MIT 2.853/2.854

Introduction to Manufacturing Systems

Inventory

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Storage

Storage is fundamental!

- Storage is *fundamental* in nature, management, and engineering.
  - ⋆ In nature, *energy* is stored. Life can only exist if the acquisition of energy can occur at a different time from the the expenditure of energy.
  - ⋆ In management, *products* are stored.
  - ⋆ In engineering, *energy* is stored (springs, batteries, capacitors, inductors, etc.); *information* is stored (RAM and hard disks); *water* is stored (dams and reservoirs); etc.

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Storage

Purpose of storage

- *The purpose of storage is to allow systems to survive even when important events are unsynchronized.* For example,
  - ⋆ the separation in time from the acquisition or production of something and its consumption; and
  - ⋆ the occurrence of an event at one location (such as a machine failure or a power surge) which can prevent desired performance or do damage at another.
- Storage improves system performance by *decoupling* parts of the system from one another.

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Storage

Purpose of storage

- Storage decouples creation/acquisition and emission.
- It allows production systems (of energy or goods) to be built with capacity less than the peak demand.
- It reduces the propagation of disturbances, and thus reduces instability and the fragility of complex, expensive systems.

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Manufacturing Inventory

Motives/benefits

Storage

- Reduces the propagation of disturbances (eg, machine failures).
- Allows economies of scale:
  - ⋆ volume purchasing
  - ⋆ set-ups
- Helps manage seasonality and limited capacity.
- Helps manage uncertainty:
  - ⋆ Short term: random arrivals of customers or orders.
  - ⋆ Long term: Total demand for a product next year.

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Manufacturing Inventory

Motives/benefits

- Financial: raw materials are paid for, but no revenue comes in until the item is sold.
- Demand risk: item loses value or is unsold due (for example) to
  - ⋆ time value (newspaper)
  - ⋆ obsolescence
  - ⋆ fashion
- Holding cost (warehouse space)
- "Shrinkage" = damage/theft/spoilage/loss

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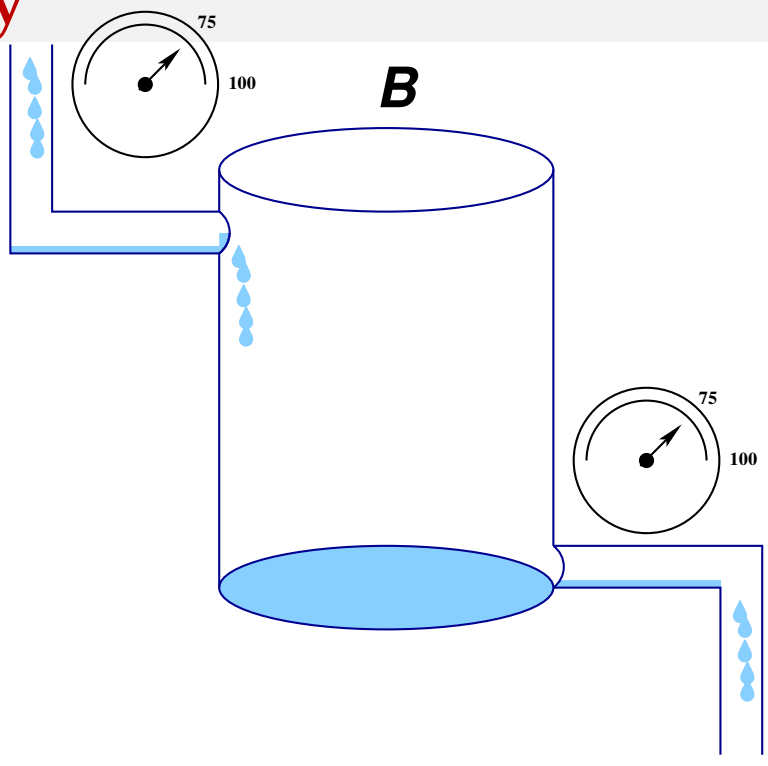
Variability and Storage

No variability



75 gal/sec in, 75 gal/sec out constantly

If we start with an empty tank, the tank is always empty (except for splashing at the bottom).

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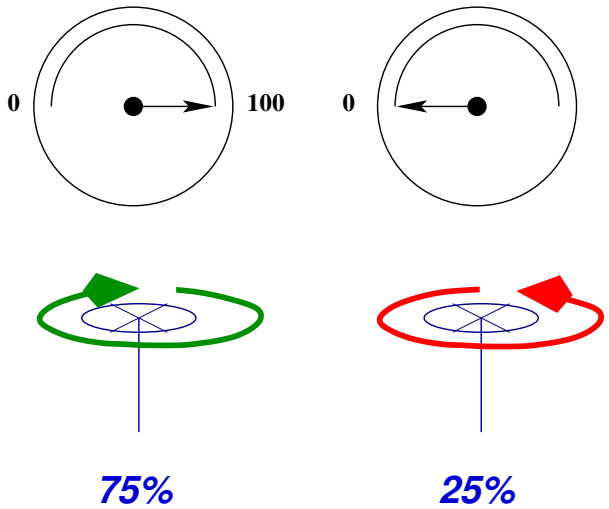
Variability and Storage

Variability from random valves

Consider a random valve:



- The average period when the valve is open is 15 minutes.
- The average period when the valve is closed is 5 minutes.
- Consequently, the *average* flow rate through it is 75 gal/sec*.

