MIT 2.853/2.854

Introduction to Manufacturing Systems

Single-part-type, multiple stage systems

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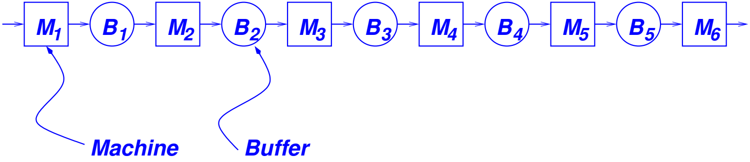
Flow Lines

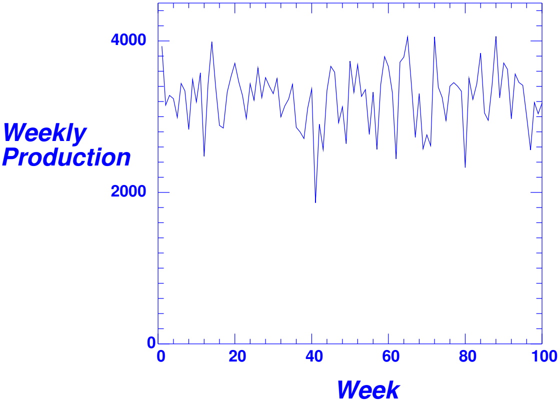
... also known as a Production or Transfer Line.



- Machines are unreliable.
- Buffers are finite.
- In many cases, the operation times are constant and equal for all machines.

Flow Lines

Output Variability



Production output from a simulation of a transfer line.

Reliable Machines

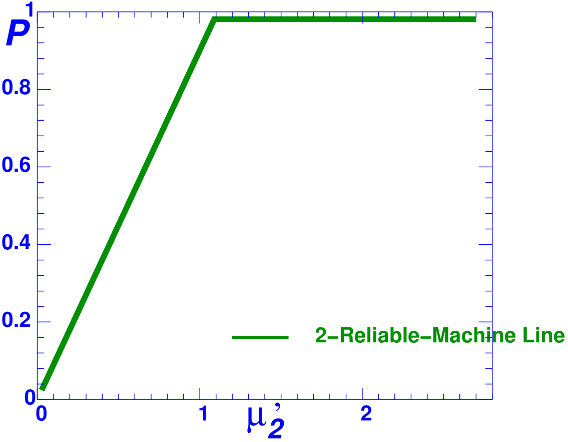
Single Reliable Machine

- If the machine is perfectly reliable, and its average operation time is τ, then its maximum production rate is $\mu = 1/\tau$.
- *Note:*
  - ⋆ Sometimes *cycle time* is used instead of *operation time,* but *BEWARE:* cycle time has two meanings!
  - ⋆ The other meaning is the time a part spends in a system. If the system is a single, reliable machine, the two meanings are the same.

Reliable Machines

Two Reliable Machines

Production rate in a two-machine reliable transfer line.

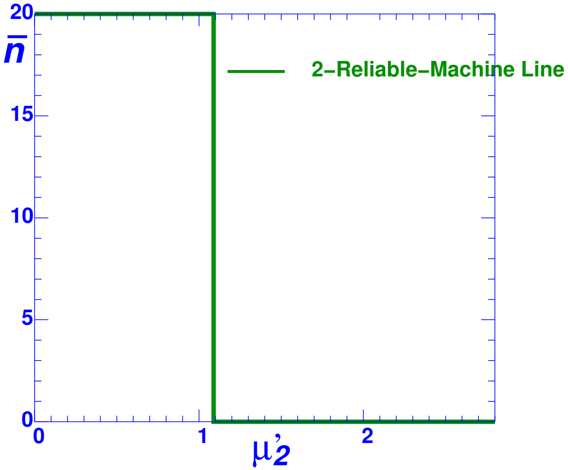


(Prime to be explained later.)

Reliable Machines

Two Reliable Machines

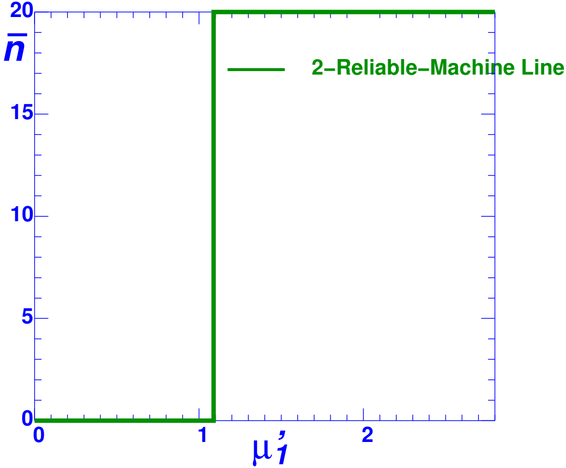
Inventory in a two-machine reliable transfer line.



Reliable Machines

Two Reliable Machines

Inventory in a two-machine reliable transfer line.



Single Unreliable Machine

Failures and Repairs

- Machine is either *up* or *down* .
- MTTF = mean time to fail.
- MTTR = mean time to repair
- MTBF = MTTF + MTTR

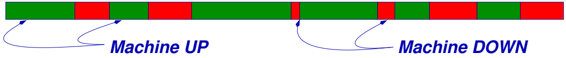
Single Unreliable Machine
Production rate

- If the machine is unreliable, and
  - ⋆ its average operation time is τ,
  - ⋆ its mean time to fail is MTTF,
  - ⋆ its mean time to repair is MTTR,

  then its maximum production rate is

$$\frac{1}{\tau}\left(\frac{\text{MTTF}}{\text{MTTF}+\text{MTTR}}\right)$$

Single Unreliable Machine
Proof



- Average production rate, while machine is up, is $1/\tau$.
- Average duration of an up period is MTTF.
- Average production during an up period is MTTF$/\tau$.
- Average duration of up-down period: MTTF $+$ MTTR.
- Average production during up-down period: MTTF$/\tau$.
- Therefore, average production rate is (MTTF$/\tau$)/(MTTF $+$ MTTR).

Single Unreliable Machine

Geometric Up- and Down-Times

- *Assumptions:* Operation time is constant (τ). Failure and repair times are *geometrically* distributed.

- Let p be the probability that a machine fails during any given operation. Then $p = \tau/\text{MTTF}$.

Single Unreliable Machine
Geometric Up- and Down-Times

- Let r be the probability that M gets repaired during any operation time when it is down. Then $r = \tau/\text{MTTR}$.

- Then the *average production rate* of M is

$$\frac{1}{\tau}\left(\frac{r}{r+p}\right).$$

- *(Sometimes we forget to say "average.")*

Single Unreliable Machine
Production Rates

- So far, the machine really has *three* production rates:
  - $1/\tau$ when it is up *(short-term capacity)* ,
  - 0 when it is down *(short-term capacity)* ,
  - $(1/\tau)(r/(r+p))$ on the average *(long-term capacity)* .

Single Unreliable Machine
ODFs

- Operation-Dependent Failures
  - A machine can only fail while it is working — not idle.
  - *(When buffers are finite, idleness also occurs due to blockage.)*
  - *IMPORTANT!* MTTF *must* be measured in working time!
  - This is the usual assumption.

