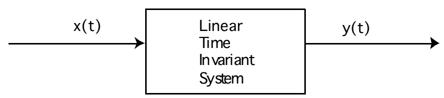
MAS.160 / MAS.510 / MAS.511 Signals, Systems and Information for Media Technology
Fall 2007

Linear Time Invariant Systems



Linearity

input $Ax_1(t)+Bx_2(t)$ ⟶ output $Ay_1(t)+By_2(t)$

scaling & superposition

Time invariance

$x(t-\tau)$ ⟶ $y(t-\tau)$

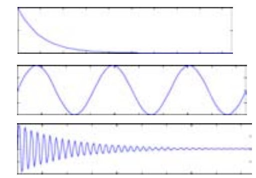
Characteristic Functions

e^{st} $s=a+jb$

complex exponentials

Complex Exponential Signals

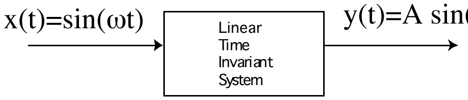
$s=a+jb$	e^{st}	
$s=\sigma$	$e^{-\sigma t}$	exponential decay
$s=\pm j\omega$	$e^{\pm j\omega t}$	sinusoids
$s=-\sigma \pm j\omega$	$e^{-\sigma \pm j\omega t}$	exponential sinusoids

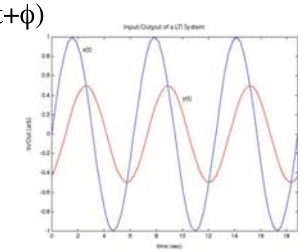


$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

characteristic functions of LTI systems

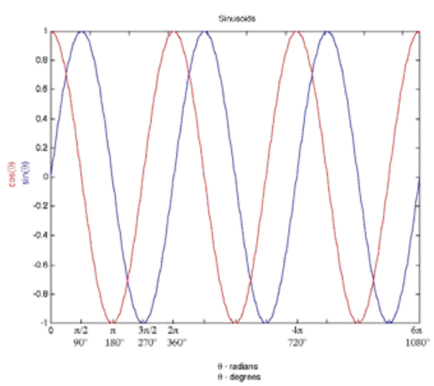
$x(t)=\sin(\omega t)$ → $y(t)=A\sin(\omega t+\phi)$





output has same frequency as input
but is scaled and phase shifted

Sinusoids



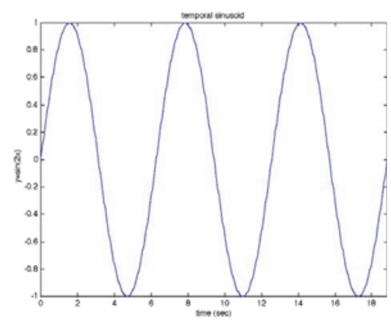
θ(rad)	$y_1=\sin(\theta)$	$y_2=\cos(\theta)$
0	0	1
π/6	.5	0.866
π /4	0.707	0.707
π /3	0.866	0.5
π /2	1	0
π	0	−1
3π/2	−1	0
2π	0	1

Periodic
$y(\theta)=y(\theta+2\pi n)$

sine odd
$\sin(-\theta)=\sin(\theta)$

cosine even
$\cos(-\theta)=-\cos(\theta)$

Continuous sinusoids $\theta=\theta(t)$



$y(t)=A\sin(\omega t+\phi)$
$y(t)=A\sin(2\pi ft+\phi)$

Parameters:

A: amplitude

ϕ: phase (radians)

ω: radian frequency (radians/sec)
or
f: frequency (cycles/sec-Hz)

Relations:

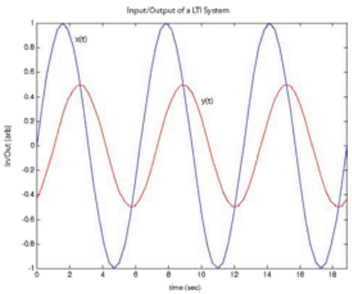
$\omega=2\pi f$ rad/sec= (2π rad/cycle)*cycle/sec

T:period (sec/cycle) $y(t)=y(t+T)$

$T = 1/f= 2\pi/\omega$

sec/cycle= 1/ (cycle/sec)= (2π rad/cycle)/(rad/sec)

Continuous sinusoids   θ=θ(t)



y(t)=A sin(ωt+ϕ)
y(t)=A sin(2πft+ϕ)

Parameters:

A: amplitude
ϕ: phase (radians)
ω: radian frequency (radians/sec)
or
f: frequency (cycles/sec-Hz)

