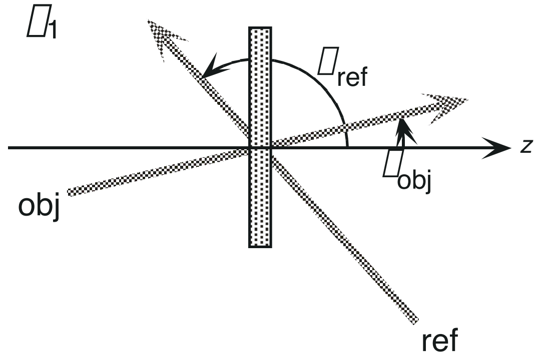
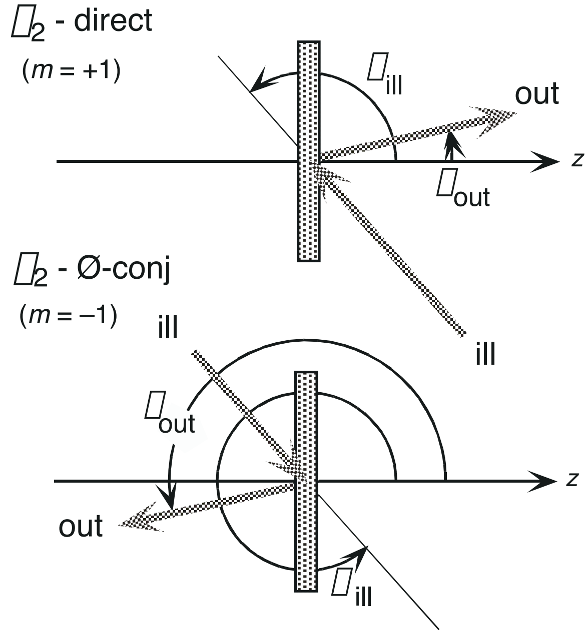
REFLECTION RAY-TRACING:

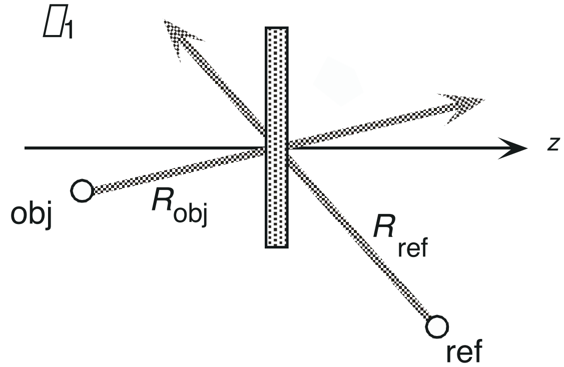
reference & illumination <u>angles</u> are measured "the long way around" and the object beam is in the +*z* direction

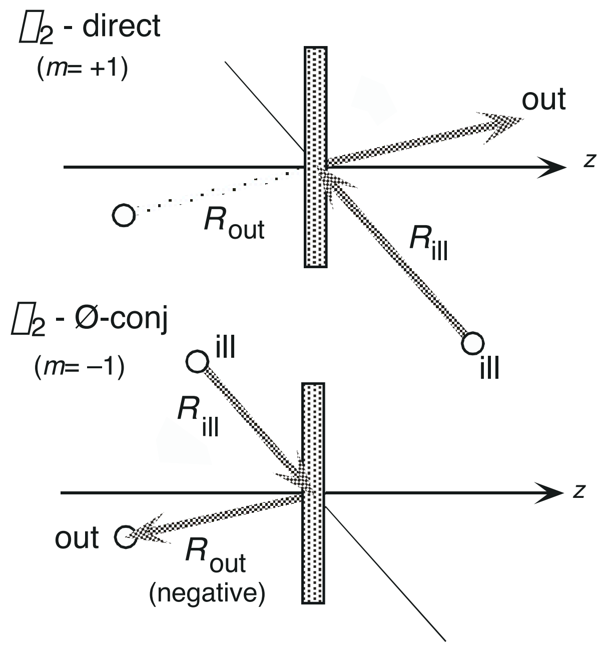




m = +1 for "direct" reconstruction
(illum & ref from **same** side)
–1 for "phase conjugation"
(illum & ref from **opposite** sides)

<u>distances</u> = radii of curvature (negative => real image)





HORIZONTAL FOCUS (<u>out</u> of the plane of the page)-

$$\frac{\frac{1}{R_{out}} - \frac{1}{R_{ill}}}{\lambda_2} = m\,\frac{\frac{1}{R_{obj}} - \frac{1}{R_{ref}}}{\lambda_1}$$

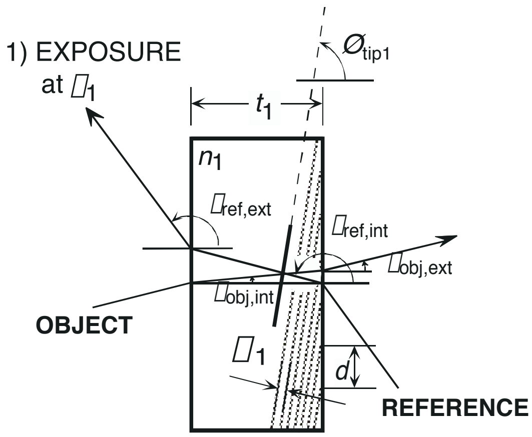
VERTICAL FOCUS (<u>in</u> of the plane of the page)-