Infinite-Buffer Lines

- Starvation: Machine M_i is starved at time t if Buffer B_{i-1} is empty at time t.

Assumptions:

- A machine is not idle if it is not starved.

- The first machine is never starved.

Infinite-Buffer Lines

Bottleneck

- The production rate of the line is the production rate of the *slowest* machine in the line — called the *bottleneck* .

- Slowest means *least average production rate* , where average production rate is calculated from one of the previous formulas.

Infinite-Buffer Lines
Bottleneck

- Production rate is therefore

$$P = \min_i \frac{1}{\tau_i} \left(\frac{\text{MTTF}_i}{\text{MTTF}_i + \text{MTTR}_i} \right)$$

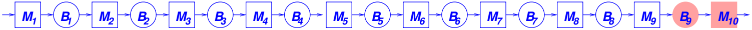
- and M_i is the bottleneck.

Infinite-Buffer Lines

Bottleneck

- The system is not in steady state.
- An increasing amount of inventory accumulates in the buffer upstream of the bottleneck.
- A finite amount of inventory appears downstream of the bottleneck.

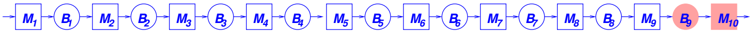
Infinite-Buffer Lines
Example 1

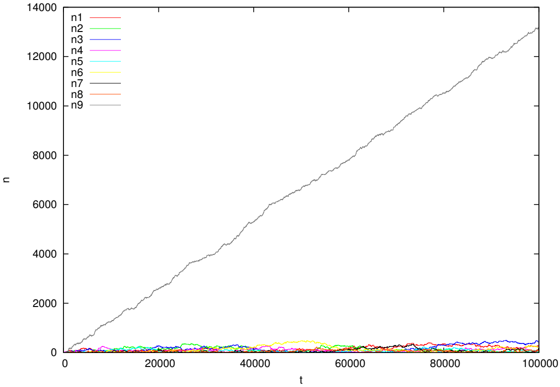


- Parameters:
  $r_i = .1, p_i = .01, i = 1, ..., 9; r_{10} = .1, p_{10} = .03.$
- Therefore, $e_i = .909, i = 1, ..., 9; e_{10} = .769.$

Infinite-Buffer Lines

Example 1





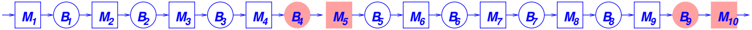
Infinite-Buffer Lines

Example 1

- *Estimate the rate of growth of* $n_9(t)$*, the inventory in* B_9*.*

Infinite-Buffer Lines

Second Bottleneck



- The *second bottleneck* is the slowest machine upstream of the bottleneck. An increasing amount of inventory accumulates just upstream of it.
- A finite amount of inventory appears between the second bottleneck and the machine upstream of the first bottleneck.
- A finite amount of inventory appears downstream of the first bottleneck.

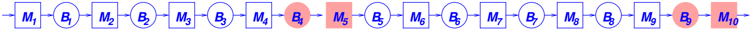
Infinite-Buffer Lines
Example 2

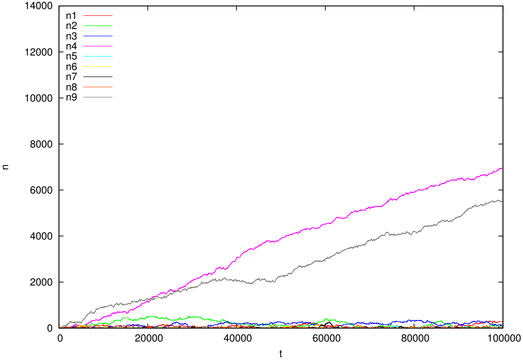
- Parameters: $r_i = .1, p_i = .01, i = 1, ..., 4, 6, ..., 9;$ $r_5 = .1, p_5 = .02, r_{10} = .1, p_{10} = .03.$

- Therefore, $e_i = .909, i = 1, ..., 4, 6, ..., 9;$ $e_5 = .833, e_{10} = .769.$

Infinite-Buffer Lines

Example 2





Infinite-Buffer Lines
Example 2

- *Estimate the rates of growth of* $n_4(t)$ *and* $n_9(t)$.

Infinite-Buffer Lines
Example 2

- Note that when t is large enough, $n_4(t) > n_9(t)$.

- Manufacturing people sometimes say that the easiest way to find the bottleneck of a line is to look for the greatest accumulation of inventory. *Is that correct?*

Infinite-Buffer Lines
Improvements

Questions:

- If we want to increase production rate, which machine should we improve?
- What would happen to production rate if we improved any other machine?

Simulation Note

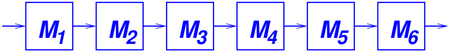
- The simulations shown here were *time-based* rather than *event-based* .
- Time-based simulations are easier to program, but less general, less accurate, and slower, than event-based simulations.
- Primarily for systems where all event times are geometrically distributed.

Simulation Note

Assume that some event occurs according to a geometric probability distribution and it has a mean time to occur of T time steps. Then the probability that it occurs in any time step is $1/T$.

- At each time step , choose a U[0,1] random number.
- If the number is less than or equal to $1/T$, the event has occurred. Change the state accordingly.
- If the number is greater than $1/T$, the event has not occurred. Change the state accordingly.

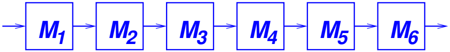
Zero-Buffer Lines



- If any one machine fails, or takes a very long time to do an operation, *all* the other machines must wait.

- Therefore the production rate is usually less — possibly much less – than the slowest machine.

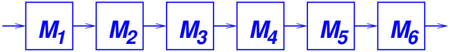
Zero-Buffer Lines



- *Example:* Constant, unequal operation times, perfectly reliable machines.
  - ⋆ The operation time of the line is equal to the operation time of the slowest machine, so the production rate of the line is *equal to* that of the slowest machine.

Zero-Buffer Lines

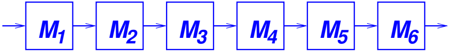
Constant, equal operation times, unreliable machines



- *Assumption:* Failure and repair times are *geometrically* distributed.
- Define $p_i = \tau/\text{MTTF}_i =$ probability of failure during an operation.
- Define $r_i = \tau/\text{MTTR}_i$ probability of repair during an interval of length τ when the machine is down.

Zero-Buffer Lines

Production Rate

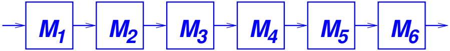


Buzacott's Zero-Buffer Line Formula:

Let k be the number of machines in the line. Then

$$P = \frac{1}{\tau} \frac{1}{1 + \sum_{i=1}^{k} \frac{p_i}{r_i}}$$

Zero-Buffer Lines
Production Rate



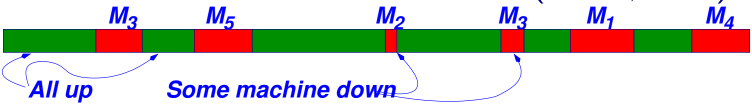
- Same as the earlier formula (Slides 9 and 12) when $k = 1$. The *isolated production rate* of a single machine M_i is

$$\frac{1}{\tau}\left(\frac{1}{1+\frac{p_i}{r_i}}\right) = \frac{1}{\tau}\left(\frac{r_i}{r_i+p_i}\right).$$

Zero-Buffer Lines

Proof of formula

- Let τ (the operation time) be the time unit.
- *Approximation:* At most, <u>one</u> machine can be down.
- Consider a long time interval of length $T\tau$ during which Machine M_i fails m_i times $(i = 1, \ldots k)$.