Phase shift

In: x(t)=1sin(t)
Out: y(t)=0.5sin(t-π/3)

x(0)=0

y(π/3)=0

Sampled Continuous Sinusoid

Continuous Sinusoid

$$y(t) = \sin(2\pi t)$$

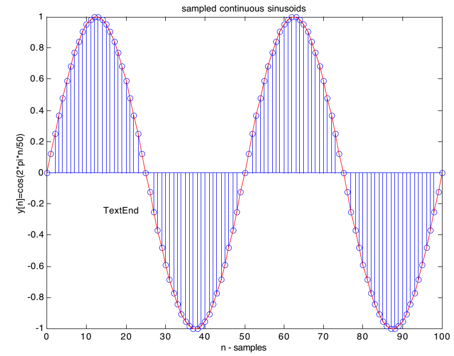
Sample rate: $T_s = \frac{1}{50}$ sec

$$t = nT_s$$

Discrete Sinusoid

$$y[n] = \sin(2\pi \cdot \tfrac{1}{50} \cdot n)$$

$$y[n] = \sin(\tfrac{\pi}{25} \cdot n)$$



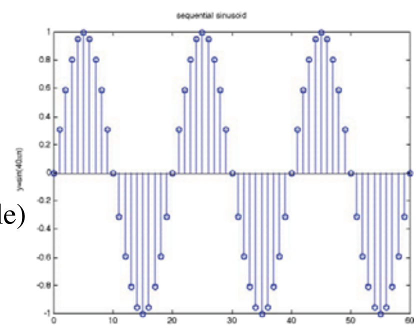
n	y[n]
0	0
1	0.125
2	0.249
3	0.368
4	0.4818

Discrete sinusoids   θ=θ[n]   n=0, 1, 2…

y[n]=A sin(ωn+ϕ)

y[n]=A sin(2πfn+ϕ)

A: amplitude
ϕ: phase (radians)
ω: radian frequency (radians/sample)
f: frequency (cycles/sample)



Relations:

ω=2πf rad/sample= (2π rad/cycle)*cycle/sample

N:period (samples/repeating cycle [integer])

Smallest integer N such that y[n]=y[n+N]

Find an integer k so N=k/f is also an integer $N \neq \frac{1}{f}$

f = k/N   (f: rational number -> k/N is ratio of integers)

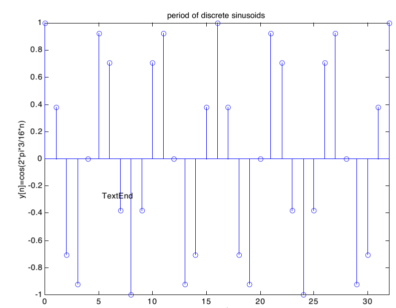
Period of discrete sinusoids

ex: y[n]=cos(2π(3/16)n)

What is the period N?

y[n]=A cos(2πfn+ϕ)

frequency: f=3/16 cycles/sample



N:period
(samples/repeating cycle [integer])

Smallest integer N such that y[n]=y[n+N]

Find an integer k so N=k/f is also an integer

f=3/16
let k=3
N=(3)*16/3=16

f = k/N   (rational number -> k/N is ratio of integers)

Period of discrete sinusoids: ex2.

$y[n] = \sin(2\pi \cdot \frac{3}{50} \cdot n)$

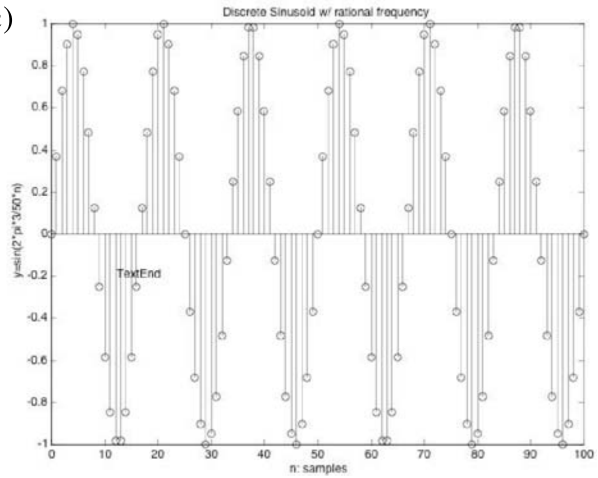
$f = \frac{3}{50} \frac{cycles}{sample}$

$y[n] = y[n+N]$

N=?? samples
N: integer

$N \neq \frac{1}{f}$

50/3 ≠ integer



Period of discrete sinusoids: ex2.

discrete function

$y[n] = \sin(2\pi \cdot \frac{3}{50} \cdot n)$

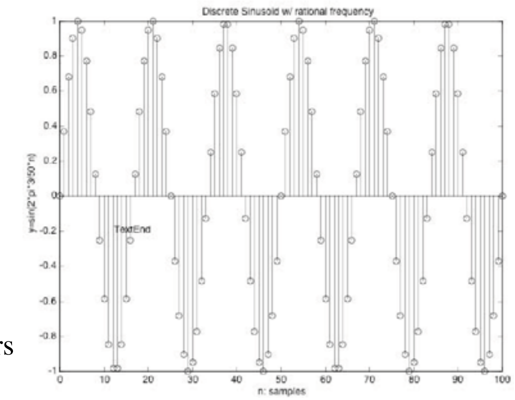
frequency: $f = \frac{3}{50} \frac{cycles}{sec}$

period: N=?? samples

$y[n] = y[n+N]$

$f \cdot N = k$ N, k:integers

$\frac{3}{50} \cdot N = k$



$\frac{N}{k} = \frac{50}{3} \frac{samples}{cycle}$ ratio of integers rational number

