MICHAEL SHORT: Hey guys, hope you enjoyed the brief break from the heavy technical stuff. Because we're going to get right back into it and develop the neutron transport equation today, the one that you see on everybody's t-shirts here in the department.

So I think multiple years folks have used this equation on the back of t-shirts just be like, we're awesome. And we do difficult math. Well, this is what you're going to start to do it. In fact, it's big enough and hard enough that we're going to spend all day today developing it, like actually writing out the terms of the equation and understanding what it actually means. Before, on Thursday and Friday, we're going to reduce it down to a much simpler equation, something that you can actually solve and do some simple reactor calculations with.

We started off the whole idea of the neutron transport equation as a way to track some population of neutrons. Let's see, I'm going to have our variable list up here. What I'll probably do is on Thursday and Friday I'll just have it back up on the screens so that we don't have to write it twice. But there's going to be a lot of variables in this equation. I'm going to do my best, again, to make the difference between V and nu very obvious, and anything else like that.

But the goal is to track some population of neutrons, at some position, at some energy, traveling in some direction omega, as a function of time. And the 3D representation of what we're looking at here is let's say we had some small volume element, which we'll call that our dV. That's got normal vectors sticking out of it. We'll call those n-hats in all directions. And inside there, let's say if this is our energy scale, we're tracking the population of neutrons that occupies some small energy group dE, and is also traveling in some small direction that we designate as d-omega.

So that's the goal of this whole equation is to track the number of neutrons at any given position. So let's call this distance, or the vector r. In this little volume, traveling in some direction omega, with some infinitesimally small energy group d. That's going to be the goal of the whole thing.

And what we'll do is write this to say the change in the population of neutrons at a given distance, energy, angle, and time, over time is just going to be a sum of gain and loss terms. And what I think we'll take all day today to do is to figure out what are the actual physical things that neutrons can do in and out of this volume, and how do we turn those into math, something that we can abstract and solve?

There's a couple other terms that we're going to put up here. We'll say that the flux of neutrons, which is usually the variable that we actually track, is just the velocity times the neutron population. And also let's define some angularly independent terms. Because in the end we've been talking about what's the probability of some neutron or particle interacting with some electron going out at some angle.

But as we're interested in how many neutrons are there in the reactor, we usually don't care in which direction they're traveling. So the first simplification that we will do is get rid of any sort of angular dependence, getting rid of two of the seven variables that we're dealing with here. So all these variables right here will be dependent on angle. And all these variables right here will be angularly independent.

So there'll be some corresponding capital N, or number of neutrons as a function of r, E, and t. We'll call this Flux. We'll call this Number.

There's going to be a number of cross sections that we need to worry about. So we'll refer to little sigma as a function of energy, as our micro-cross-section, and big sigma of E as a macroscopic cross-section.

Then you want to remember the relation between these two.

AUDIENCE: Solid angle?

MICHAEL SHORT: The solid angle, not quite. That's, let's see. There's a difference between-- and so what these physically mean is little sigma means the probability of interaction with one particle. And this is just the total probability of interaction with all the particles that may be there. So yeah, Chris?

AUDIENCE: Number density?

MICHAEL SHORT: There's number density. Already we have another variable conflict. How do we want to resolve this? Let's see. We'll have to change the symbol somehow. Let's make it cursive. Don't know what else to do. I don't want to give it a number other than n since we're talking about neutrons, or that right here it's going to be number density.

And in the end, we're worried about some sort of reaction rate, which is always going to equal some flux, or let's just stick with some angularly dependent flux, that r, E omega t, times some cross-section as a function of energy. And it's these reaction rates that are the rates of gains and losses of neutrons out of this volume, out of this little angle, out of this energy group, and out of that space, or into that volume energy group and space.

So let's see, other terms that we'll want to define include nu, like last time. We'll call this neutron multiplication. In other words, this is the number of neutrons made on average during each fission event. And we give it energy dependence because as we saw on the Janis libraries on Friday, I think it was. What's today, Tuesday? I don't even know anymore. I think as we saw on Friday, that depends on energy for the higher energy levels.

And there's also going to be some Kai spectrum, or some neutron birth spectrum, which tells you the average energy at which neutrons are born from fission. So regardless of what energy goes in to cause the fission, there's some probability distribution of a neutron being born at a certain energy. And it looks something like this, where that's about 1 MeV. That's about 10 MeV. And that average right there is around 2 MeV.