* *... as long as flow is not impeded upstream or downstream.*

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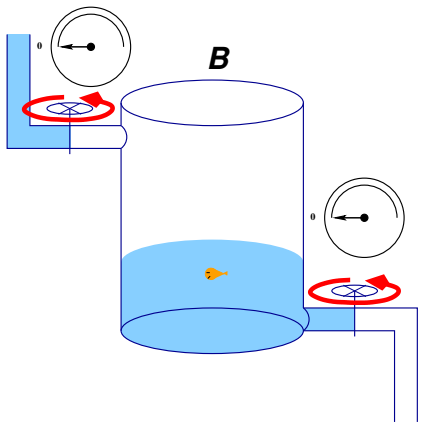
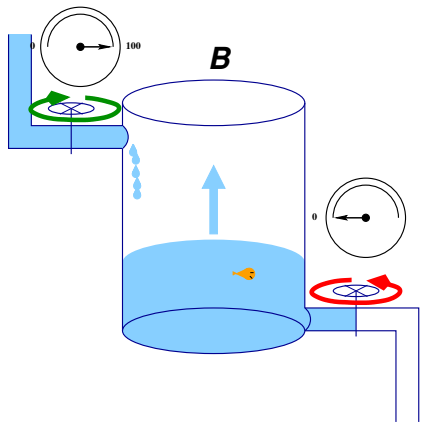
Variability and Storage

Variability from random valves

Four possibilities for two *unsynchronized* valves:

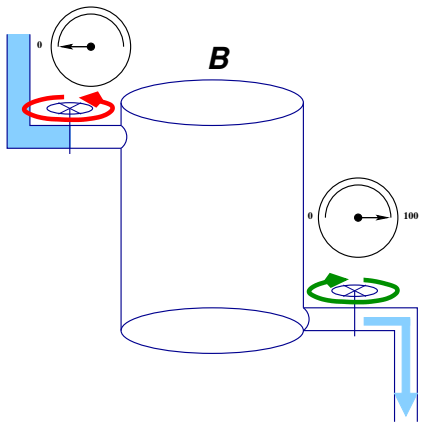
0 in, 0 out

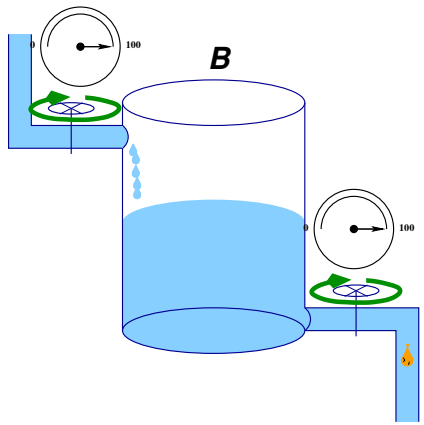




100 gal/sec in, 0 out

0 in, 100 gal/sec out





100 gal/sec in, 100 gal/sec out

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Variability and Inventory

Observation:

- There is *never* any water in the tank when the flow is constant.
- There is *sometimes* water in the tank when the flow is variable.

Conclusions:

1. You can't always replace random variables with their averages.
2. *Variability causes inventory!!*

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Variability and Inventory

- To be more precise, *non-synchronization causes inventory.*
  - ⋆ Living things do not acquire energy at the same time they expend it. Therefore, they must store energy in the form of fat or sugar.
  - ⋆ Rivers are dammed and reservoirs are created to control the flow of water — to reduce the variability of the water supply.
  - ⋆ For solar and wind power to be successful, energy storage is required for when the sun doesn't shine and the wind doesn't blow.

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Inventory Topics

- Newsvendor Problem — demand risk
- Economic Order Quantity (EOQ) — economies of scale (deterministic demand)
- Base Stock Policy — manage a simple factory to avoid stockout
- Q, R policy — economies of scale (random demand)
- Inventory Position — nonzero lead time for raw material
- Inventory Inaccuracy

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Newsvendor Problem

... formerly called the "Newsboy Problem".

Motive: demand risk.

- Newsguy buys x newspapers at c dollars per paper (*cost*).
- Demand for newspapers, at *price* $r > c$ is w.
- Unsold newspapers are redeemed at *salvage price* $s < c$.

Demand W is a continuous random variable with distribution function $F(w) = \text{prob}(W \leq w)$; $f(w) = dF(w)/dw$ exists for all w.

Note that $w \geq 0$ so $F(w) = 0$ and $f(w) = 0$ for $w \leq 0$.

Problem: Choose x to maximize expected profit.

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Newsvendor Problem

Model

$$\text{Revenue} = R = \begin{cases} rx & \text{if } x \leq w \\ rw + s(x - w) & \text{if } x > w \end{cases}$$

$$\text{Profit} = P = \begin{cases} (r - c)x \text{ if } x \leq w \\ rw + s(x - w) - cx \\ \qquad = (r - s)w + (s - c)x \text{ if } x > w \end{cases}$$