periodic
N=50 samples, k=3 cycles

Aperiodic discrete sinusoids

continuous function

$y(t) = \sin(2\pi \cdot \sqrt{2} \cdot t)$

$T = \frac{1}{\sqrt{2}}$ sec periodic

sample

$t = nT_s$ $T_s = \frac{1}{25}$ sec

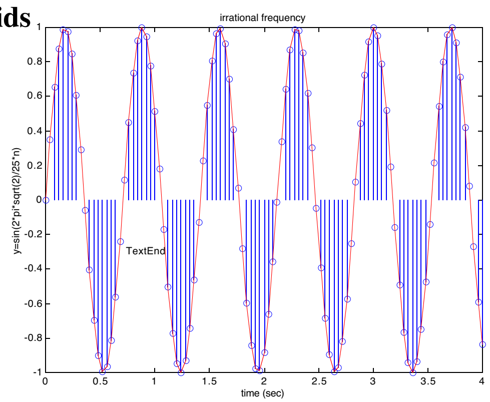
discrete function

$y[n] = \sin(2\pi \cdot \frac{\sqrt{2}}{25} \cdot n)$

period?

$y[n] = y[n+N]$

N=?? samples
(integer)



$f = \frac{\sqrt{2}}{25}$

$f \cdot N = k$ N,k: integers

$\frac{N}{k} = \frac{25\sqrt{2}}{2}$ not a ratio of integers irrational number

sampled discrete sinusoid aperiodic

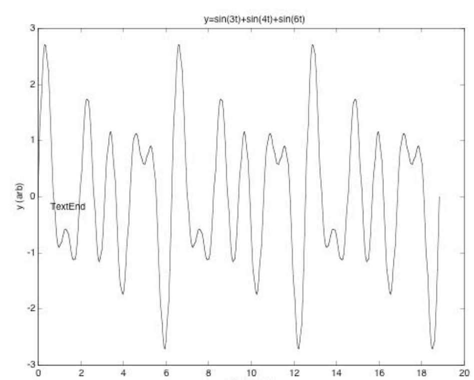
Periodicity

arbitrary continuous signal

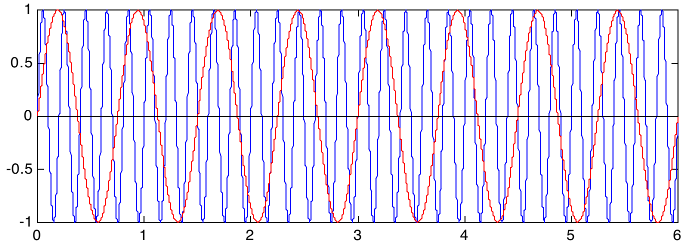
T:period (sec/cycle)

y(t+T)=y(t)

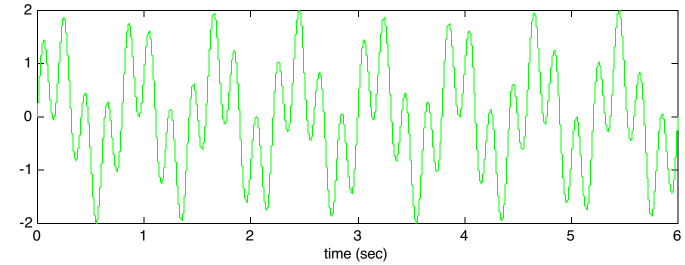
After what interval does the signal repeat itself?



Ex: Period of sum of sinusoids



T1=0.2 seconds, T2=0.75 seconds



Tsum=? seconds

Least common multiple

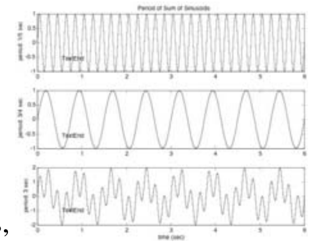
seconds to complete cycles

T1=1/5 seconds

1/5s, 2/5s, 3/5s …

4/20s, 8/20s, 12/20s, 16/20s, 20/20s, 24/20s, 28/20s, 32/20s, 36/20s, 40/20s, 44/20s, 48/20s, 52/20s, 56/20s, 60/20s

15 cycles



seconds to complete cycles

T2=3/4 seconds

3/4s, 6/4s, …

15/20s. 30/20s, 45/20s, 60/20s

4 cycles

1/5*k=3/4*l
k/l=15/4

rational number

Tsum=15*T1=15/5=3 seconds

Tsum=4*T2=3/4*4=3 seconds

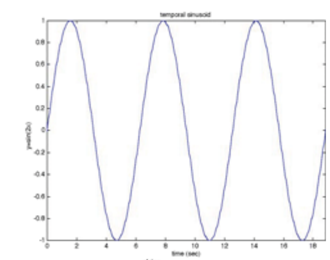
Tsum=3 seconds

Instantaneous frequency

$y(\theta)=\sin(\theta)$
$\theta=\theta(t)$ time varying argument

instantaneous frequency
$\omega=d\theta/dt$

sinusoid constant frequency
$y(t)=A\sin(\omega t+\phi)$
$\theta=\omega t+\phi$
$d\theta/dt=\omega$