- Without failures, the line would produce T parts.

Zero-Buffer Lines

Proof of formula

- The average repair time of M_i is τ / r_i each time it fails, so the total system down time is close to

$$D\tau = \sum_{i=1}^{k} \frac{m_i \tau}{r_i}$$

where D is the number of operation times in which a machine is down.

Zero-Buffer Lines

Proof of formula

- The total up time is approximately

$$U\tau = T\tau - \sum_{i=1}^{k} \frac{m_i \tau}{r_i}$$

- where U is the number of operation times in which all machines are up.

Zero-Buffer Lines

Proof of formula

- Since the system produces one part per time unit while it is working, it produces U parts during the interval of length $T\tau$.
- Note that, approximately,

$$m_i = p_i U$$

because M_i can only fail while it is operational.

Zero-Buffer Lines

Proof of formula

- Thus,

$$U\tau = T\tau - U\tau \sum_{i=1}^{k} \frac{p_i}{r_i},$$

or,

$$\frac{U}{T} = E_{ODF} = \frac{1}{1 + \sum_{i=1}^{k} \frac{p_i}{r_i}}$$

Zero-Buffer Lines

p_i and r_i and p_i/r_i

and

$$P = \frac{1}{\tau} \frac{1}{1 + \sum_{i=1}^{k} \frac{p_i}{r_i}}$$

- Note that P is a function of the *ratio* p_i/r_i and not p_i or r_i separately.
- The same statement is true for the infinite-buffer line.
- However, the same statement is *not* true for a line with finite, non-zero buffers.

Zero-Buffer Lines

Improvements

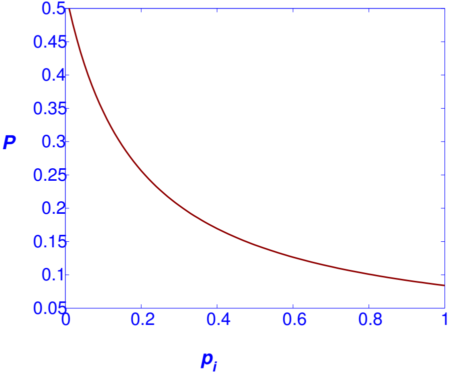
Questions:

- If we want to increase production rate, which machine should we improve?
- What would happen to production rate if we improved any other machine?

Zero-Buffer Lines

P as a function of p_i

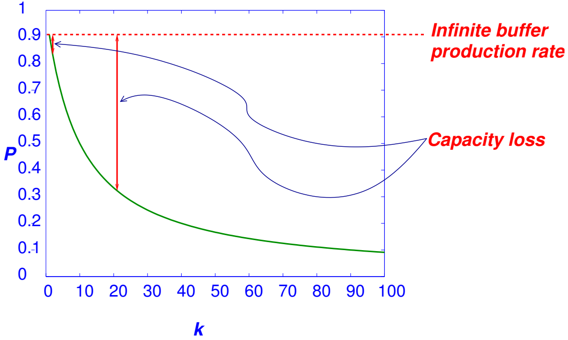
All machines are the same except M_i. As p_i increases, the production rate decreases.



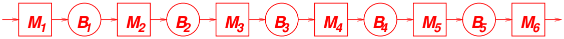
Zero-Buffer Lines

P as a function of p_i

All machines are the same. As the line gets longer, the production rate decreases.



Finite-Buffer Lines



- Motivation for buffers: recapture some of the lost production rate.
- Cost
  - ⋆ in-process inventory/lead time
  - ⋆ floor space
  - ⋆ material handling mechanism

Finite-Buffer Lines

- Infinite buffers: delayed downstream propagation of disruptions(*starvation*) and *no* upstream propagation.
- Zero buffers: instantaneous propagation in both directions.
- Finite buffers: delayed propagation in both directions.
  - ⋆ New phenomenon: *blockage* .
- Blockage: Machine M_i is blocked at time t if Buffer B_i is full at time t.

Finite-Buffer Lines

- Difficulty:
  - ⋆ No simple formula for calculating production rate or inventory levels.
- Solution:
  - ⋆ Simulation
  - ⋆ Analytical approximation
  - ⋆ *Exact analytical solution for two-machine lines only.*

Two Machine, Finite-Buffer Lines
Markov Chain Model

- Exact solution *is* available to Markov process model of a two-machine line.
- *Discrete time-discrete state Markov process:*

$$\text{prob}\{X(t+1) = x(t+1) | X(t) = x(t), X(t-1) = x(t-1), X(t-2) = x(t-2), \ldots\} =$$

$$\text{prob}\{X(t+1) = x(t+1) | X(t) = x(t)\}$$

Two Machine, Finite-Buffer Lines

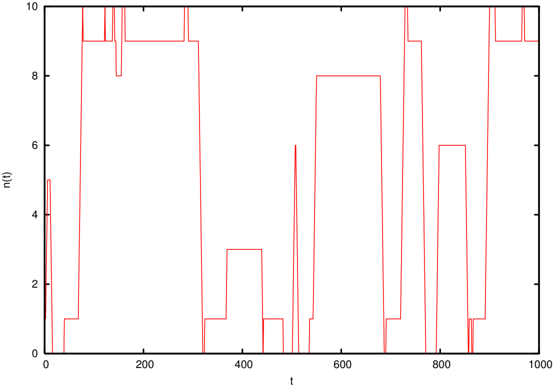
State Space

Here, $X(t) = (n(t), \alpha_1(t), \alpha_2(t))$, where

- n is the number of parts in the buffer;
  $n = 0, 1, ..., N$.

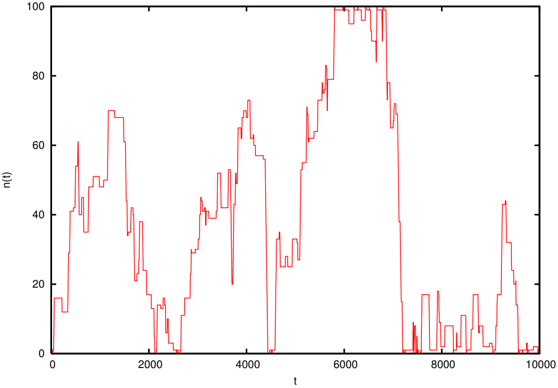
- α_i is the repair state of M_i; $i = 1, 2$.
  - $\alpha_i = 1$ means the machine is *up* or *operational*;
  - $\alpha_i = 0$ means the machine is *down* or *under repair*.

