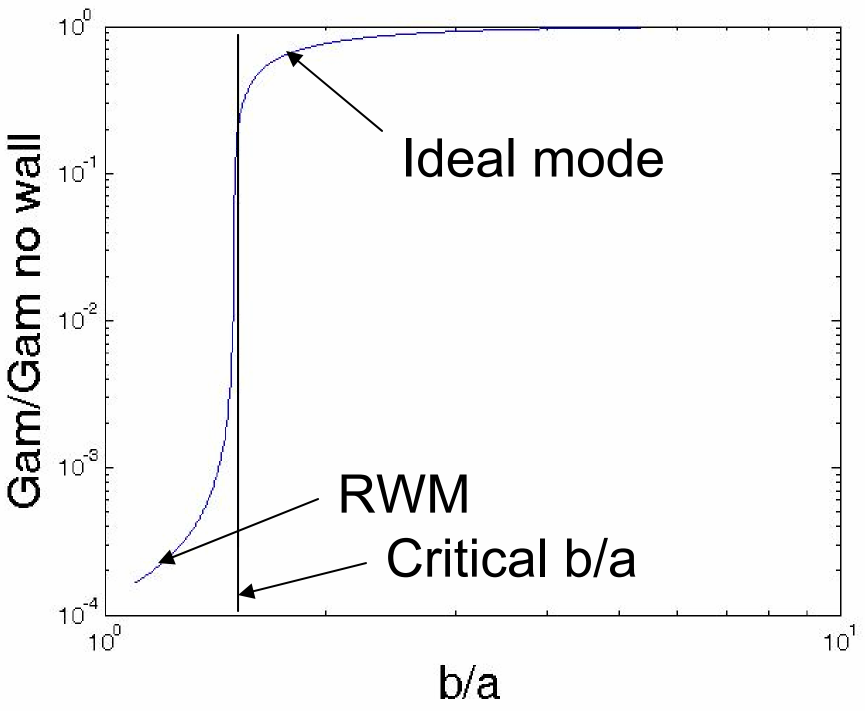
- Hollow J means less I_M

- A conducting wall raises the β limit by as much as a factor of 2



But the wall has a finite conductivity

- A finite σ wall slows down γ, but leaves β_{crit} the same as without a wall
- This slow growing mode is known as the resistive wall mode
- It is a <u>major impediment</u> to steady state operation



Can the resistive wall mode be stabilized?

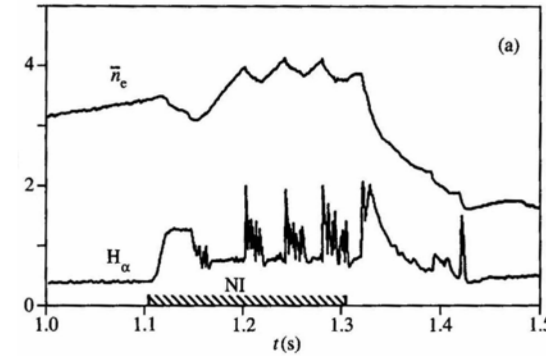
- Feedback may work but is somewhat complicated
- Plasma flow can stabilize the mode but high flow velocities $v \sim v_{thermal}$ are needed and are difficult to initiate and maintain
- Plasma kinetic effects may also play an important role
- This is a crucial unresolved problem in tokamak research

Any other MHD reactor problems?

- Edge localized modes (ELMs)
- These are bursts of plasma energy from the plasma edge that occur when the pressure gets too high
- MHD modes driven by the edge ∇p and J
- The edge acts like a pressure relief valve
- This should be a good way to control and stabilize the edge plasma pressure

But

- There are several types of ELMs
- Most are bad
- Type I = bad
- Type II = good
- Type III = bad
- Difficult to predict which type ELMs will be present
- Another very important MHD problem



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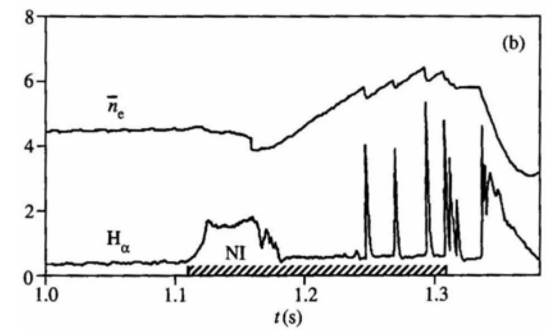