And so it's important to note that neutrons are born at different energies. Because we want to track every single possible dE throughout this control volume, which we'll also call a reactor.

Let's see, what other terms will we need to know? The different types of cross sections, or the different interactions that neutrons can have with matter. What are some of the ones that we had talked about? What can neutrons do when they run into stuff?

AUDIENCE: Scatter.

MICHAEL SHORT: They can scatter. So there's going to be some scattering cross-section. And when they scatter, the important part here is they're going to change in energy. What else can they do? Yeah?

AUDIENCE: Absorbed.

MICHAEL SHORT: They can be absorbed. So we'll have some sigma absorption. What are some of the various things that can happen when a neutron is absorbed?

AUDIENCE: Fission.

MICHAEL SHORT: Yeah, so one of them is fission. What are some of the other ones?

AUDIENCE: Capture.

MICHAEL SHORT: Yep, capture. What were some of the ones that we talked about during the Chadwick paper?

AUDIENCE: Neutron [INAUDIBLE].

MICHAEL SHORT: Yep, so there can be some-- we'll call it n,in, which means one neutron goes in, i neutrons come out, so 1 to i neutrons, sure. Anything else? Encompassed in absorption?

Well when we refer to scatter here, what type of scattering are we talking about?

AUDIENCE: Compton's scatter?

MICHAEL SHORT: Compton's for photons. It's OK. Was it elastic or inelastic scattering?

AUDIENCE: Elastic.

MICHAEL SHORT: Elastic scattering. So another thing you could call an absorption event, depending on what bin you put things in, is inelastic scattering, which is that kind of-- we call it scattering because one neutron goes in, one neutron comes out. But in reality, you have a compound nucleus forming and a neutron emitted from a different energy level. So it doesn't follow the simple ballistic laws of, and kinematic laws of inelastic scattering.

What else can neutrons do? Now we're getting into the real esoteric stuff. But I want to see if you guys have any idea. Did you know that neutrons can decay? A low neutron is actually not a stable particle.

If you look up on the Kyrie table of nuclides, it's got a half life of 12 minutes. So if you happen to be able to have neutrons in a bottle or something, which we actually can do. There's centers for ultra-cold neutrons and atoms. There's one at North Carolina State where they actually cool down neutrons to cryogenic temperatures to the point where they can actually confine them. They only live on average 12 minutes.

And then there would also be what we call a neutron-neutron interactions. There is a finite, non-zero but very small probability that neutrons can hit other neutrons. But the mean-free path for these is on the order of 10 to the 8th centimeters. So this is not something we have to consider. But it's interesting to know that yes, neutrons can run into other neutrons. And these sorts of things have been measured.

We won't have to worry about this. We won't have to worry about neutron decay. But it's interesting to note that a low neutron is not a stable particle. It will spontaneously undergo beta decay, into a proton and an electron. Pretty neat, huh?

Anyway, if we sum up all these possible interactions, we have one other cross-section, which we're going to call the total cross-section, the probability of absolutely any interaction occurring at all. Because any sort of interaction of that neutron is going to cause removal from this group of energy position, angle, location, whatever.

Whether it's absorption, or fission, or elastic scattering, or inelastic scattering, any sort of event-- except for forward scattering, which means nothing happens-- is going to result in this neutron either leaving the volume.

So it might scatter out of our little volume. Or it might change direction, scatter out of our d-omega. Or it will lose some energy, or gain some energy, in some cases, leaving our little dE, which is what we're trying to track. Because we're actually tracking what's the population of neutrons in this little dE, in this direction, in this position, at this time. And supposedly if we know this term fully, we can solve for all the neutrons everywhere, anywhere in the reactor with full information.

So what we'll spend the rest of today doing is figuring out what are all the possible gain and loss terms. So let's start just putting them out physically, or in words. And then we'll put them to math.

So what are some of the ways in which neutrons can enter our little group of volume, angle, and energy? How are neutrons created? Yeah, Luke?

AUDIENCE: From fission, or a neutron emission.

MICHAEL SHORT: From fission, yeah, so that's one big source. So we'll call this a gains. This is a losses. And you said a neutron source. Can you be more specific?