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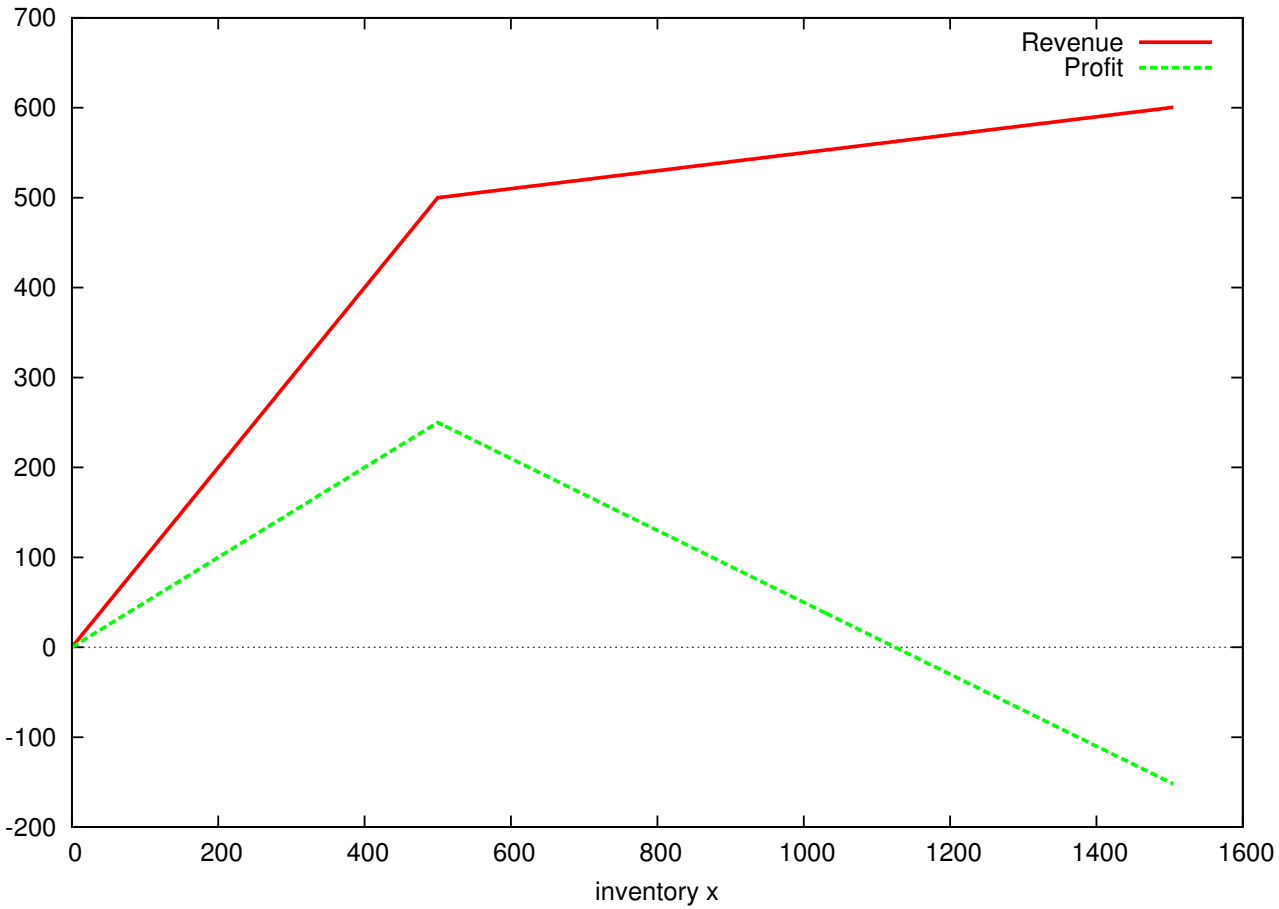
Newsvendor Problem

Model



$$r = 1.; c = .5; s = .1; w = 500.$$

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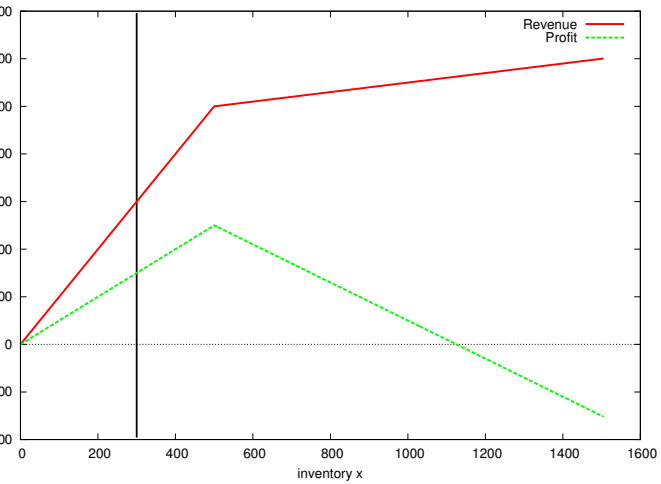
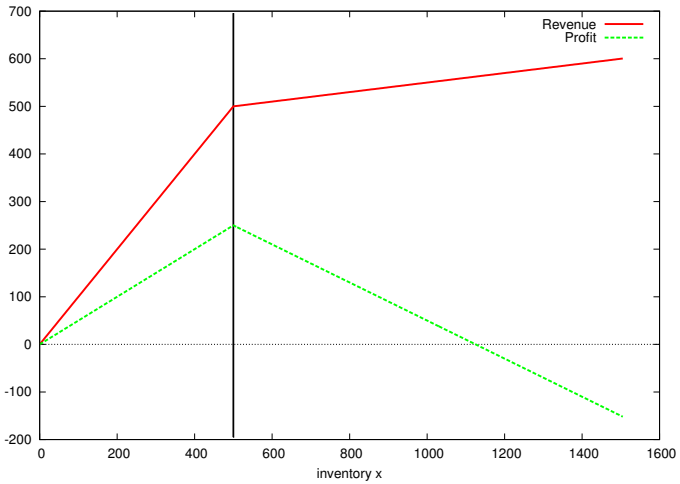
Newsvendor Problem