chirp linearly swept frequency

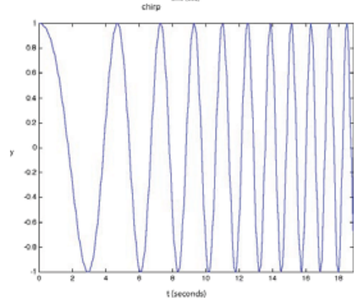
$\omega=d\theta/dt$
$\omega=((\omega_1-\omega_0)/T)t+\omega_0$

integrate

$\theta=(\omega_1-\omega_0)/2T\, t^2+\omega_0 t+C$

t	ω
0	ω_0
T	ω_1

$y_{chirp}(t)=A\sin((\omega_1-\omega_0)/(2T)\, t^2+\omega_0 t+\phi)$



Representations of a sinusoid

$y(t)=A\cos(\omega t+\phi)$ trig function

$y(t)=Ae^{j\phi}\frac{\left[e^{j\omega t}+e^{-j\omega t}\right]}{2}$ complex conjugates

$y(t)=\mathrm{Re}\{Ae^{j\phi}e^{j(\omega t)}\}$ real part of complex exponential

$$e^{j(\theta)}=e^{j(\omega t+\phi)}=e^{j\phi}e^{j\omega t}$$

$X=Ae^{j\phi}$ complex amplitude (constant)

$y(t)=\mathrm{Re}\{Xe^{j(\omega t)}\}$ rotating phasor

Euler's relations

$e^{j\theta}=\cos(\theta)+j\sin(\theta)$

$$\cos\theta=\frac{e^{j\theta}+e^{-j\theta}}{2}\qquad \sin\theta=\frac{e^{j\theta}-e^{-j\theta}}{2j}$$

Complex Exponentials

Why use complex exponentials?

Trigonometric manipulations -> algebraic operations on exponents

Trigonometric identities

$$\cos(x)\cos(y) = \tfrac{1}{2}\left[\cos(x-y) - \left[\cos(x+y)\right]\right]$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

Properties of exponentials

$re^x e^y = re^{x+y}$

$(re^x)^n = r^n e^{nx}$

$$\sqrt[n]{x} = x^{1/n}$$

$$\frac{1}{x} = x^{-1}$$

Vector representation (graphical)

Complex Exponentials

Amplitude modulation (multiply two sinusoids of different frequencies)

$$A\cos(\omega_1 t)\, B\cos(\omega_2 t+\phi) = C(\cos(\omega_3 t + \phi_2) + \cos(\omega_4 t + \phi_2))$$

$$A\cdot\cos(\omega 1)\cdot(B\cdot\cos(\omega 2 + \phi 2))$$

$$A\cdot B\cdot\cos(\omega 1)\cdot((\cos(\omega 2)\cdot\cos(\phi 2) - \sin(\omega 2)\cdot\sin(\phi 2)))$$

*sum formula for sin & cos trig id

$$A\cdot B\cdot\cos(\omega 1)\cdot\cos(\phi 2) - A\cdot B\cdot\cos(\omega 1)\cdot\sin(\omega 2)\cdot\sin(\phi 2)$$

$$A\cdot B\cdot\frac{\cos(\omega 2+\omega 1)+\cos(\omega 2-\omega 1)}{2}\cdot\cos(\phi 2) - A\cdot B\cdot\frac{\sin(\omega 2+\omega 1)+\sin(\omega 2-\omega 1)}{2}\cdot\sin(\phi 2)$$

*product formula for sin & cos trig id

$$A\cdot B\cdot\frac{\cos(\omega s)+\cos(\omega d)}{2}\cdot\cos(\phi 2) - A\cdot B\cdot\frac{\sin(\omega s)+\sin(\omega d)}{2}\cdot\sin(\phi 2)$$

$$\frac{1}{2}\cdot A\cdot B\cdot(\cos(\phi 2)\cdot\cos(\omega s) + \cos(\phi 2)\cdot\cos(\omega d) - \sin(\phi 2)\cdot\sin(\omega s) - \sin(\phi 2)\cdot\sin(\omega d))$$

$$\frac{1}{2}\cdot A\cdot B\cdot(\cos(\omega s+\phi 2) + \sin(\omega d + \phi 2))$$

*sum formula for sin & cos trig id

or

$$A\cdot\cos(\omega 1)\cdot(B\cdot\cos(\omega 2 + \phi 2))$$

$$A\cdot\left[\frac{e^{j\cdot\omega 1} + e^{-(j\cdot\omega 1)}}{2}\right]\cdot\left[B\cdot\left[\frac{e^{j\cdot(\omega 2+\phi)} + e^{-j\cdot(\omega 2+\phi)}}{2}\right]\right]$$

cos = complex conj

mult. exponentials

$$\frac{1}{4}\cdot A\cdot B\cdot\left[\exp[j\cdot(\phi+\omega 2+\omega 1)] + \exp[-j\cdot(\phi+\omega 2+\omega 1)] + \exp[-j\cdot(\phi+\omega 2-\omega 1)] + \exp[j\cdot(\phi+\omega 2-\omega 1)]\right]$$

$$\frac{1}{4}\cdot A\cdot B\cdot(2\cdot\cos(\phi+\omega 2+\omega 1) + 2\cdot\cos(\phi+\omega 2-\omega 1))$$

cos = complex conj

$$\frac{1}{2}\cdot A\cdot B\cdot(\cos(\phi+\omega 2+\omega 1) + \cos(\phi+\omega 2-\omega 1))$$