AUDIENCE: A neutron emission, like [INAUDIBLE].

MICHAEL SHORT: OK, so we'll say n,in reactions, right? OK, cool. How else can we gain neutrons?

AUDIENCE: Fusion?

MICHAEL SHORT: Fusion? OK. That is true. Although fusion reactors don't really operate on the principle of neutron criticality, or neutron balance. So this discussion for now is going to be limited to fission reactors. But yeah, good point. Fusion does make neutrons. What else? Yeah?

AUDIENCE: They could enter from one of the adjacent volume?

MICHAEL SHORT: Yeah, they could come from somewhere else, right? Let's just call that an external source. In the books and in your reading, you'll just see them treat this external source as some variable s of r, E, omega, t.

So you'll just see this treated as s, a source, with no further explanation. It's like, oh, math says that there could be external sources. But I want to tell you where they really come from. Most reactors nowadays don't just start up when you throw a bunch of uranium into a pool and pull out the control rods.

You actually have to stick in-- if this is your little reactor right here-- you actually have to stick in a little piece of californium-- I think the isotope is 252-- as what we call a kickstarter source. So californium is made mostly in the HFIR, or the High Flux Isotope Reactor, at the Oakridge National Lab in Tennessee, where they have a really, really high power reactor.

It's 85 megawatts. It's about that big around and this tall. It's really, really small. For reference, that's about the size of the MIT reactor, except our reactor's 6 megawatts. Theirs is 85 megawatts. And it's designed to be an incredibly high flux, to go by neutron capture, and neutron capture reactions, to build up californium 252, which is spontaneously giving off neutrons like crazy.

And this right here, that's your external source. And this helps get reactors going. Because you can either very slowly wait for the fission reaction to build up in a controlled manner. Or you can give it a kick in the pants and get it going.

This HFIR reactor is pretty cool. Like I said, it's 85 megawatts. And it's about as dense as it can get. The fuel is actually made by explosively bonding sheets of uranium in a certain sort of semi-cylindrical configuration. And it produces so much decay heat in so little space that if it were to lose cooling, the reactor would melt in 8 seconds. You usually have days or so before that happens in a conventional reactor because the power density just isn't that high.

So you can actually see down to the tank that contains HFIR if you go for a tour at Oakridge National Lab. And it's way down below this gigantic like, not quite Olympic, but getting there sized pool of water, just to make sure that there is adequate cooling for this thing. It's intense. But that's just a notice that these external sources, these are real things that we use in power reactors to get them going.

What are some other ways that one could make neutrons, or that neutrons could enter into our energy group? And the silence is expected because this is usually the hardest part of developing this equation. And I want to introduce it. Yeah, Luke?

AUDIENCE: [INAUDIBLE] scattering, too.

MICHAEL SHORT: That's exactly it. They can scatter in. So when we develop this neutron transport equation, we're not just tracking the neutrons in this little energy group dE, direction, d-omega, and volume dV. You actually have to know what's the population of neutrons in every single group. Because you might have a neutron at a higher energy level that undergoes scattering from some different energy, E-prime into our energy group.

So continuing with our gigantic list of variables, we're going to call E is, we'll say r energy. And this vector omega is our direction, the one that we're tracking. And E-prime is going to be some other energy. And omega-vector-prime is going to be some coming from some other direction.

And like Luke said, this is what we would refer to as in-scattering, which means some neutron comes, that was going in a different direction, that did have a different energy, and has now entered into the single group that we're tracking. Eventually we're going to integrate over all energies to track all energy groups. So that's where we're going.

And there's one more term that I want to introduce right now. It's what's called the scattering kernel. Don't ask me why it's called kernel. But this is just the terminology I want you guys to get used to. And there's going to be some sort of probability function where a neutron starts off at a different energy, E-prime, and in a different direction, omega-prime. And it enters into our group energy E and direction omega.

Right now we'll leave it as a highly general function. What we're going to find later is there's just some sort of simple line to it.

If you guys remember, if some neutron starts off, let's see, probability of entering into some energy group. If you notice, if you remember from last time, the neutron, when it undergoes any sort of scattering reaction, can end up with any energy between its original energy for the case of theta equals 0, and this parameter, alpha energy, for the case theta equals pi, where alpha is A minus 1, over A plus 1 squared, where A is the atomic mass.