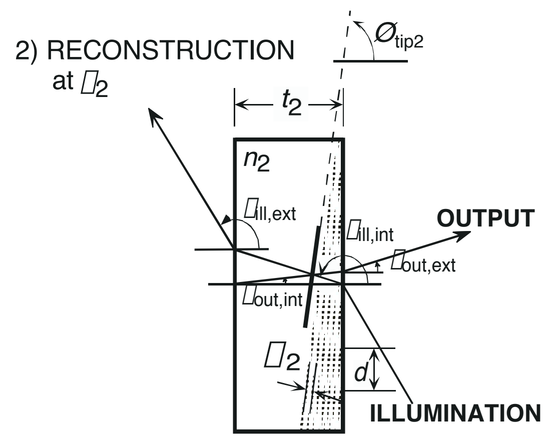
$$\frac{\frac{\cos^2\theta_{out}}{R_{out}} - \frac{\cos^2\theta_{ill}}{R_{ill}}}{\lambda_2} = m\,\frac{\frac{\cos^2\theta_{obj}}{R_{obj}} - \frac{\cos^2\theta_{ref}}{R_{ref}}}{\lambda_1}$$

<u>All</u> angles on this page are the usual **external** angles.
The usual time that internal angles (the θ') are used is in the "fringe-tip and -separation" calculations, and the "*z*-equation" for the allowed angles (if you use that approach).

The angles and wavelengths are first determined by those calculations, and are then plugged in to these focus equations to solve the imaging questions. Recall that negative radii of curvature mean converging waves for all directions of travel!

Off-Axis Reflection Holography
(direct, forward, *m*=+1 reconstruction)





recall: Snell's Law: $\sin\theta_{xxx,\mathrm{ext}} = n_i \cdot \sin\theta_{xxx,\mathrm{int}}$ also: $n_{\mathrm{ext}}\cdot\lambda_{\mathrm{ext}} = n_{\mathrm{int}}\cdot\lambda_{\mathrm{int}}$

<u>tilted-stacked-mirror representation:</u>

$$t_1 \cdot \tan\phi_{\mathrm{tip1}} = t_2 \cdot \tan\phi_{\mathrm{tip2}}$$

$$\phi_{\mathrm{tip1}} = \frac{\theta_{\mathrm{obj,int}}+\theta_{\mathrm{ref,int}}}{2}\ ,\quad \phi_{\mathrm{tip2}} = \frac{\theta_{\mathrm{out,int}}+\theta_{\mathrm{ill,int}}}{2}$$

$$\frac{t_1}{\Lambda_1}\sin\phi_{\mathrm{tip1}} = \frac{t_2}{\Lambda_2}\sin\phi_{\mathrm{tip2}}$$

$$\frac{1}{\Lambda_1} = \frac{2}{\lambda_{1,\mathrm{int}}}\cos\left(90^\circ + \frac{\theta_{\mathrm{obj,int}}-\theta_{\mathrm{ref,int}}}{2}\right)\ ,\quad \frac{1}{\Lambda_2} = \frac{2}{\lambda_{2,\mathrm{int}}}\cos\left(90^\circ + \frac{\theta_{\mathrm{out,int}}-\theta_{\mathrm{ill,int}}}{2}\right)$$

<u>*x*-, *z*-grating representation (all *m*):</u>

$$\frac{\sin\theta_{\mathrm{out,ext}}-\sin\theta_{\mathrm{ill,ext}}}{\lambda_{2,\mathrm{ext}}} = \frac{1}{d} = m\ \frac{\sin\theta_{\mathrm{obj,ext}}-\sin\theta_{\mathrm{ref,ext}}}{\lambda_{1,\mathrm{ext}}}$$ ⇒ means that $1/R$ and $\cos^2\theta/R$ still work!

$$n_2 \cdot t_2\ \frac{\cos\theta_{\mathrm{out,int}}-\cos\theta_{\mathrm{ill,int}}}{\lambda_{2,\mathrm{ext}}} = m\cdot n_1 \cdot t_1 \frac{\cos\theta_{\mathrm{obj,int}}-\cos\theta_{\mathrm{ref,int}}}{\lambda_{1,\mathrm{ext}}}$$ (±1, Goodman-Heisenberg Uncertainty)

Special Case: On-Axis Reflection "Denisyuk" Holography
(direct, forward, *m*=+1 reconstruction)

$\theta_{\mathrm{ref,ext}} = 180^\circ - \theta_{\mathrm{obj,ext}}$, so $\phi_{\mathrm{tip1}} = \phi_{\mathrm{tip2}} = 90^\circ$ (conformal fringes)

so that: $\theta_{\mathrm{out,ext}} = 180^\circ - \theta_{\mathrm{ill,ext}}$ (mirror reflection)

$$\frac{1}{\Lambda_1} = \frac{2\cdot n_1}{\lambda_{1,\mathrm{ext}}}\cos\left(\theta_{\mathrm{obj,int}}\right)$$

$$\frac{t_1}{\Lambda_1} = \frac{t_2}{\Lambda_2}$$

$$\frac{1}{\Lambda_2} = \frac{2\cdot n_2}{\lambda_{2,\mathrm{ext}}}\cos\left(\theta_{\mathrm{out,int}}\right),$$

or pulling it together: $$n_2 \cdot t_2 \frac{\cos\theta_{\mathrm{out,int}}}{\lambda_{2,\mathrm{ext}}} = n_1 \cdot t_1 \frac{\cos\theta_{\mathrm{obj,int}}}{\lambda_{1,\mathrm{ext}}}$$ (±1, GHU)