Two Machine, Finite-Buffer Lines
Simulations



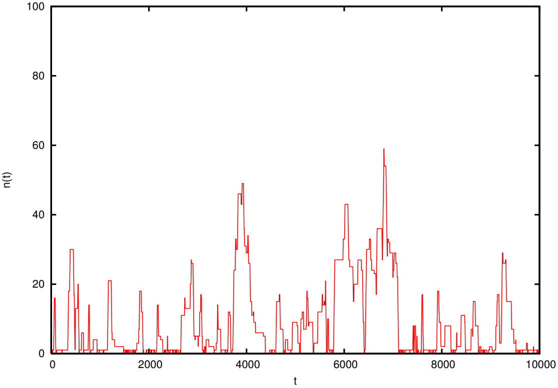
$$r_1 = .1, p_1 = .01, r_2 = .1, p_2 = .01, N = 10$$

Two Machine, Finite-Buffer Lines
Simulations



$$r_1 = .1, p_1 = .01, r_2 = .1, p_2 = .01, N = 100$$

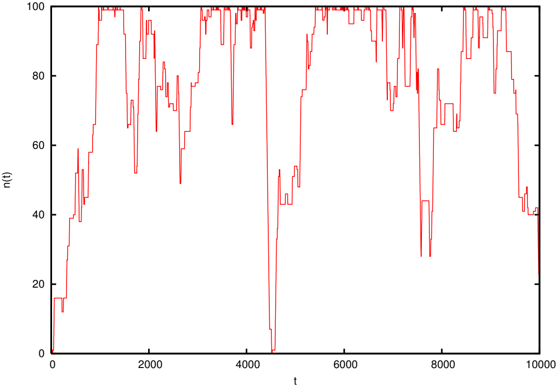
Two Machine, Finite-Buffer Lines
Simulations



$$r_i = .1, i = 1, 2, p_1 = .02, p_2 = .01, N = 100$$

Two Machine, Finite-Buffer Lines

Simulations



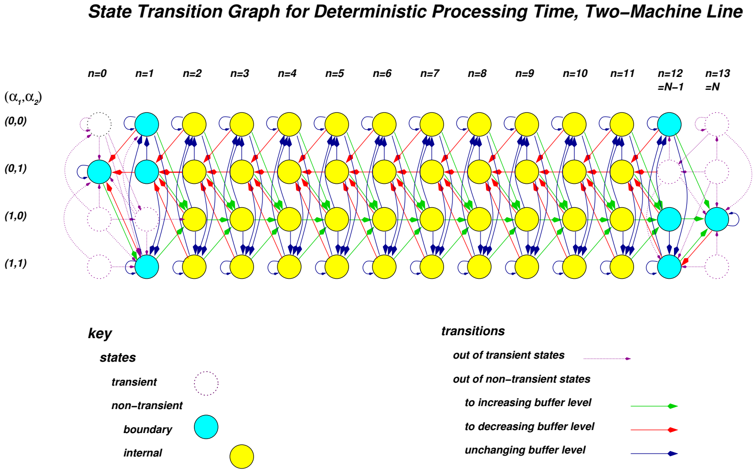
$$r_i = .1, i = 1, 2, p_1 = .01, p_2 = .02, N = 100$$

Two Machine, Finite-Buffer Lines

Several models available:

- *Deterministic processing time* , or *Buzacott model:* deterministic processing time, geometric failure and repair times; discrete state, discrete time.

Two Machine, Finite-Buffer Lines

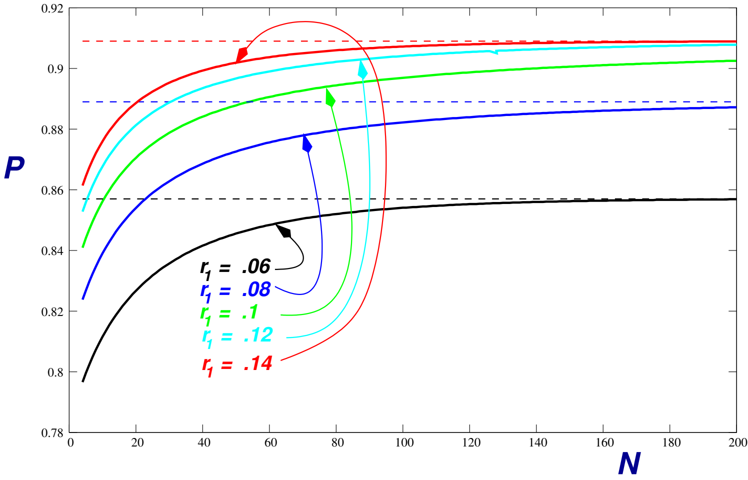


Two Machine, Finite-Buffer Lines

- *Exponential processing time:* exponential processing, failure, and repair time; discrete state, continuous time.

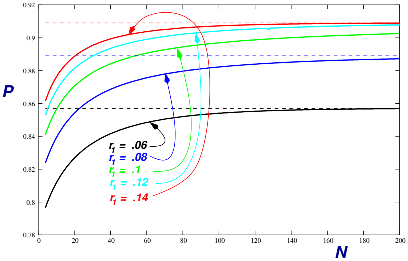
- *Continuous material,* or *fluid:* deterministic processing, exponential failure and repair time; mixed state, continuous time.

Two Machine, Finite-Buffer Lines



$\tau = 1.$
$p_1 = .01$
$r_2 = .1$
$p_2 = .01$

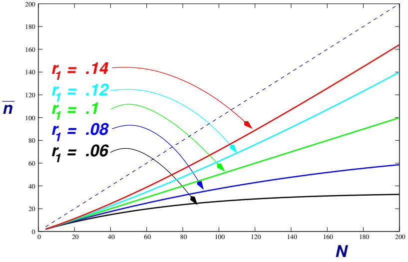
Two Machine, Finite-Buffer Lines



Discussion:

- What is P when $N = 0$?
- Why are the curves increasing?
- Why do they reach an asymptote?
- What is the limit of P as $N \to \infty$?
- Why are the curves with smaller r_1 lower?

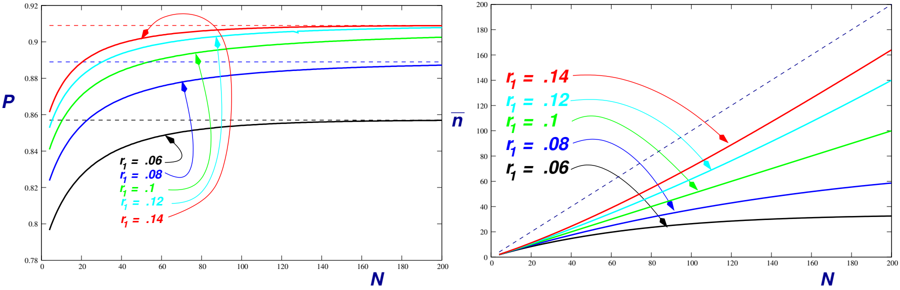
Two Machine, Finite-Buffer Lines



Discussion:

- Why are the curves increasing?
- Why *different* asymptotes?
- What is $\bar{n}$ when $N = 0$?
- What is the limit of $\bar{n}$ as $N \to \infty$?
- Why are the curves with smaller r_1 lower?