You guys remember this from back in the Q equation days, when we were finding out what's the probability that a neutron coming in with energy E ends up at any energy E-prime? Actually I'll just write this as the scattering kernel.

What it ends up looking like, in most cases, is just a flat line. There's an equal probability of the neutron ending up anywhere between energy E and anywhere between energy alpha-E. It's actually a pretty simple function. It's just a constant value here and 0 everywhere else.

What that means is that if, let's say, a neutron hits a uranium atom, there is no way in hell that it can transfer all of its energy to a uranium atom because of conservation of energy and momentum, like we've been harping on for kind of this whole class.

What's the only time that this alpha-E could actually extend all the way to 0? What case would that be?

AUDIENCE: [INAUDIBLE].

AUDIENCE: You're hitting another neutron.

MICHAEL SHORT: You're hitting another neutron, which, as we said, is a very rare event. That is true. Or what else could you be hitting?

AUDIENCE: A proton?

MICHAEL SHORT: A proton, hydrogen. That's right. So it can only be, let's say you can only have the probability of the neutron ending up with any energy for the case of hydrogen. Incidentally, this is why we fill light water reactors with hydrogen. The goal is to get the neutrons as slow as possible as quick as possible. Interesting sentence to say there, right?

We want the neutrons to be as low energy as possible as rapidly as possible. And the best way to do that is to fill the reactor with hydrogen because then any collision could, in theory, get the neutron down to zero energy. Without water, or something with the same mass as a neutron, like another neutron, there is no way that that neutron can slow down by very much.

So even though we're going to keep it as this generalized function, note that in reality it's this pretty simple function. It changes a little bit, as there can be a forward scattering bias for some neutron reactions. But we are not going to deal with that this year. You will deal with that next year in 22.05.

So I've been saying a lot, oh, well, we're not going to go into this topic because you're going to see it in 22.02, which is quantum. Now I switched gears to say, we're not going to go into the way this function changes because you'll see it next year in 22.05, which is neutron physics. But for now I want you to be prepared for 22.05. So we'll put on in-scattering as one of our gains.

There's a last one I want to make you aware of. We very briefly touched upon it. But I wouldn't be surprised if no one remembers because it was for like 10 seconds. It's what's called photo fission. What this means is you have some reaction that would, in comes a gamma, and out goes fission.

This actually does start to happen around 3 or 4 MeV, for isotopes like uranium 235. And in our reactor, whatever shape we decide it is, there are tons of gamma rays flying about in all directions at very high energy. Does anyone remember where they come from? Anyone remember the fission timeline that we drew on Friday?

So what we said there was right away, let's say fission happens. And almost instantly, you get your fission product one and fission product two. And they move around for a little while. And then some of them will emit some neutrons. And then some of them will start to emit gamma rays, betas, and whatever else they're going to do until they finally lose all their kinetic energy and stop in the surrounding fuel, creating the heat that actually powers the turbine and make steam to make electricity.

And so it's from these gammas, as well as any of the gammas from the decay products of the fission products that lead to a huge flux of gamma rays firing out from all sides in the reactor. That's one of the main things that you actually have to shield in a nuclear reactor.

Since we talked about all sorts of different shielding, and all sorts of ways that you have to shield things, you know from seeing the MIT reactor-- which you all did-- that there's like six feet of lead and concrete shielding around the reactor. It's not there to shield the alphas and the betas, because those don't really make it out of the water. It's not there to shield the soft X-rays that betas make from bremsstrahlung.

It's also not there to shield the neutrons because the neutrons don't really get out. They bounce around, or get absorbed in the water, or the fuel, the reflector. It's there to shield the high-energy gamma rays. Because the only thing that stops high energy gamma rays is lots of mass in between the source and you.

So we know there's tons of gammas all about. So let's say there's also going to be some gamma ray flux. There'll be some gamma ray energy. And there'll be some cross-section for photo fission as a function of the incoming gamma ray energy spectrum.

Now I'm adding terms to the ones that you'll see in the reading because drawing them out in math is actually fairly instructive. They all follow the same pattern. So instead of just showing you one of each and saying memorize, we'll develop a whole lot of these. And you'll find out that they all actually look almost the same.

Can anyone else think of any possible gains of neutrons? Where else could they come from? Yeah?