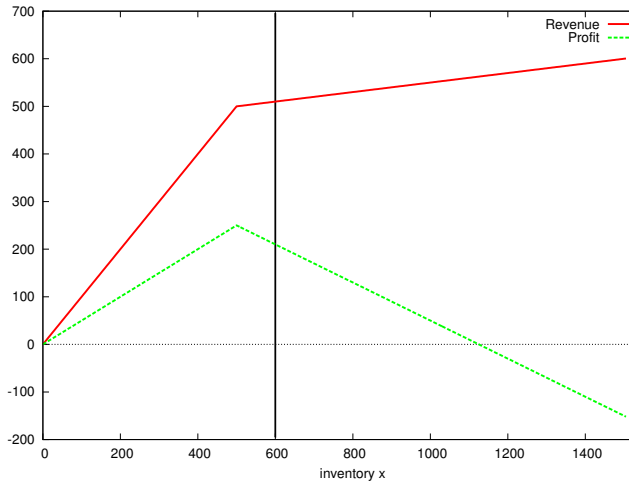
Model



x too little



x just right



x too much

$$r = 1.; c = .5; s = .1; w = 500.$$

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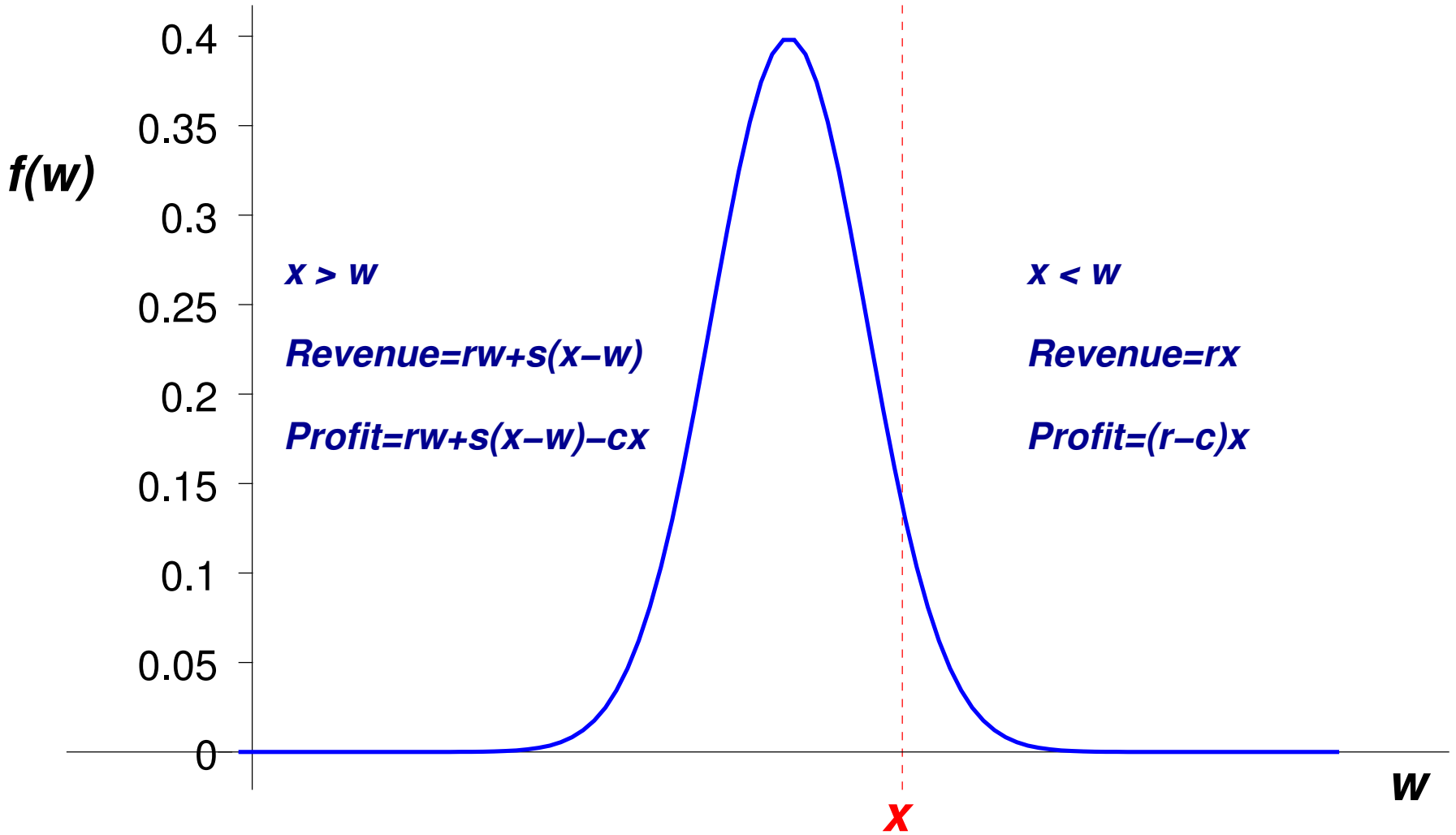
Newsvendor Problem