Representations of a sinusoid

$y(t) = A\cos(\omega t+\phi)$ — trig function

$y(t) = Ae^{j\phi}\dfrac{\left[e^{j\omega t} + e^{-j\omega t}\right]}{2}$ — complex conjugates

$$e^{j(\theta)} = e^{j(\omega t+\phi)} = e^{j\phi}e^{j\omega t}$$

$y(t) = \mathrm{Re}\{Ae^{j\phi}e^{j(\omega t)}\}$ — real part of complex exponential

$X = Ae^{j\phi}$ complex amplitude (constant)

$y(t) = \mathrm{Re}\{Xe^{j(\omega t)}\}$ — rotating phasor

Euler's relations

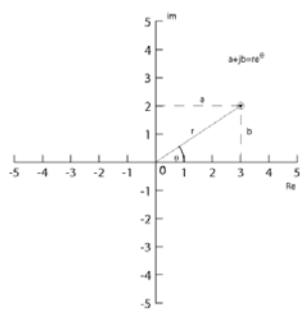
$e^{j\theta} = \cos(\theta) + j\sin(\theta)$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \qquad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

complex numbers

cartesian $s = a + jb$ $\quad j = \sqrt{-1}$

polar $s = re^{j\theta}$



conversion

$$a = r\cos(\theta)$$
$$b = r\sin(\theta)$$

$$r = \sqrt{a^2 + b^2}$$
$$\theta = a\tan 2\left(\frac{b}{a}\right)$$

quadrants!

$e^{j0} = 1$

$e^{j\pi/2} = i$

$e^{j\pi} = -1$

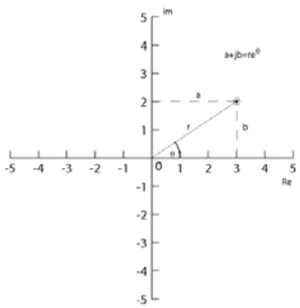
conjugate

$s^* = a - jb = re^{-j\theta}$

complex numbers

cartesian s=a+jb $j = \sqrt{-1}$

polar s=re$^{j\theta}$



conversion

$$a = r\cos(\theta)$$
$$b = r\sin(\theta)$$

$$r = \sqrt{a^2 + b^2}$$
$$\theta = a\tan\left(\frac{b}{a}\right)$$

remember:

$e^{j0}=1$

$e^{j\pi/2}=j$

$e^{j\pi}=-1$

ex.

$1^j = (e^{j0})^j$
$= e^{j(j)(0)}$
$= e^{-1(0)}$
$= e^0 = 1$

ex. cos(j)?
1/j?

conjugate

s*=a-jb=re$^{-j\theta}$

complex arithmetic

Addition Subtraction
Multiplication Division
Powers Roots

complex numbers

cartesian

$s_1=3+j2$

$=\sqrt{3^2+2^2}\cdot e^{j\cdot \mathrm{atan}\left[\frac{2}{3}\right]}$

$= \sqrt{13}\ e^{j\,0.588}$

$s_2=-2+j1$

$= \sqrt{(-2)^2+1^2}\cdot e^{j\cdot\left[\mathrm{atan}\left[\frac{1}{-2}\right]+\pi\right]}$

$= \sqrt{5}\ e^{j2.678}$

polar

$s_1=\sqrt{13}\ e^{j\,0.588}$

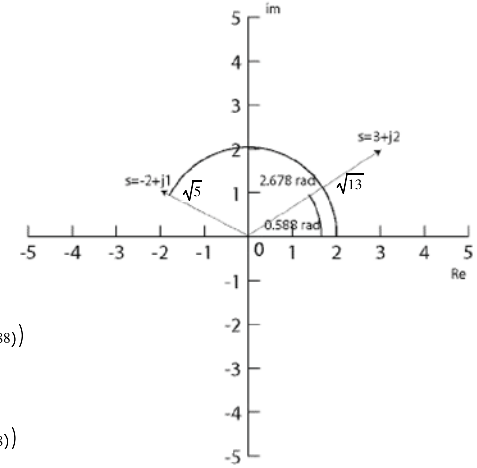
$=\sqrt{13}\cdot\cos(0.588) + j\cdot\left(\sqrt{13}\cdot\sin(0.588)\right)$