AUDIENCE: Neutron birth spectrum, is that?

MICHAEL SHORT: So the neutron birth spectrum is included in fission. So our nu is in there. Our chi of E is in there. And that's a nu of E. That's all accounted for in the fission term. And we'll see how we put that together to math. And if no one else has any ideas, that's good. Because neither do I.

Now what about the lost terms? There aren't too many of these, as long as you lump them correctly. So what sort of ways could neutrons be lost from our energy group? Yep?

AUDIENCE: Scatter out.

MICHAEL SHORT: Scatter out, yep. They can undergo any kind of scattering reaction. And they will probably change direction and energy. What else? Well, we've got to list up on the board right there, right? Capture, fission, because in order to undergo a vision you actually have to lose a neutron, and so on, and so on, and so on.

What I want to do to simplify things is this. It's a lot simpler just to track the total cross-section, the probability of any interaction at all whatsoever, because any interaction will cause the neutron to either change energy and angle, or disappear, even if it makes some other ones. So we can simplify this to just the total cross-section term. And there's only one other way that neutrons can leave our energy angle and volume group. What would that be?

So any reaction takes care of energy and angle. What about volume? How do neutrons leave the control volume? It's simpler than it may sound. They just go. They just move. The neutrons are always moving, right? We'll call that leakage. Because every neutron's got a speed, like we showed up here, where the flux of neutrons, the number of neutrons moving through some surface per second, is just their velocity times the number that are there.

For there to be a neutron flux there has to be a velocity, which means the neutrons are moving. So the neutrons, even without undergoing any reaction, could just move out of our control volume. And then they're gone.

And that's all there is for gain and loss terms. So let's see if we can do this all on one board. I want to start putting this table right here into math that we'll be able to abstract, simplify, and then solve, but not today, not solve today.

So if we want to track the change in the number of neutrons as a function of time, let's start writing down the gain terms. So how do we describe the number of neutrons produced from fission? What sort of terms do we have to include? And Jared started kicking us off, so what would you say?

AUDIENCE: Neutron birth?

MICHAEL SHORT: Yep, so the neutron birth spectrum, there's going to be some probability that a neutron is born in our energy group E. Because we're tracking how many neutrons are in our little dE energy group. What else matters in terms of fission?

AUDIENCE: Number of fissions?

MICHAEL SHORT: Yep, number of fissions. So if we want to write number of fissions, we have to write that as a reaction rate. So let's take those two terms right there. So we'll have sigma fission. In this case, we're going to write E-prime times flux of r E-prime, omega-prime, t. Why did I write E and omega prime here? Just from a physical reason. Yeah?

AUDIENCE: So you're going to be coming from another energy group.

MICHAEL SHORT: Precisely. That's right. So the neutrons are going to be produced from some other energy group. For example, the fission birth spectrum right here starts out-- where did it go? I knew I drew it somewhere-- at one MeV. But most of the neutrons that cause fission to happen are way down below 1 eV. So it's different energy neutrons that cause neutrons to be born in our energy group.

That's why we're using E-prime and not E. It's some other energy group. And so we also have to account for all possible other energy groups. So if we want to write this, right, we'll say this could be as low as 0 eV, to our maximum energy. And there's going to be some d-omega-prime, dE-prime, dV. We'll also have to account for all possible angles and integrate over our entire volume.

It's going to look ugly quick, but it's all going to be understandable. So what this says is that we have to account for the reaction rate of fission from all other energy neutrons inside our volume from other energies and other angles, and account for every other possible energy. Because they can all make fission happen.

What else is missing in terms of describing the number of neutrons made from fission?

AUDIENCE: Neutron multiplication.

MICHAEL SHORT: Yep, there's the number of neutrons made per fission. So we have to put in our neutron multiplication factor. And in this case, normalize-- I think someone had mentioned solid angle-- we normalize over all possible angles with an over 4 pi in there. And this right here is the fission term.

So this tells us the number of neutrons gained in terms of a reaction rate, times the number of neutrons for each of those reactions, times the probability that there just happened to be born in the energy group that we're tracking.

So is there any term here that's unclear to folks? Yeah?

AUDIENCE: So what's the lower bound on the first integral?

MICHAEL SHORT: On the first integral? That 0 electron volts.

AUDIENCE: Oh, OK.