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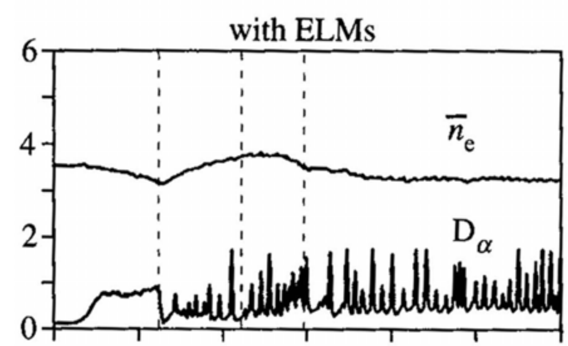


II

Any more problems?

- The neoclassical tearing mode (NTM)
- Resistive tearing mode including toroidal trapped particle effects
- Requires a finite seed island to grow (e.g. due to sawteeth)
- NTMs can be excited at lower β than ideal MHD modes
- The m=3/n=2 mode can lead to enhanced transport
- The m=2/n=1 mode can lead to disruptions

Preventing NTMs

- Preventing NTMs #1: Reduce seed island by sawtooth destabilization (e.g. ICCD at the q = 1 surface)

- Preventing NTMs #2: Reduce island width by external control (e.g. ECCD at q = 3/2 surface)

Happy endings for AT problems?

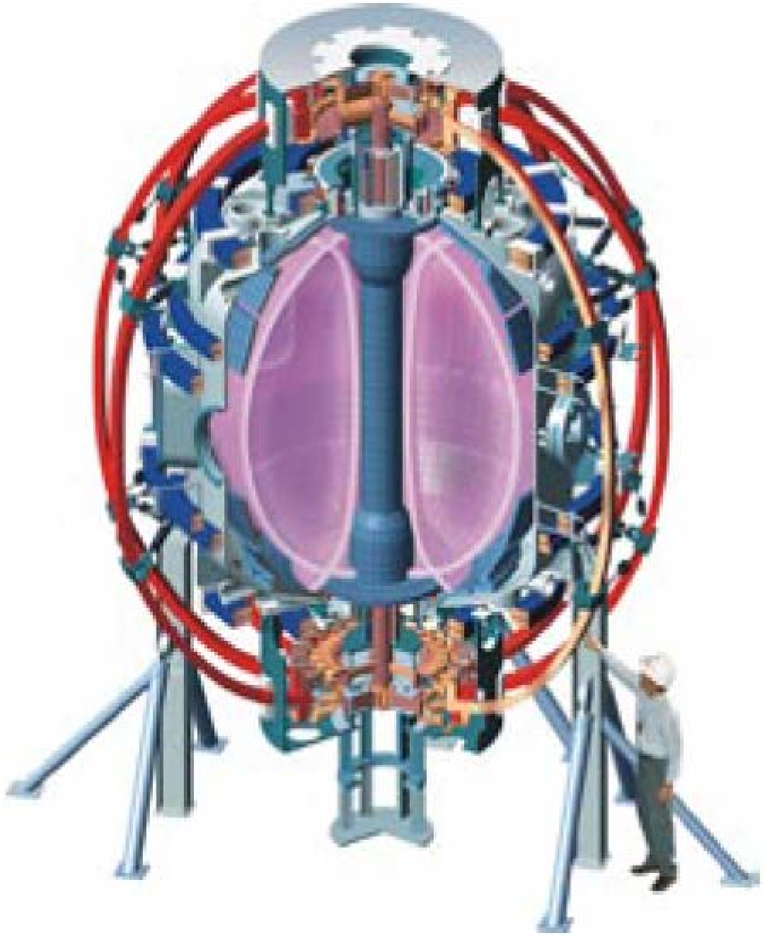
- Stabilize the resistive wall mode (feedback, rotation, kinetic effects)
- Control ELMs (edge ∇p driven modes, affected by shear flow in the edge)
- Prevent neoclassical tearing modes (eliminate seed island, limit island growth)

What about the spherical tokamak (ST)?