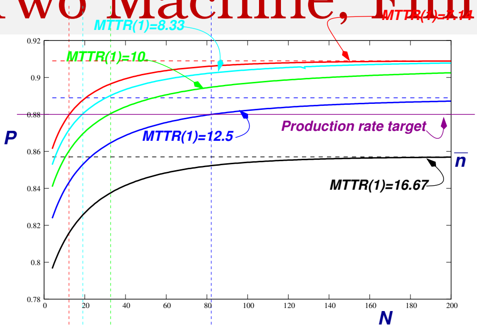
Two Machine, Finite-Buffer Lines

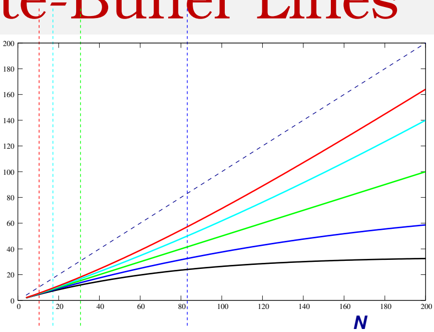


Problem: Select M_1 and N to maximize profit (revenue-[capital cost+inventory cost])

- What can you say (qualitatively) about the optimal buffer size for a given M_1?
- How should it be related to r_i, p_i?

Two Machine, Finite-Buffer Lines



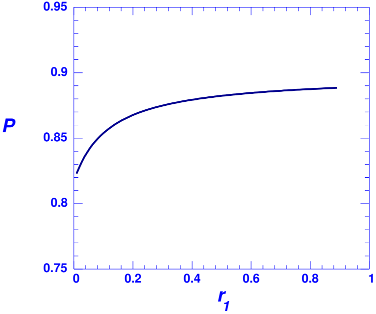


Problem: Select M_1 and N so that $P = .88$ and profit is maximized

- *Observation:* If M_1 is better, N can be smaller.

Two Machine, Finite-Buffer Lines

Should we prefer short, frequent, disruptions or long, infrequent, disruptions?



- $r_2 = 0.8$, $p_2 = 0.09$, $N = 10$
- r_1 and p_1 vary together and $\frac{r_1}{r_1+p_1} = .9$
- *Answer:* evidently, short, frequent failures.
- *Why?*

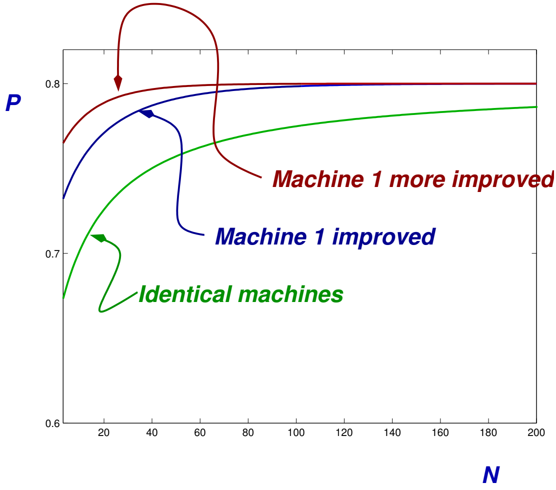
Two Machine, Finite-Buffer Lines
Improvements

Questions:

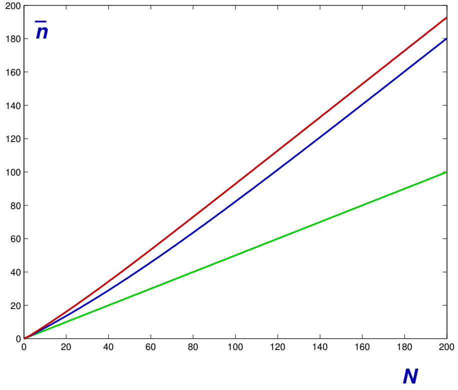
- If we want to increase production rate, which machine should we improve?
- What would happen to production rate if we improved any other machine?

Two Machine, Finite-Buffer Lines

Improvements to *non-bottleneck* machine.



Two Machine, Finite-Buffer Lines



- Inventory increases as the (non-bottleneck) *upstream* machine is improved and as the buffer space is increased.
- If the *downstream* machine were improved, the inventory would be less and it would increase much less as the space increases.

Two Machine, Finite-Buffer Lines

Exponential — discrete material, continuous time

- $\mu_i \delta t =$ the probability that M_i completes an operation in $(t, t + \delta t)$;
- $p_i \delta t =$ the probability that M_i fails during an operation in $(t, t + \delta t)$;
- $r_i \delta t =$ the probability that M_i is repaired, while it is down, in $(t, t + \delta t)$;

Two Machine, Finite-Buffer Lines

Continuous — continuous material, continuous time

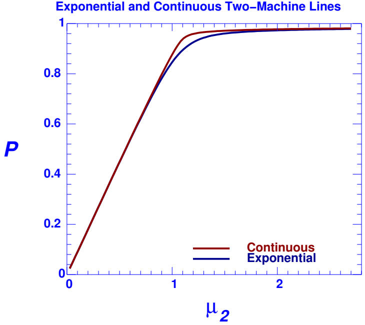
- $\mu_i \delta t =$ the amount of material that M_i processes, while it is up, in $(t, t + \delta t)$;

- $p_i \delta t =$ the probability that M_i fails, while it is up, in $(t, t + \delta t)$;

- $r_i \delta t =$ the probability that M_i is repaired, while it is down, in $(t, t + \delta t)$;

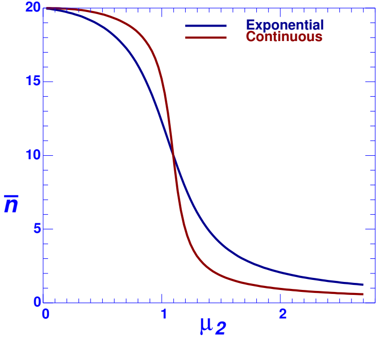
Two Machine, Finite-Buffer Lines

- $r_1 = 0.09$, $p_1 = 0.01$, $\mu_1 = 1.1$
- $r_2 = 0.08$, $p_2 = 0.009$
- $N = 20$
- *Explain the shapes of the graphs.*