Normally distributed demand



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Newsvendor Problem

Expected profit

Expected Profit $= EP(x) =$

$$\int_{-\infty}^{x}[(r-s)w+(s-c)x]f(w)dw+$$

$$\int_{x}^{\infty}(r-c)xf(w)dw$$

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Newsvendor Problem

Expected profit

or $EP(x)$

$$= (r-s)\int_{-\infty}^{x} wf(w)dw + (s-c)x\int_{-\infty}^{x} f(w)dw$$

$$+(r-c)x\int_{x}^{\infty} f(w)dw$$

$$= (r-s)\int_{-\infty}^{x} wf(w)dw$$

$$+(s-c)xF(x) + (r-c)x(1-F(x))$$

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Newsvendor Problem

Expected profit

- When $x = 0$, EP is 0.
- When $x \to \infty$,
  - ⋆ the first term goes to a finite constant, $(r - s)Ew$;
  - ⋆ the last term goes to 0;
  - ⋆ the middle term, $(s - c)xF(x) \to -\infty$.

Therefore when $x \to \infty$, $EP \to -\infty$.

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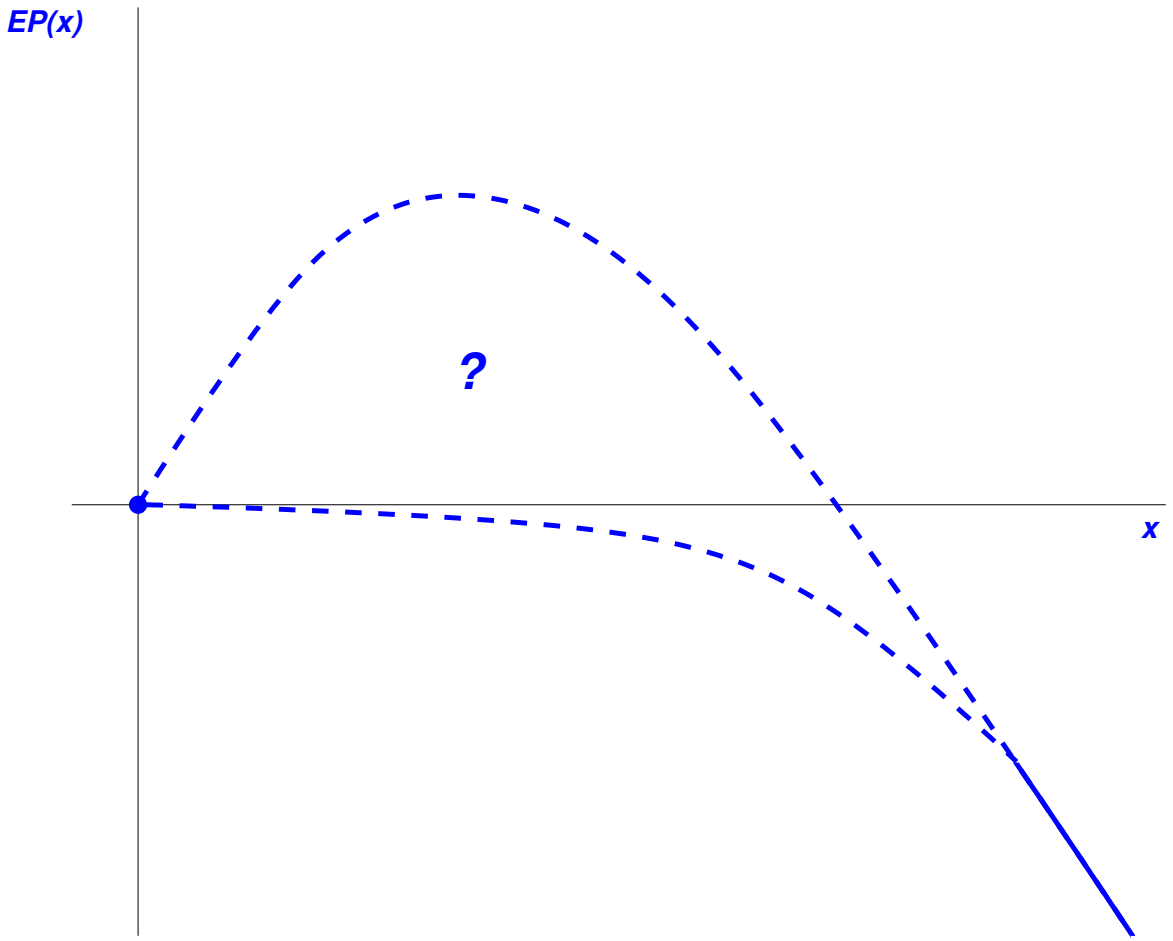
Newsvendor Problem

Expected profit



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Newsvendor Problem

Expected profit

$$EP(x) = (r-s)\int_{-\infty}^{x} wf(w)dw + (s-c)xF(x) + (r-c)x(1-F(x))$$

$$\begin{aligned}\frac{dEP}{dx} &= (r-s)xf(x) + (s-c)F(x) + (s-c)xf(x) \\ &\quad +(r-c)(1-F(x)) - (r-c)xf(x) \\ &= xf(x)(r-s+s-c-r+c) \\ &\quad +r-c+(s-c-r+c)F(x) \\ &= r-c+(s-r)F(x)\end{aligned}$$

Note also that $\dfrac{d^2EP}{dx^2} = (s-r)f(x)$.