- The ST is an ultra-tight aspect ratio tokamak

- $\varepsilon = a/R_0 \rightarrow 1$

- Capable of high beta since $\beta_{crit} \propto a/R_0$



Source: Princeton Plasma Physics Laboratory.

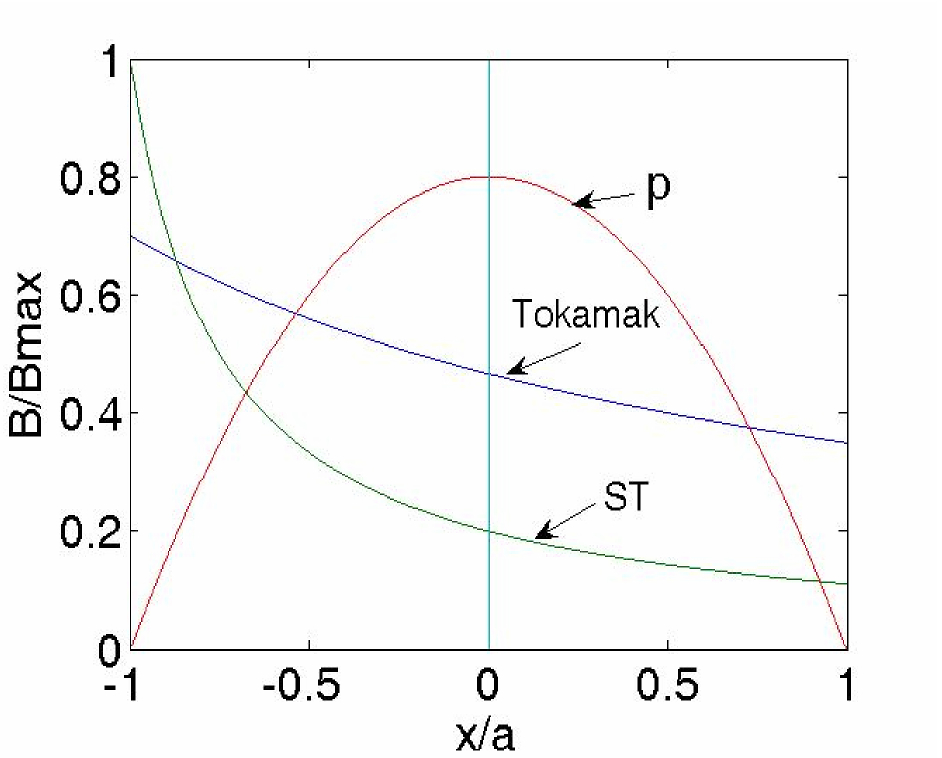
But

- $p \propto \beta\,(B_{max})^2\,(1\text{-}\varepsilon)^2$

- Pressure is not that high because of 1/R effect

- Large *I* is required – tough bootstrap problem

- Central TF leg must be copper: low B_{max} (7.5T) because of joule losses



My not so happy conclusion

- ST is very interesting plasma physics experiment
- The ST does not solve any of the difficulties of the standard tokamak
- The ST probably generates more new problems than in the standard tokamak.

The Stellarator

- An inherently 3-D configuration (a toroidal-helix)

- An inherently steady state device – resolves the current drive problem

- No toroidal *I* is required – should greatly reduce the kink driven disruption problem

Are there any problems?

- Stellarators are much more complicated technologically
- The magnet design in particular is complex
- 3-D equilibrium with closed flux surfaces can be calculated but with difficulty
- 3-D stability can be calculated but also with difficulty
- Stellarators are very flexible before they are built – many options. But are inflexible once they are built

The LHD (Japan $1B)

- Continuous wound superconducting coil
- Engineering marvel
- Technology doesn’t extrapolate well into a reactor

Diagram of Large Helical Device (LDH) reactor
removed for copyright reasons.

Types of new stellarators

- Stellarator geometries are based largely on reducing 3-D neoclassical transport losses
- This has lead to the concept of quasi-symmetry
- Use of modular coils for reactor viability

What is quasi symmetry?