$=3+j2$

$s_2=\sqrt{5}\ e^{j2.678}$

$= \sqrt{5}\cdot\cos(2.678) + j\cdot\left(\sqrt{5}\cdot\sin(2.678)\right)$

$=-2+j1$



Addition

cartesian

$s_1=a_1+jb_1$
$s_2=a_2+jb_2$

$s_1+s_2=a_1+jb_1 + a_2+jb_2$
$=(a_1 + a_2) +j(b_1+b_2)$

polar

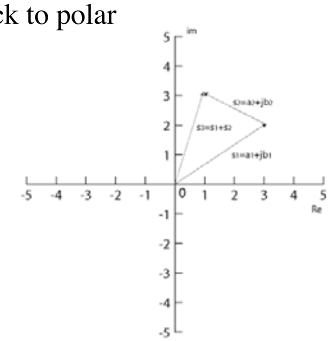
$s_1=r_1e^{j\theta 1}$
$s_2=r_2e^{j\theta 2}$

convert to cartesian
add
convert back to polar

vector

place tail of s_2 at head of s_1

connect origin to s_2



Addition - example

cartesian

$s_1=3+j2$

$s_2=-2+j1$

$$s_1+s_2= a_1 + a_2 +j(b_1+b_2) = 3 -2 +j(2+1) = 1 +j3$$

polar

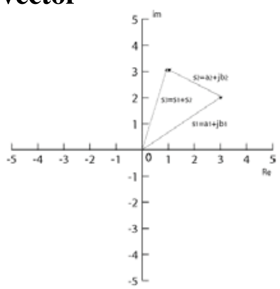
$s_1=\sqrt{13}\ e^{j\,0.588}$

$s_2=\sqrt{5}\ e^{j2.678}$

$$s_1= \sqrt{13}\cos(0.588)+j\sqrt{13}\ \sin(0.588)= 3+j2$$

$$s_2= \sqrt{5}\ \cos(2.678)+j\sqrt{5}\ \sin(2.678)= -2+j1$$

vector



$$s_1+s_2 = 1 +j3 = \sqrt{1^2+3^2}\cdot e^{j\cdot \operatorname{atan}(3)} = \sqrt{10}\cdot e^{j\cdot 1.249}$$

Subtraction

cartesian

$s_1=a_1+jb_1$

$s_2=a_2+jb_2$

$$s_1-s_2=a_1+jb_1 - (a_2+jb_2) =(a_1 - a_2) + j(b_1-b_2)$$

polar

$s_1=r_1e^{j\theta1}$

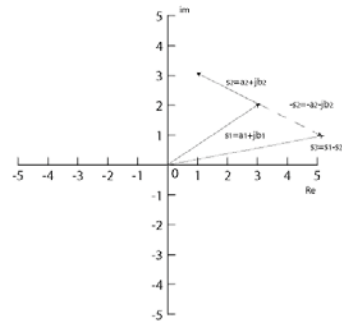
$s_2=r_2e^{j\theta2}$

convert to cartesian
add
convert back to polar

vector

rotate s_2 180° ($-s_2$)
place tail of $-s_2$ at head of s_1

connect origin to s_2



Subtraction - example

cartesian

$s_1=3+j2$

$s_2=-2+j1$

$$s_1-s_2 =(a_1 - a_2) + j(b_1-b_2) =(3 - (-2)) + j(2-1) =5 + j1$$

polar

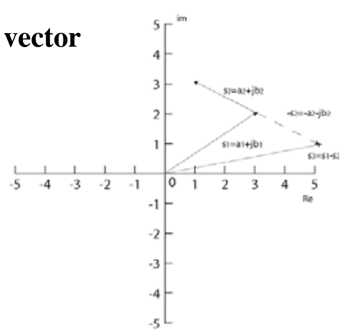
$s_1=\sqrt{13}\ e^{j\,0.588}$

$s_2=\sqrt{5}\ e^{j2.678}$

$$s_1= \sqrt{13}\cos(0.588)+j\sqrt{13}\ \sin(0.588)= 3+j2$$

$$s_2= \sqrt{5}\ \cos(2.678)+j\sqrt{5}\ \sin(2.678)= -2+j1$$

vector



$$s_1-s_2 = 5 +j1 = \sqrt{1^2+5^2}\cdot e^{j\cdot \operatorname{atan}\left[\frac{1}{5}\right]} = \sqrt{26}\cdot e^{j\cdot 0.197}$$

Multiplication

cartesian

$s_1=a_1+jb_1$

$s_2=a_2+jb_2$

$$s_1s_2=(a_1+jb_1)(a_2+jb_2) = a_1a_2 + ja_1b_2 + ja_2b_1 + jb_1jb_2 = a_1a_2 + j^2b_1b_2 + j(a_1b_2+a_2b_1) = a_1a_2 -b_1b_2 + j(a_1b_2+a_2b_1)$$

polar

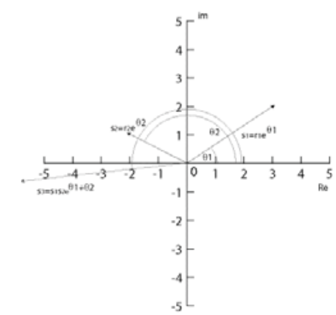
$s_1=r_1e^{j\theta1}$

$s_2=r_2e^{j\theta2}$

$$s_1s_2 =r_1e^{j\theta1}\, r_2e^{j\theta2} =r_1r_2e^{j\theta1}\, e^{j\theta2} =r_1r_2e^{j\theta1+\theta2}$$