MICHAEL SHORT: Because supposedly you could have a neutron at 0 eV, which has a very high cross-section.

So it should probably induce fission. In reality, there might be some actual minimum temperature. But there is a non-zero probability that you could have a neutron at rest. It's just not very large.

AUDIENCE: And the top bound?

MICHAEL SHORT: The top bound as E max, whatever your maximum neutron energy is. This is usually around 10 MeV, for most fission reactors. That E max is going to be this point right here, the highest energy at which neutrons can be born by any process.

And so this term right here is going to serve as a template for all the other gain and loss terms. So I think this is the hardest one that we had to develop from the beginning. Now let's develop the term for, let's just go with external sources, pretty easy. There's going to be some source making neutrons. It's something that you would just impose. Like say, all right, I have a californium source giving off this many neutrons. Well then you know how many neutrons it's giving off. And that one's done. That's easy.

So we've done fission. We've done external.

Now that we've done fission let's tackle photo fission. So what would be photo fission cross-section look like? It's going to look awfully similar. So what sort of things do you need to know if it's a fission reaction? Well, what do we have up here? Just start reading things off. I heard a murmur. What was that?

AUDIENCE: [INAUDIBLE] flux.

MICHAEL SHORT: Yeah, so you're going to have to have some flux. In this case, we want to know what's the flux of gamma rays because photo fission starts off with a gamma, then ends up with a fission. And it's also going to be in our volume. It's going to matter what the energy of those gammas is. They'll all be traveling in some direction at some time. What else do we need?

AUDIENCE: [INAUDIBLE] the 4 pi [INAUDIBLE].

MICHAEL SHORT: Yeah, if we're going to be going over all angles, you need the 4 pi. What else? Do we have a reaction rate yet?

AUDIENCE: No.

MICHAEL SHORT: No, well what's missing?

AUDIENCE: The cross-section.

MICHAEL SHORT: That's right. We need a cross-section. And in this case, instead of just fission, or neutron fission, we'll put in the gamma fission cross-section. And so now we have a reaction rate for a single reaction. We've got to integrate over all of our gamma ray energies, over all angles, over our volume. What else is missing besides our d-omega, dE gamma, d-Volume. It should look awfully similar because the terms are basically exactly the same, with just different cross sections and energies in there. So what's missing between the photo fission and the neutron fission one?

AUDIENCE: [INAUDIBLE]

MICHAEL SHORT: Sure, there might be some different birth spectrum for gammas. And there might be some different multiplication factor for gammas between neutron fission and photo fission. But these terms should look exactly the same because in every case you're looking at some reaction rate between either the neutrons and fission or the gamma and fission. And you need to know at what energy they're born, how many are made, all the angles, and integrate overall the variables that we care about.

And this is part of why I'm adding these extra terms because they end up looking all exactly the same. It's the integral of a reaction rate times some stuff. That's all that every single one of these terms is going to be.

So we've got photo fission. Now let's tackle in-scattering. So how do we represent scattering? In the same way that we represented fission, what do we start with inside the integral?

AUDIENCE: Reaction rate?

MICHAEL SHORT: Reaction rate, yes. So we're going to have some scattering cross-section. And if it's in the scattering, it means it's coming from a different energy, hence the prime. We'll have our flux. And here's where we're going to bring in our scattering kernel. Because there's some probability that the neutron scatters in from a different group.

And then we'll have our d-omega, dE-prime, dV. Is this complete yet? We've now accounted for one other energy, E-prime. Now how do we account for all possible other energies scattering into our energy group?

AUDIENCE: Integrals.

MICHAEL SHORT: Yep, again, same integrals. We integrate over all possible energies, over all possible angles, and over our volume. Hopefully these terms are looking very similar. In every case it's a volume, angle, energy integral of a reaction rate. And all that's saying is there's some rate that these reactions are occurring, which is either a gain rate or a loss rate. We integrate over whatever volume, energy, and angle we're tracking. And that's all there is to it.

So we've got in-scattering. Now let's tackle the n,in reactions. There's only going to be one little difference. But I want you guys to tell me what sort of things are going to be the same as all the other terms that we have here. So what do we start off with inside the integral?

AUDIENCE: Reaction rate.

MICHAEL SHORT: A reaction rate. And how do we write that?

AUDIENCE: [INAUDIBLE].