- It has been shown that the guiding center particle drift off a flux surface depends only on |**B**| not **B**

$$\mathbf{n} \cdot \mathbf{V}_{\mathbf{GC}} = f\left(\left|\mathbf{B}\right|, \psi\right)$$

Three symmetries

- General stellarator field

$$\mathbf{B} = B_0 \frac{R_0}{R} \mathbf{e}_\phi + \sum \mathbf{b}_{m,n}(r) \exp\left[i\left(m\theta + n\phi \right) \right]$$

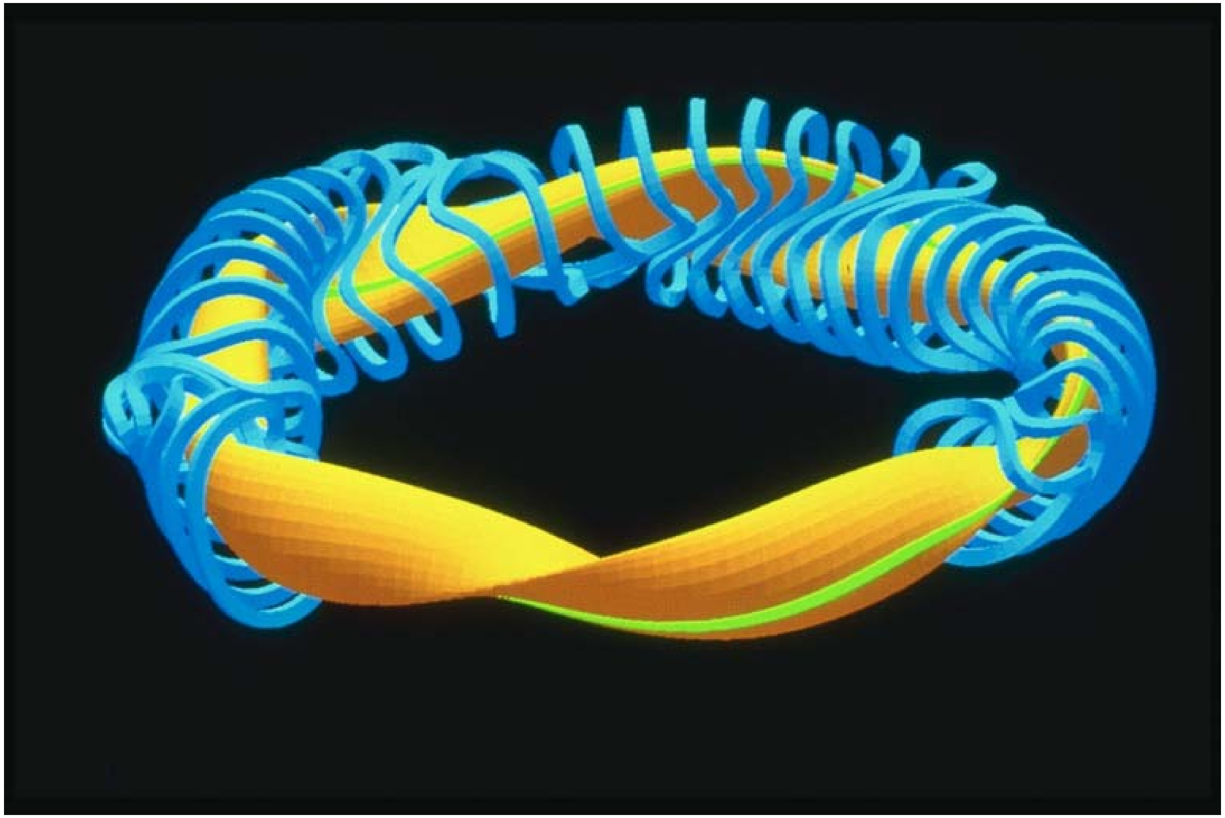
- Quasi-poloidal symmetry (W7-X): $|\mathbf{B}| \approx f\,(\,\phi,\ \psi)$

- Quasi-toroidal symmetry (NCSX): $|\mathbf{B}| \approx f\,(\,\theta,\ \psi)$

- Quasi-helical symmetry (HSX): $|\mathbf{B}| \approx f\,(\,M\theta + N\phi,\ \psi)$