vector

magnitudes multiply
angles add



$x^ax^b=x^{a+b}$

Multiplication - example

cartesian

$s_1=3+j2$
$s_2=-2+j1$

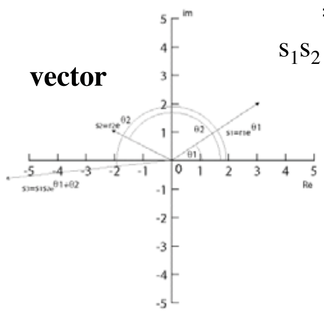
$$s_1 s_2 = a_1a_2 - b_1b_2 + j(a_1b_2+a_2b_1)$$
$$= 3(-2) - 2(1) + j(3(1)+(2)(2))$$
$$= -6 - 2 + j(3-4)$$
$$= -8 - j1$$

polar

$s_1=\sqrt{13}\,e^{j\,0.588}$
$s_2=\sqrt{5}\,e^{j2.678}$

$$s_1 s_2 = r_1 r_2 e^{j\theta 1+\theta 2}$$
$$= \sqrt{13}\ \sqrt{5} e^{j(0.588+2.678)}$$
$$= \sqrt{65}\, e^{j3.266}$$
$$s_1 s_2 = \sqrt{65}\cos(3.266) + \sqrt{65}\, j\sin(3.266)$$
$$= -8 - j1$$

vector



Division

cartesian

$s_1=a_1+jb_1$
$s_2=a_2+jb_2$

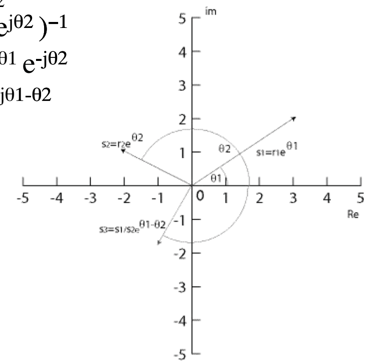
$$s_1/s_2=(a_1+jb_1)/(a_2+jb_2)$$
$$=(a_1+jb_1)(a_2-jb_2)/(a_2+jb_2)(a_2-jb_2)$$
$$=(a_1+jb_1)(a_2-jb_2)/(a_2^2+b_2^2)$$
$$=s_1 s_2^{*}/|s_2|^2$$

polar

$s_1=r_1e^{j\theta 1}$
$s_2=r_2e^{j\theta 2}$

$$s_1/s_2 = r_1e^{j\theta 1}/r_2e^{j\theta 2}$$
$$=r_1e^{j\theta 1}(r_2e^{j\theta 2})^{-1}$$
$$=(r_1/r_2)e^{j\theta 1}e^{-j\theta 2}$$
$$=(r_1/r_2)e^{j\theta 1-\theta 2}$$

vector



divide magnitudes
angles subtract

$x^a/x^b=x^{a-b}$

Division - example

cartesian

$s_1=3+j2$
$s_2=-2+j1$

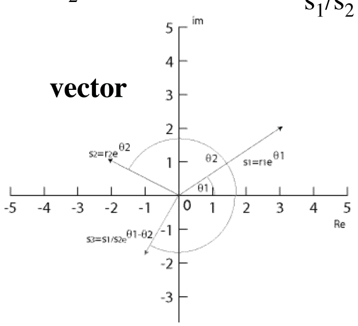
$$s_1/s_2 = s_1 s_2^{*}/|s_2|^2$$
$$=(a_1+jb_1)(a_2-jb_2)/(a_2^2+b_2^2)$$
$$=(3+j2)(-2-j1)/(2^2+1^2)$$
$$=(3(-2)-j3(1)+j2(-2)+2(1))/(5)$$
$$=(-6-j3-j4+2)/(5)$$
$$=(-4-j7)/(5)$$

polar

$s_1=\sqrt{13}\,e^{j\,0.588}$
$s_2=\sqrt{5}\,e^{j2.678}$

$$s_1/s_2=(r_1/r_2)e^{j\theta 1-\theta 2}$$
$$=(\sqrt{13}/\sqrt{5})\,e^{j0.588-2.678}$$
$$=(\sqrt{13}/\sqrt{5})\,e^{j0.588-2.678}$$
$$=\sqrt{\frac{13}{5}}\cdot e^{-j\cdot 2.09}$$
$$=\sqrt{\frac{13}{5}}\cos(-2.09)+j\sqrt{\frac{13}{5}}\sin(-2.09)$$
$$=-0.8-j1.4$$

vector



Powers

cartesian

$s=a+jb$

$$s^n=(a+jb)^n$$

Binomial expansion

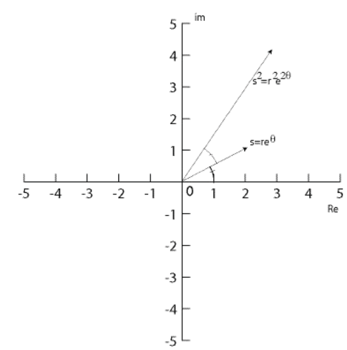
$$=\sum_{k=0}^{n}\binom{n}{k}a^{n-k}(jb)^k \quad \text{where} \quad \binom{n}{k}=\frac{n(n-1)(n-2)\ldots(n-(k-1))}{k!}$$