MICHAEL SHORT: Yep, a cross-section, there's going to be some cross-section, for, let's call it n,in reaction as a function of energy, times a flux. There'll be our normal integrated over all angles, all energies, and our volume. So we have a reaction rate. What do we have to then integrate that reaction right over to get all the neutrons in?

AUDIENCE: [INAUDIBLE] same things.

MICHAEL SHORT: Same things as everything else, exactly. Integrate over all possible energies, integrate over all possible angles, integrate over our volume, looks quite similar. The only thing we haven't dealt with is this i term right here. Because there can be, there are actually n,2n reactions, n,3n, n,4n, and so on. So there are probabilities that, let's say you, in goes 1 neutron, out comes 3 neutrons. But no fission actually happened. You just blast a few of them out.

So I think all we'd really have to do is sum over i equals 1. Oh, I'm sorry, i equals 2, because a neutron going in and then the same neutron going out, that's just scattering. And what would be the maximum? Probably 4. Because the probability of an increasing i or more neutrons coming out gets lower and lower as you go. In fact, these reactions don't even turn on until-- the n,2n reaction turns on and around like 1 MeV. This one turns on and around like 5 MeV. This one turns on at like 12 MeV. I was just looking up these cross-sections before class.

So if you have a reaction that doesn't happen beyond your highest neutron energy, you probably don't need to worry about it. But the reason I had us write all these extra equations-- and I think that the t-shirt for this department needs some updating to include these extra terms-- is because they're all the same term. It is in every case, it's an integral over all of our stuff of a reaction rate, d-stuff, times a multiplier.

Every single term in this equation follows the exact same pattern. So what I hope, and I would expect out of you guys, is that if I were to give you this table of possible reactions, you would be able to recreate this neutron transport equation using this template to know that every single reaction is just multiplier, times integral of stuff of a reaction rate, d-stuff, where the reaction rate is just a cross-section times a flux. That's all there is to it.

Not bad when you see that everything follows the same pattern, right? That's the basis behind most of the hideous equations that you see in all of physics everywhere, is if there are additive or subtractive terms, they'd better be in the same units. And so they're going to follow some sort of a similar template. Not too scary when you think of it that way.

So let's now come up with the loss terms. I should have planned these boards better. Keep these ones here so we keep a template. And we'll have a minus, well, how do we write the anything reaction using this template?

How many neutrons undergo a reaction when one neutron undergoes a reaction? Yeah, 1. So our multiplier is 1. We don't have to worry about it.

We have our integral of stuff. So we'll have to integrate over all possible volumes, angles, and energy. And what's on the inside?

AUDIENCE: [INAUDIBLE] total cross-section.

MICHAEL SHORT: Yep, total cross-section as a function of energy times the flux, d-stuff to save time. So don't worry, even though the boards are laid out funny, on the pictures of the blackboard that we'll put on the Stellar site, I'll Photoshop these and arrange them so that they're all in sequence. And you can see everything.

And then there's the last term to account for. That's the leakage term. This one is a little different. It's the only one that's a little different. And in this case, we're going to say that our little volume element also has a surface to it. And if the neutrons leave the surface, then they leave the volume.

So in this case, we'll have a surface integral of our neutron flux, say our neutron flux dS. Because there's no reaction happening when neutrons just move, right. They just go. And so, well, we'd also have to multiply by our normal vector. Because every flux is going to have a certain number of neutrons moving in a certain direction.

Let's say we were tracking the flow of neutrons through this surface right here. And if we had a flux going in exactly this direction, through this surface, and this is the normal vector, in this case, flux, which is a vector dotted with the normal vector, is just the flux. Which is to say that if the flux and the normal vector are aligned in the same way, then every neutron going through the surface is tracked as going through the surface.

To take the opposite example, what about the situation where you have a surface here and you have a mono-directional flux of neutrons in this direction. And that is your surface normal. What does the flux dotted with the surface normal vector equal? 0, it's just a dot product between the direction that your neutrons are moving and the normal vector saying, does it go out of the surface at all?

So for these two limiting cases, in this case, the fluxes just let's say, what is it, the number of neutrons leaving the surface is the flux. In this case, no neutrons leave the surface because they're not actually going through the surface. It's a good time to mention, again, that these units of flux are in neutrons per centimeter squared per second, which is to say the number of particles traveling through this area in centimeter squared every second.

