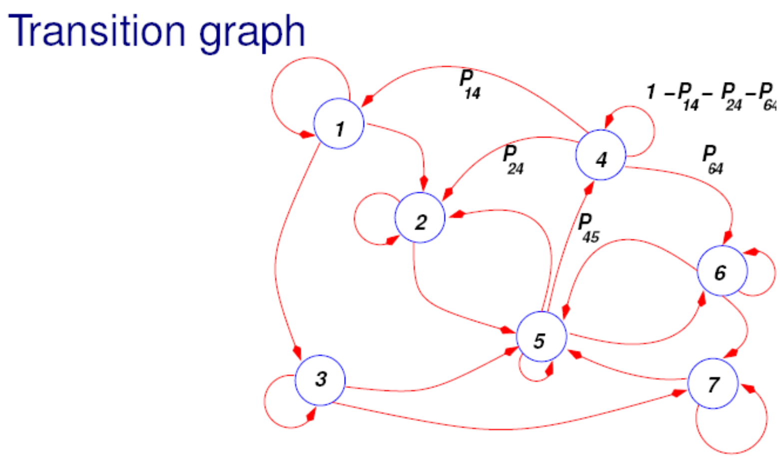
An example of the Transition Equations

Transition equations: $\pi_i(t+1) = \sum_j P_{ij}\pi_j(t)$



For node 2, by total probability theorem, we have

$$\begin{aligned} P(X(t+1)=2) = & P(X(t+1)=2 \mid X(t)=1)\cdot P(X(t)=1) \\ & + P(X(t+1)=2 \mid X(t)=2)\cdot P(X(t)=2) \\ & + P(X(t+1)=2 \mid X(t)=4)\cdot P(X(t)=4) \\ & + P(X(t+1)=2 \mid X(t)=5)\cdot P(X(t)=5) \end{aligned} \quad (1)$$

Note the definition of the notation $\pi_i(t) = prob\{X(t)=i\}$, we have

$$\begin{aligned} & P(X(t+1)=2) = \pi_2(t+1) \\ & P(X(t)=1) = \pi_1(t) \\ & P(X(t)=2) = \pi_2(t) \\ & P(X(t)=4) = \pi_4(t) \\ & P(X(t)=5) = \pi_5(t) \end{aligned}$$

and

$$\begin{aligned} & P(X(t+1)=2 \mid X(t)=1) = P_{21} \\ & P(X(t+1)=2 \mid X(t)=2) = P_{22} \\ & P(X(t+1)=2 \mid X(t)=4) = P_{24} \\ & P(X(t+1)=2 \mid X(t)=5) = P_{25} \end{aligned}$$

Thus, Equation 1 is equal to

$$\pi_2(t+1) = P_{21}\pi_1(t) + P_{22}\pi_2(t) + P_{24}\pi_4(t) + P_{25}\pi_5(t)$$

where $P_{22} = 1 - P_{52}$.

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