W7-X (Germany $1B)

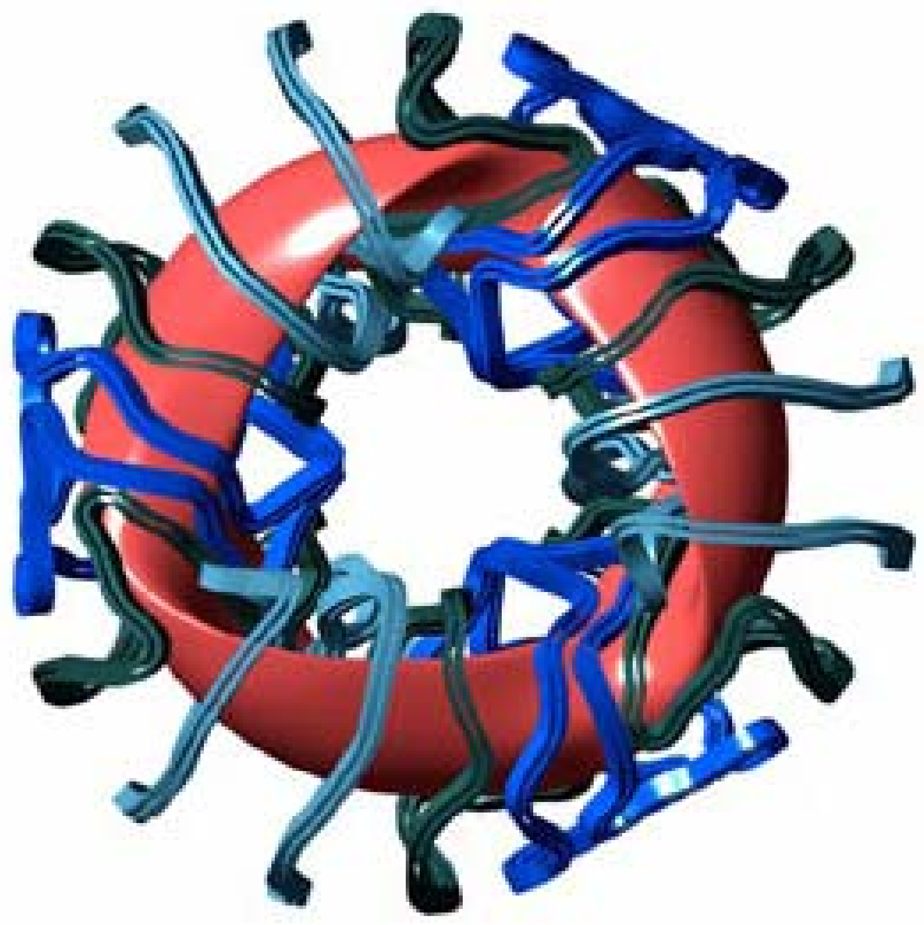
- Modular superconducting coil for reactor viability
- Very low bootstrap current
- Large aspect ratio: $R_0/a = 10$



Courtesy Max-Planck-Institut für Plasmaphysik. Source: Wikipedia.

NCSX (USA $100M)

- Modular copper coils
- Significant bootstrap current
- Tight aspect ratio: $R_0/a = 4$



Source: Princeton Plasma Physics Laboratory.

General behavior

- Confinement approaching that of a tokamak
- Beta limits not yet tested
- Heating seems to work but not yet at tokamak levels

A happy ending to stellarator problems?

- More efficient 3-D equilibrium codes
- More efficient 3-D stability codes
- Good coil design codes
- Less complicated, less expensive magnets

Summary of Talk

- We have accomplished a lot in MHD
- But there is still a lot more to do
- More in inventing new ideas than developing new tools

New Ideas Needed

- Stabilize the resistive wall mode
- Optimize the use of flow stabilization
- Predict and control ELMs
- Stabilize the neoclassical tearing mode
- Invent ever cleverer stellarator geometries
- Develop less expensive stellarator magnets

New Theory Tools Needed

- More efficient 3-D equilibrium codes
- More efficient 3-D stability codes
- Development of hybrid MHD-kinetic codes