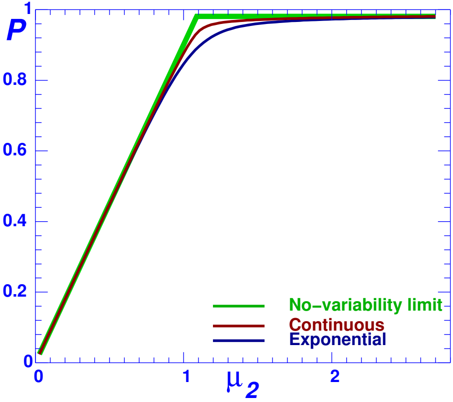


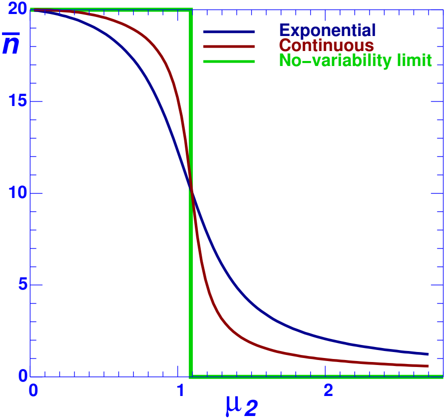
Two Machine, Finite-Buffer Lines

- *Explain the shapes of the graphs.*



Two Machine, Finite-Buffer Lines



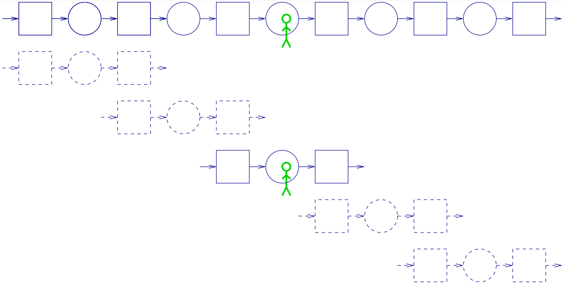


No-variability limit: a continuous model where both machines are reliable, and processing rate μ_i' of machine i in the no-variability is the same as the isolated production rate of machine i in the other cases. That is, $\mu_i' = \mu_i r_i/(r_i + p_i)$.

Long Lines

- Difficulty:
  - ⋆ No simple formula for calculating production rate or inventory levels.
  - ⋆ State space is too large for exact numerical solution.
    - If all buffer sizes are N and the length of the line is k, the number of states is $S = 2^k(N+1)^{k-1}$.
    - if $N = 10$ and $k = 20$, $S = 6.41 \times 10^{25}$.
  - ⋆ *Decomposition* seems to work successfully.

Decomposition



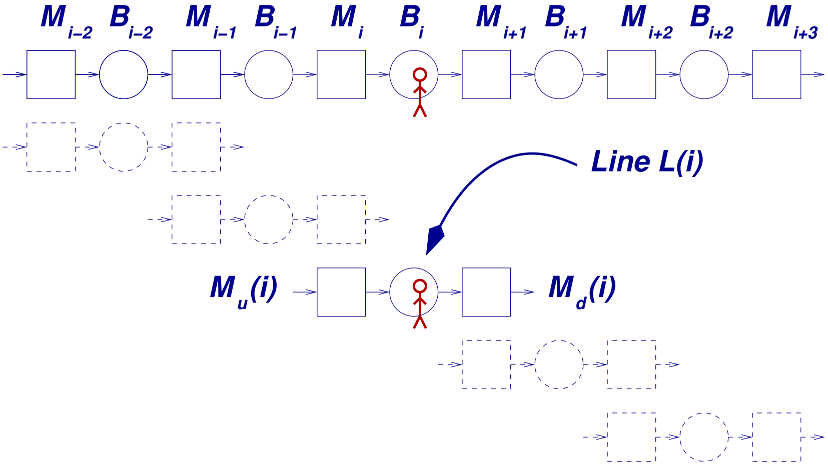
- Decomposition breaks up systems and then reunites them.
- Conceptually: put an observer in a buffer, and tell him that he is in the buffer of a two-machine line.
- Question: *What would the observer see, and how can he be convinced he is in a two-machine line? Construct the two-machine line. Construct all the two-machine lines.*

Decomposition

- Consider an observer in Buffer B_i.
  - ⋆ Imagine the material flow process that the observer sees *entering* and the material flow process that the observer sees *leaving* the buffer.
- We construct a two-machine line $L(i)$
  - ⋆ ie, we find machines $M_u(i)$ and $M_d(i)$ with parameters $r_u(i)$, $p_u(i)$, $r_d(i)$, $p_d(i)$, and $N(i) = N_i$)

  such that an observer in its buffer will see almost the same processes.
- The parameters are chosen as functions of the behaviors of the *other* two-machine lines.

Decomposition



Decomposition

There are $4(k-1)$ unknowns. Therefore, we need

- $4(k-1)$ equations, and
- an algorithm for solving those equations.

Decomposition

Equations

- *Conservation of flow,* equating all production rates.
- *Flow rate/idle time,* relating production rate to probabilities of starvation and blockage.
- *Resumption of flow,* relating $r_u(i)$ to upstream events and $r_d(i)$ to downstream events.
- *Boundary conditions,* for parameters of $M_u(1)$ and $M_d(k-1)$.

Decomposition
Equations

- This is a set of $4(k-1)$ equations.
- All the quantities in these equations are
  - ⋆ specified parameters, or
  - ⋆ unknowns, or
  - ⋆ functions of parameters or unknowns derived from the two-machine line analysis.

Decomposition

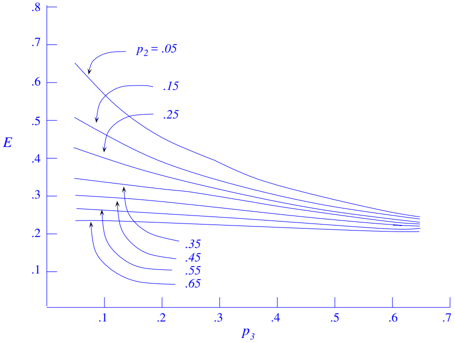
Algorithm

DDX algorithm : due to Dallery, David, and Xie (1988).

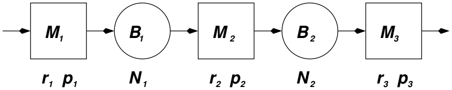
1. Guess the downstream parameters of $L(1)$ $(r_d(1), p_d(1))$. Set $i = 2$.
2. Use the equations to obtain the upstream parameters of $L(i)$ $(r_u(i), p_u(i))$. Increment i.
3. Continue in this way until $L(k-1)$. Set $i = k-2$.
4. Use the equations to obtain the downstream parameters of $L(i)$. Decrement i.
5. Continue in this way until $L(1)$.
6. Go to Step 2 or terminate.

Examples

Three-machine line – production rate.

