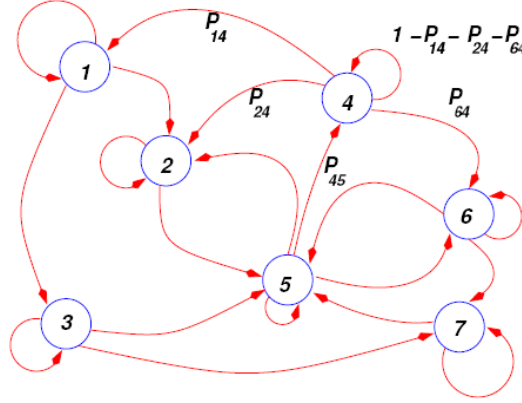


An example of the Transition Equations

Transition equations: $\pi_i(t+1) = \sum_j P_{ij} \pi_j(t)$

Transition graph



For node 2, by total probability theorem, we have

$$\begin{aligned}
 P(X(t+1) = 2) &= P(X(t+1) = 2 | X(t) = 1) \cdot P(X(t) = 1) \\
 &\quad + P(X(t+1) = 2 | X(t) = 2) \cdot P(X(t) = 2) \\
 &\quad + P(X(t+1) = 2 | X(t) = 4) \cdot P(X(t) = 4) \\
 &\quad + P(X(t+1) = 2 | X(t) = 5) \cdot P(X(t) = 5)
 \end{aligned} \tag{1}$$

Note the definition of the notation $\pi_i(t) = \text{prob}\{X(t) = i\}$, we have

$$\begin{aligned}
 P(X(t+1) = 2) &= \pi_2(t+1) \\
 P(X(t) = 1) &= \pi_1(t) \\
 P(X(t) = 2) &= \pi_2(t) \\
 P(X(t) = 4) &= \pi_4(t) \\
 P(X(t) = 5) &= \pi_5(t)
 \end{aligned}$$

and

$$\begin{aligned}
 P(X(t+1) = 2 | X(t) = 1) &= P_{21} \\
 P(X(t+1) = 2 | X(t) = 2) &= P_{22} \\
 P(X(t+1) = 2 | X(t) = 4) &= P_{24} \\
 P(X(t+1) = 2 | X(t) = 5) &= P_{25}
 \end{aligned}$$

Thus, Equation 1 is equal to

$$\pi_2(t+1) = P_{21} \pi_1(t) + P_{22} \pi_2(t) + P_{24} \pi_4(t) + P_{25} \pi_5(t)$$

where $P_{22} = 1 - P_{52}$.

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Fall 2016

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