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Newsvendor Problem

Expected profit

Note that

- $d^2EP/dx^2 = (s-r)f(x) \leq 0$. Therefore EP is concave and has a maximum.
- $dEP/dx > 0$ when $x = 0$. Therefore EP has a maximum which is greater than 0 for some $x = x^* > 0$.
- x^* satisfies $\dfrac{dEP}{dx}(x^*) = 0$.

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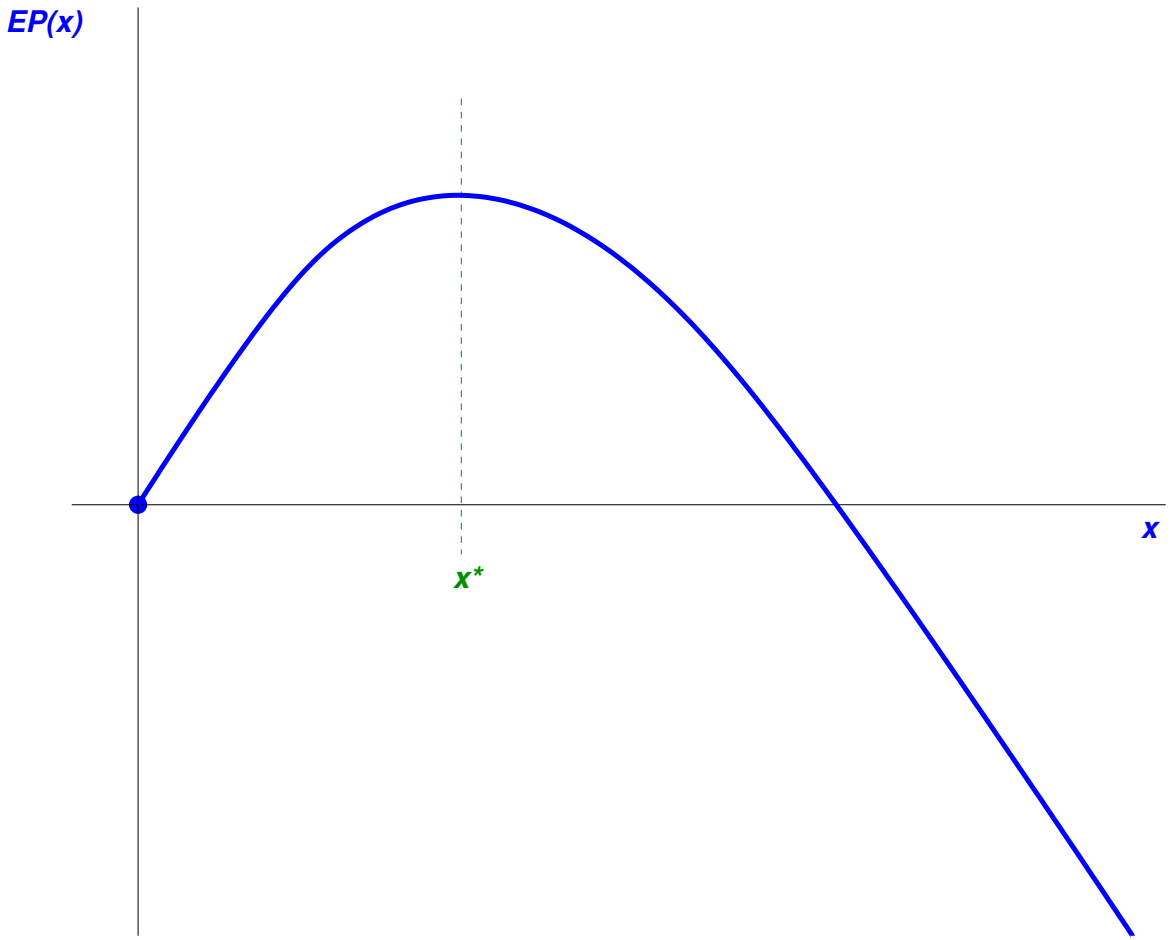
Newsvendor Problem

Expected profit



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Newsvendor Problem

Expected profit

Therefore $$\boxed{F(x^*) = \frac{r-c}{r-s}.}$$

(Note that $0 \leq \frac{r-c}{r-s} \leq 1$.)

Recall the definition of $F()$: $F(x^*)$ is the probability that $W \leq x^*$. So the equation above means