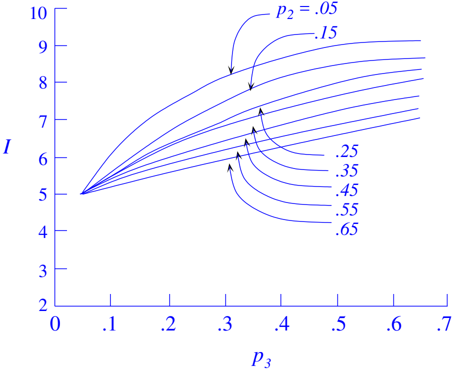


$$r_1 = r_2 = r_3 = .2$$
$$p_1 = .05$$
$$N_1 = N_2 = 5$$



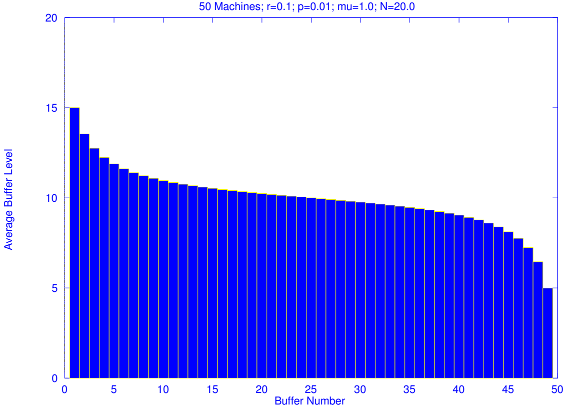
Examples

Three-machine line – total average inventory



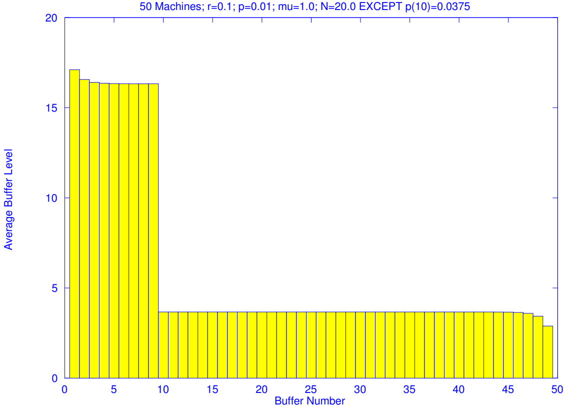
$$r_1 = r_2 = r_3 = .2$$
$$p_1 = .05$$
$$N_1 = N_2 = 5$$

Examples



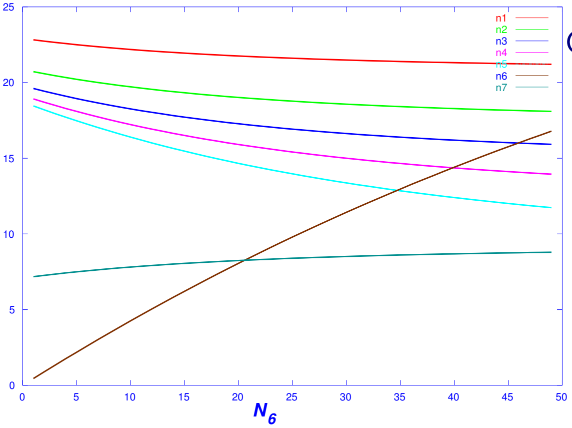
Distribution of material in a line with identical machines and buffers. *Explain the shape.*

Examples



Effect of a bottleneck. Identical machines and buffers, except for M_{10}.

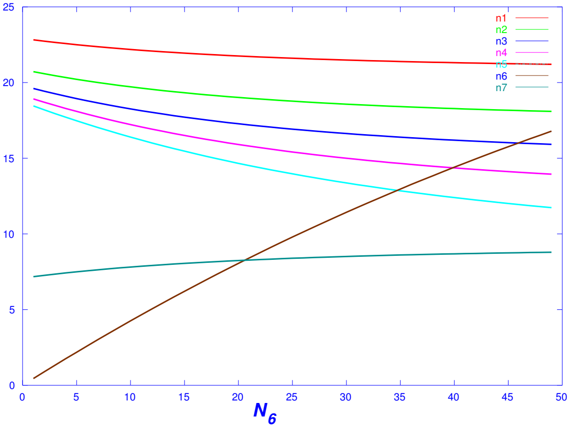
Examples



Continuous material model.

- Eight-machine, seven-buffer line.
- For each machine, $r = .075$, $p = .009$, $\mu = 1.2$.
- For each buffer *(except Buffer 6)*, $N = 30$.

Examples



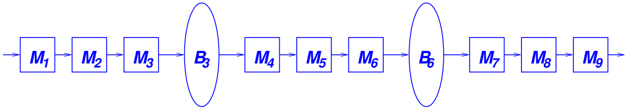
- Which $\bar{n}_i$ are decreasing and which are increasing?
- Why?

Examples

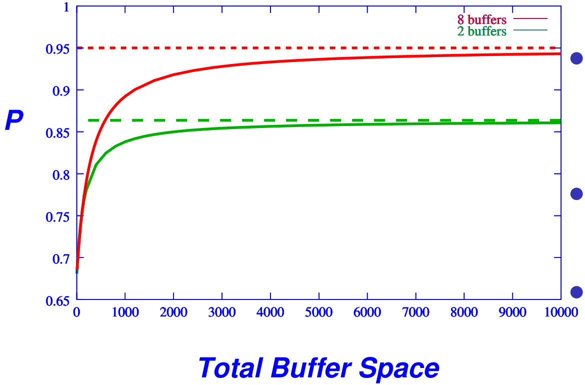
Which has a higher production rate?

- 9-Machine line with two buffering options:
  - ⋆ 8 buffers equally sized; or

  - ⋆ 2 buffers equally sized.



Examples



- Continuous model; all machines have $r = .019$, $p = .001$, $\mu = 1$.
- What are the asymptotes?
- Is 8 buffers *always* faster?

Optimal buffer space distribution

- Design the buffers for a 20-machine production line.
- The machines have been selected, and the only decision remaining is the amount of space to allocate for in-process inventory.
- *The goal is to determine the smallest amount of in-process inventory space so that the line meets a production rate target.*

Optimal buffer space distribution

- The common operation time is one operation per minute.

- The target production rate is .88 parts per minute.

Optimal buffer space distribution

- *Case 1* MTTF= 200 minutes and MTTR = 10.5 minutes for all machines ($P = .95$ parts per minute).
- *Case 2* Like Case 1 except Machine 5. For Machine 5, MTTF = 100 and MTTR = 10.5 minutes ($P = .905$ parts per minute).
- *Case 3* Like Case 1 except Machine 5. For Machine 5, MTTF = 200 and MTTR = 21 minutes ($P = .905$ parts per minute).

Optimal buffer space distribution

Are buffers really needed?

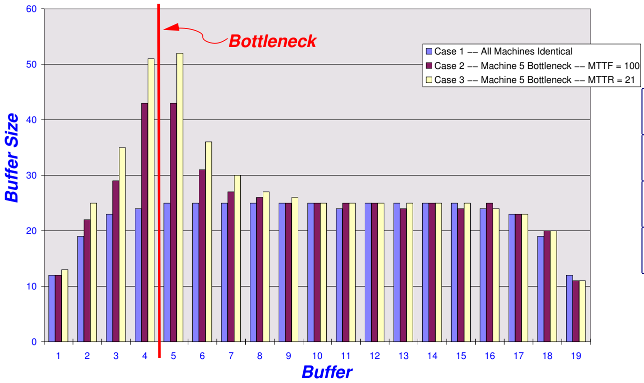
Line	Production rate with no buffers, parts per minute
Case 1	.487
Case 2	.475
Case 3	.475

Yes.

How were these numbers calculated?