I know we've gone over it before, but I want you to keep these units in mind. Because now they actually have a little more physical significance. And that's why we have this flux times normal vector dS. That describes the number of neutrons that get through the surface.

The last thing we'll do, because everything else is a volume integral, we want this to be a volume integral because we're going to simplify this in terms of getting rid of all the volume stuff. We're going to use what's called the divergence theorem. I hear some snickering. Because I remember this is probably something where you were told in 1801 or 1802, this exists. Use it in a few problems. Moving on. That sound about right? This is when you actually use it.

So the divergence theorem says that the integral of some variable F dS, through some volume element of surface, is the same as the volume integral of-- how does this go-- del dot F dV.

And this is going to be quite important because one, it gives us a volume integral. So this is, it will be a volume integral of our del dot flux dV, so now everything's in the same units.

And if we were to say forget about our little volume element. Let's just assume an infinite reactor. Every single volume integral in term just instantly disappears. Because we wrote these equations to be identical for any dV anywhere inside this reactor. If the reactor is then infinite, then all of those volume terms disappear. And that's the first simplification that we'll make in the next class.

But right here on these five boards, we've developed the neutron transport equation, which is the absolute, most general, highest escalated form of how do you track neutrons through any volume, any direction, any energy, at any time. And we'll spend Thursday and Friday simplifying this to something that we can solve.

The other reason that we use this divergence theorem is because we're going to make an approximation. This crazy looking thing right here, we will make an approximation called the diffusion approximation where we assume that neutrons are like a gas that just diffuse away from each other. And that's going to make solving this really, really easy. It's going to go from some second order differential or differential integral equation to just the equation that you can solve with algebra. Yep?

AUDIENCE: Do you need the dE d-omega for the last term?

MICHAEL SHORT: Probably, yeah, over all E, over all omega. And that flux is going to be of r, E omega, t. Absolutely.

Just to make sure, everything is in the same units, every term has a fairly similar template. The only difference is leakage, there's no reaction here. Every single other term constitutes a reaction. And they all follow this template.

So I will stop here because it is five of. See if anyone has any quick questions on what you've got here. I'll make sure to get all of this on the board images so you guys can take a look at it. And I'll projected up on the screen so that we can make some simplifications based on what we see here on Thursday. Yeah?

AUDIENCE: What is the neutron birth spectrum?

MICHAEL SHORT: The neutron birth spectrum says that if you have any old fission event, what's the probability of those neutrons being born at different energies? What this says is that they're born between 1 and 10 MeV, with a peak at around 2 MeV. But if you want to track the number of neutrons in every energy group, you need to know where they begin.

Good question. So if any of the terms here are unclear what they physically mean, because that's what I'm most interested in you guys knowing, please do ask either on Piazza, on email, on Thursday. Yeah?

AUDIENCE: What's the difference between the big N and the little n again?

MICHAEL SHORT: The big N and the little n, which one? The cursive one, or this one?

AUDIENCE: The little n up top there and then the non-cursive one.

MICHAEL SHORT: OK, the little n and the non-cursive one. The little n is the number of neutrons in a volume, at a certain energy, going in a direction, at a certain time. Big N right here is just number density, number of atoms per centimeter cubed. Cursive n is the number of neutrons at an energy, in a volume. We don't care where they're going.

And the reason I write these terms up here is we are going to switch from lowercase to capital, or angularly-dependent to angularly-independent by making a simple approximation to say, we don't care what direction they're going. We just care if they're there.

But in real complex neutron physics problems, like the one solved at the computational reactor physics group, you need to know all the angles. And you need to know the probability or the cross-section that a neutron coming in at this angle leaves at that angle and imparts a certain energy. Because they're all different.

For the purposes of this class, I just want you to know that they exist. And the first thing we will do is simplify them away. But this way, you'll be fully prepared for 22.05 and a lifetime of reactor physics, if you so choose.

Who here is done a year op in the computational reactor physics group? Just one, OK. I recommend more. They tend to be the biggest group in the department. They've got like 20 grad students and probably more year ops than that. So try it out. It's what makes us us, us nukes, right, is neutrons and tracking them to ridiculous proportions.

OK, definitely want to let you guys go it's one of. So I'll see you all on Thursday.