Buy enough stock to satisfy demand $100K\%$ of the time, where

$$K = \frac{r-c}{r-s}.$$

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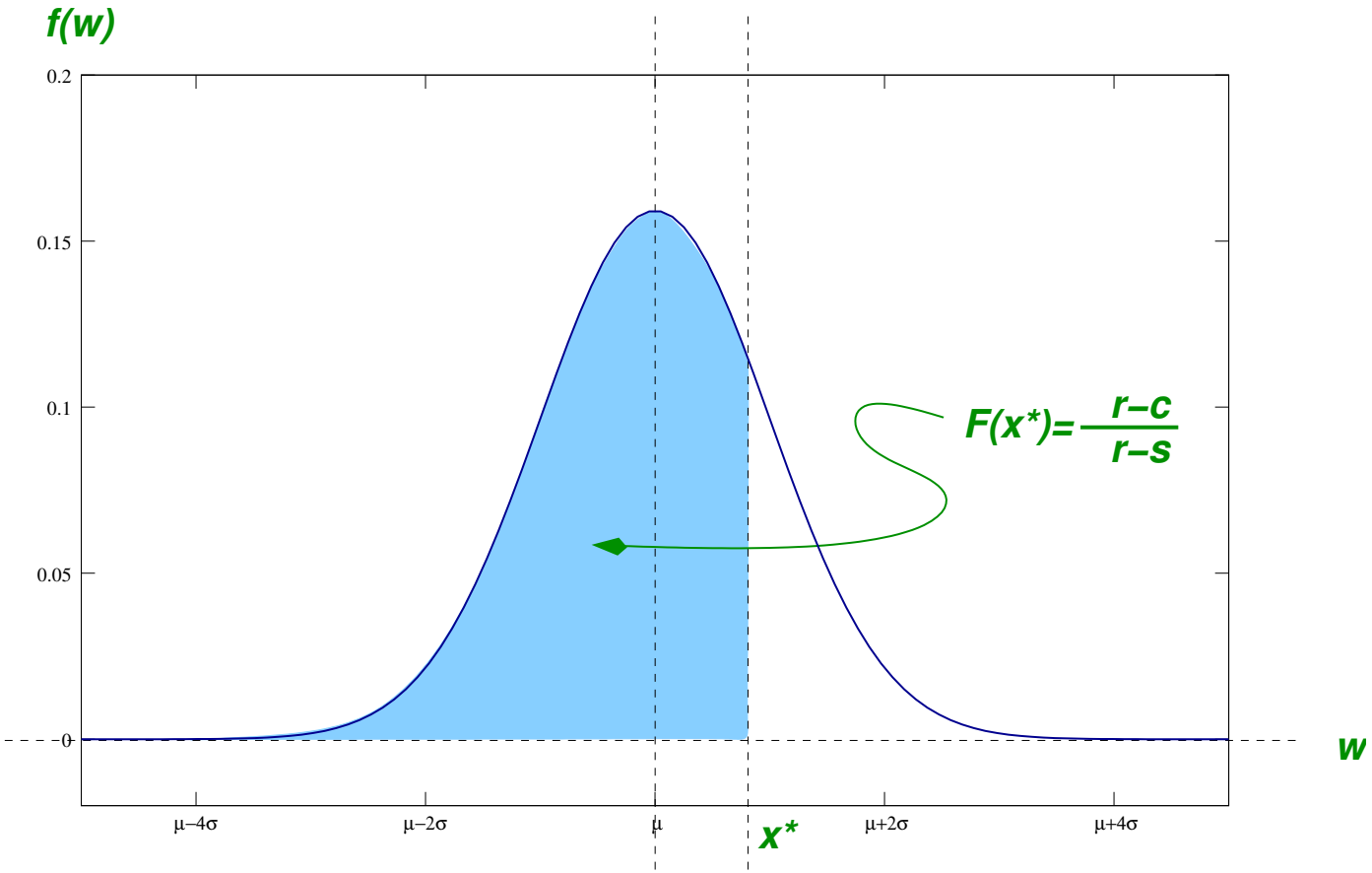
Newsvendor Problem

Expected profit



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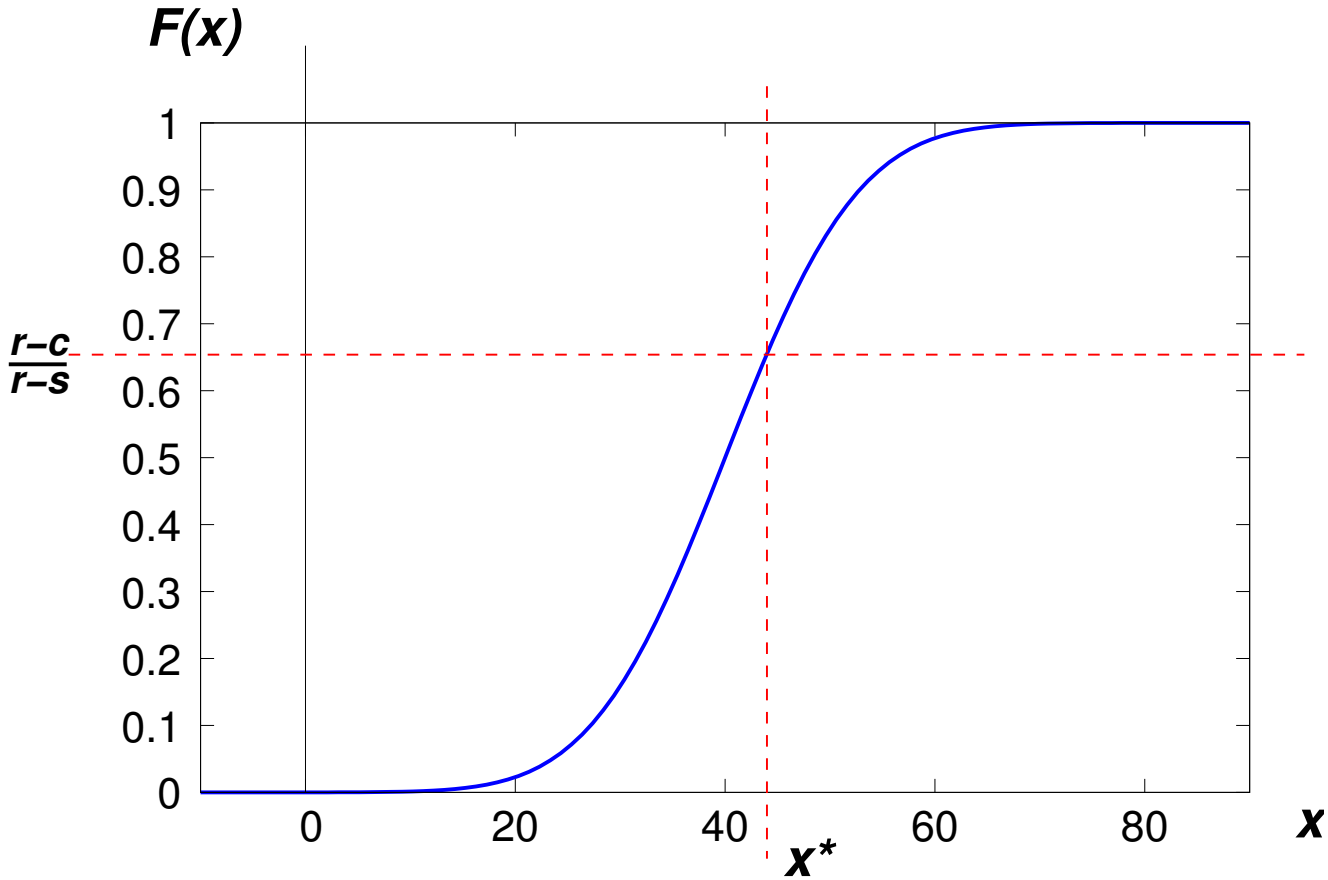
Newsvendor Problem