Optimal buffer space distribution

Solution



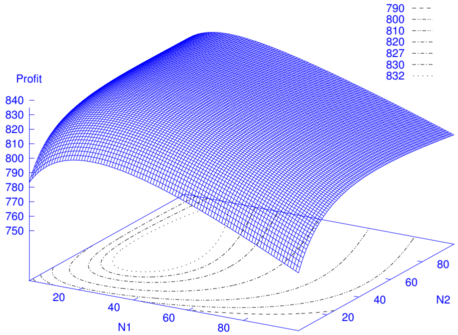
Line	Space
Case 1	430
Case 2	485
Case 3	523

Optimal buffer space distribution

- Observation from studying buffer space allocation problems:

  - *Buffer space is needed most where buffer level variability is greatest!*

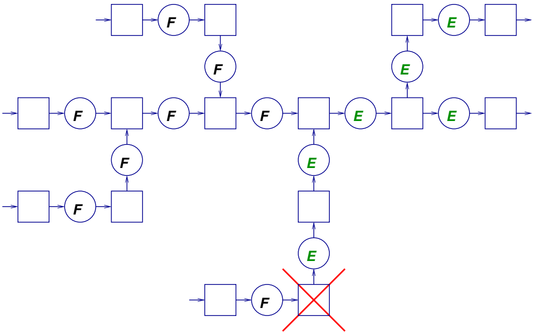
Profit as a function of buffer sizes



- Three-machine, continuous material line.
- $r_i = .1$, $p_i = .01$, $\mu_i = 1$.
- $\Pi = 1000P(N_1, N_2) - (\bar{n}_1 + \bar{n}_2)$.

Assembly

- Decomposition can be extended to assembly systems.
- Propagation of disturbances is more complex:

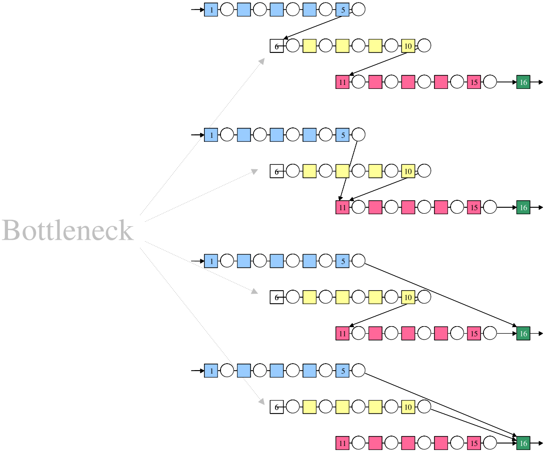


Assembly

Question: How should an assembly system be structured?

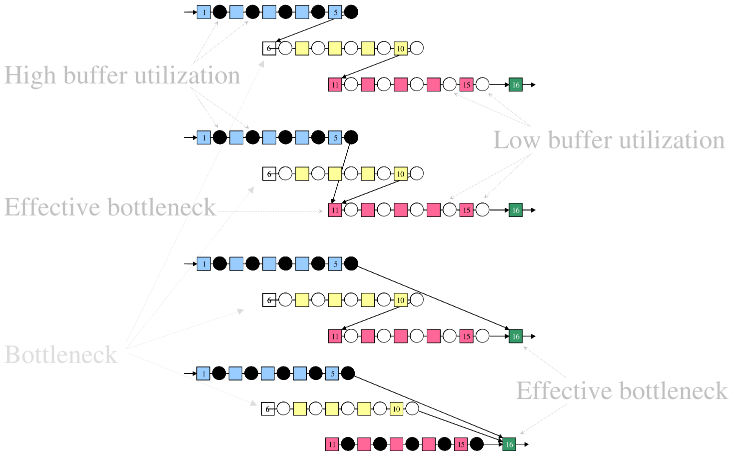
- Add parts to a growing assembly *or* form subassemblies and then assemble them?
- Production rates are roughly the same, but inventories can be affected.

Assembly



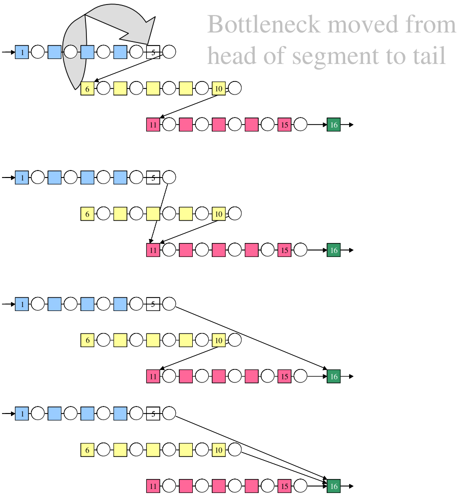
Schick/Gershwin 4 38

Assembly



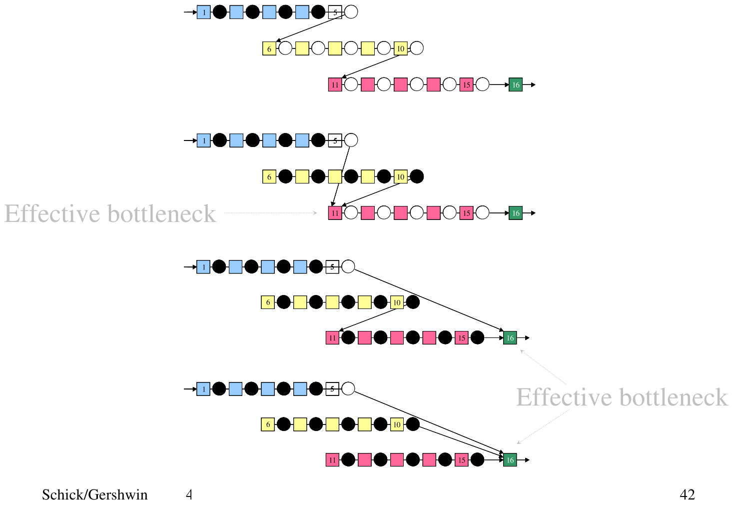
Schick/Gershwin 4 40

Assembly

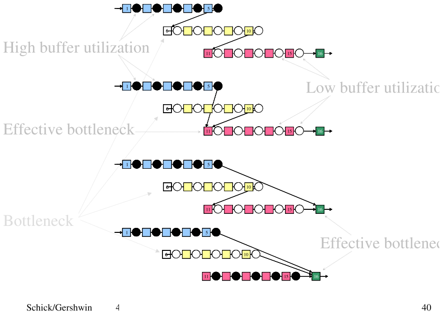


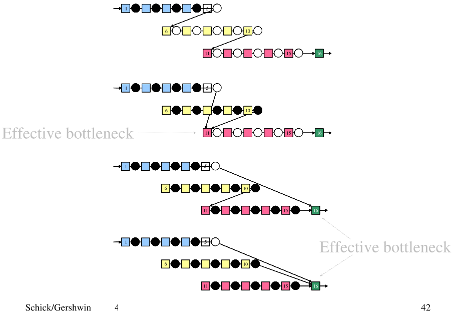
Schick/Gershwin 4 41

Assembly


1
5
6
10
11
15
16
Effective bottleneck
Effective bottleneck
Schick/Gershwin
4
42

Assembly


High buffer utilization
Low buffer utilizatic
Effective bottleneck
Bottleneck
Effective bottlenec
Schick/Gershwin
4
40


Effective bottleneck
Effective bottleneck
Schick/Gershwin
4
42