polar

$s=re^{j\theta}$

$$s^n=(re^{j\theta})^n$$
$$s^n=r^ne^{jn\theta}$$

vector



magnitude raised to power n
angle multiplied by n

$(x^a)^n=x^{an}$

Powers - example $s^2=(2+j1)^2$

cartesian

$s=2+j1$

$=\sqrt{2^2+1^2}\cdot e^{j\cdot \operatorname{atan}\left[\frac{1}{2}\right]}$

$=\sqrt{5}\cdot e^{j\cdot 0.464}$

polar

$s=re^{j\theta}$

$=\sqrt{5}\cdot e^{j\cdot 0.464}$

$s^n=r^ne^{jn\theta}$

$s^2=r^2e^{j2\theta}$

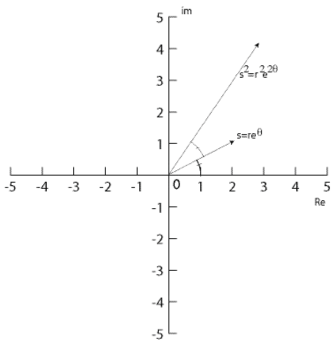
$s^2=\sqrt{5}^{\,2}e^{j2(0.464)}$

$s^2=5e^{j0.927}$

$=5\cos(0.927)+j5\sin(0.927)$

$=3+j4$

vector



Roots

cartesian

$s=a+jb$ $\qquad s^{1/n}=(a+jb)^{1/n}$ $\qquad$???

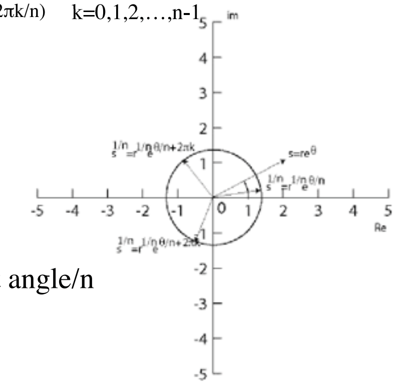
$$= a^{1/n}\left(1+j\frac{b}{a}\right)^{1/n} = 1+j\frac{1}{n}\frac{b}{a}+\binom{1/n}{2}\left(j\frac{b}{a}\right)^2+\binom{1/n}{3}\left(j\frac{b}{a}\right)^3+\ldots$$

polar

$s=re^{j\theta}$ $\qquad s^{1/n}=(re^{j\theta})^{1/n}$

$s^{1/n}=r^{1/n}e^{j(\theta/n+2\pi k/n)}$ $\quad k=0,1,2,\ldots,n-1$

vector



nth positive root of magnitude

circle evenly divided by n starting at angle/n

$\sqrt[n]{x}=x^{1/n}$

Roots - example $s^{1/3}=(a+jb)^{1/3}$

cartesian

$s=2+j1$

$s^{1/n}=r^{1/n}e^{j(\theta/n+2\pi k/n)}$ $\quad k=0,1,2,\ldots,n-1$

$s^{1/3}=\sqrt{5}^{\,1/3}e^{j(0.464/3+2\pi k/3)}$ $\quad k=0,1,2$

polar

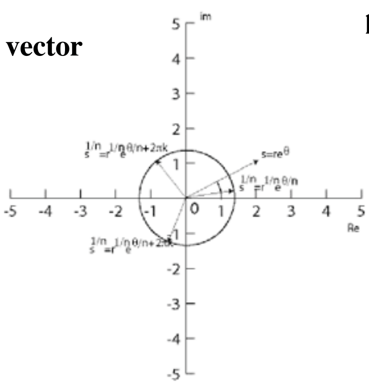
$=\sqrt{5}\cdot e^{j\cdot 0.464}$

$s^{1/3}=1.308\ e^{j(0.464/3)}$

$=1.308\ e^{j(0.464/3+2\pi 1/3)}$

$1.308\ e^{j(0.464/3+2\pi 2/3)}$

vector



$s^{1/3}=1.308\ e^{j0.155}=1.292+j0.202$

$=1.308\ e^{j2.249}=-0.821+j1.019$

$=1.308\ e^{j4.343}=-0.472-j1.22$

Complex Conversions

cartesian → polar: $s=a+jb$ $\quad s=\sqrt{a^2+b^2}\,e^{j\cdot a\tan(b/a)}$

polar → cartesian: $s=re^{j\theta}$ $\quad s=r\cos\theta+jr\sin\theta$

Complex Arithmetic		
Addition	cartesian	$(a_1+jb_1)+(a_2+jb_2)=(a_1+a_2)+j(b_1+b_2)$
Subtraction	cartesian	$(a_1+jb_1)-(a_2+jb_2)=(a_1-a_2)+j(b_1-b_2)$
Multiplication	polar	$r_1e^{j\theta_1}\cdot r_2e^{j\theta_2}=r_1r_2e^{j(\theta_1+\theta_2)}$
Division	polar	$\frac{r_1e^{j\theta_1}}{r_2e^{j\theta_2}}=\frac{r_1}{r_2}e^{j(\theta_1-\theta_2)}$
Powers	polar	$\left(re^{j\theta}\right)^n=r^ne^{jn\theta}$
Roots	polar	$z^n=s=re^{j\theta}$ $z=s^{1/n}=r^{1/n}e^{j(\theta/n+2\pi k/n)}$ $\quad k=1,2\ldots n-1$