Expected profit



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Newsvendor Problem

Expected profit

This can also be written $F(x^*) = \dfrac{r-c}{(r-c)+(c-s)}$.

$r - c > 0$ is the marginal profit when $x < w$.

$c - s > 0$ is the marginal loss when $x > w$.pause

Choose x^* so that the fraction of time you do not buy too much is

$$\frac{\text{marginal profit}}{\text{marginal profit} + \text{marginal loss}}$$

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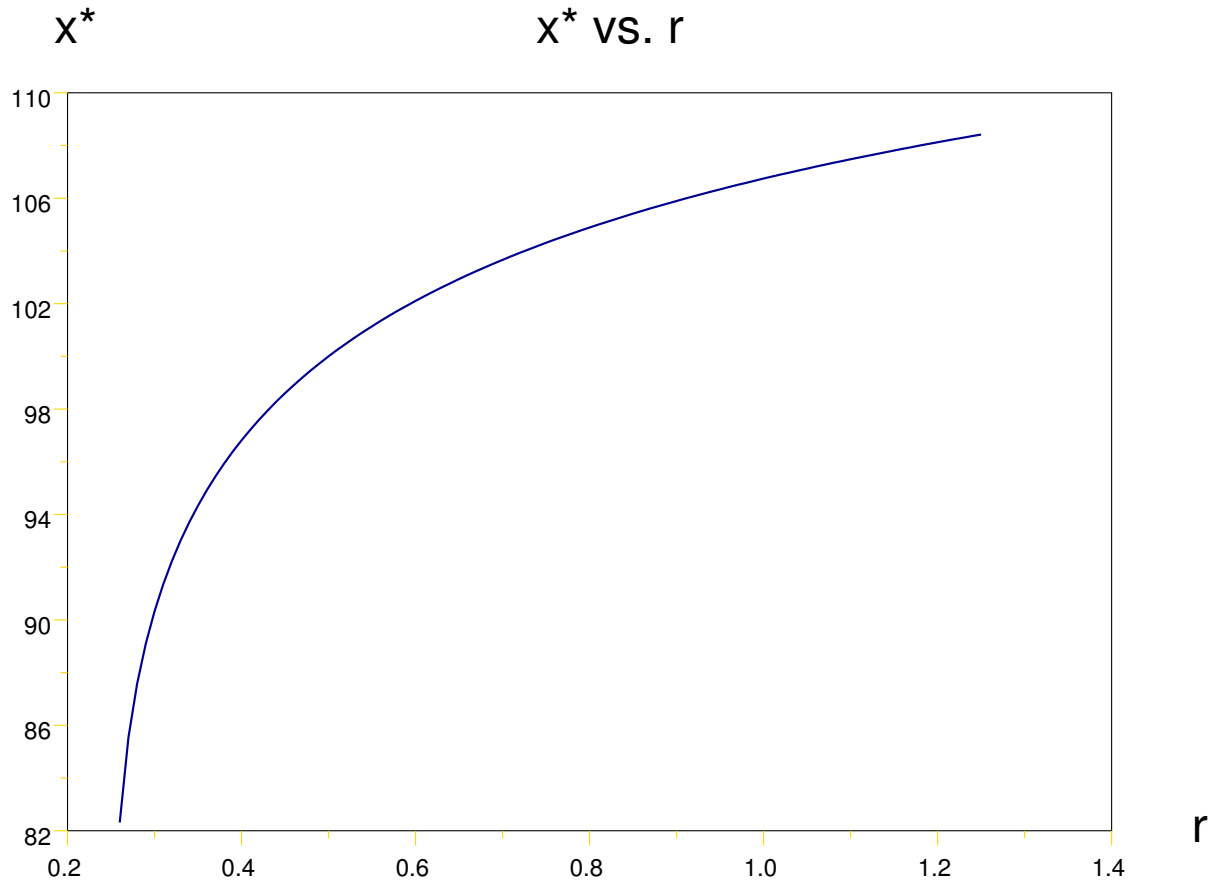
Newsvendor Problem

Example 1: $r = 1, c = .25, s = 0, \mu_w = 100, \sigma_w = 10$



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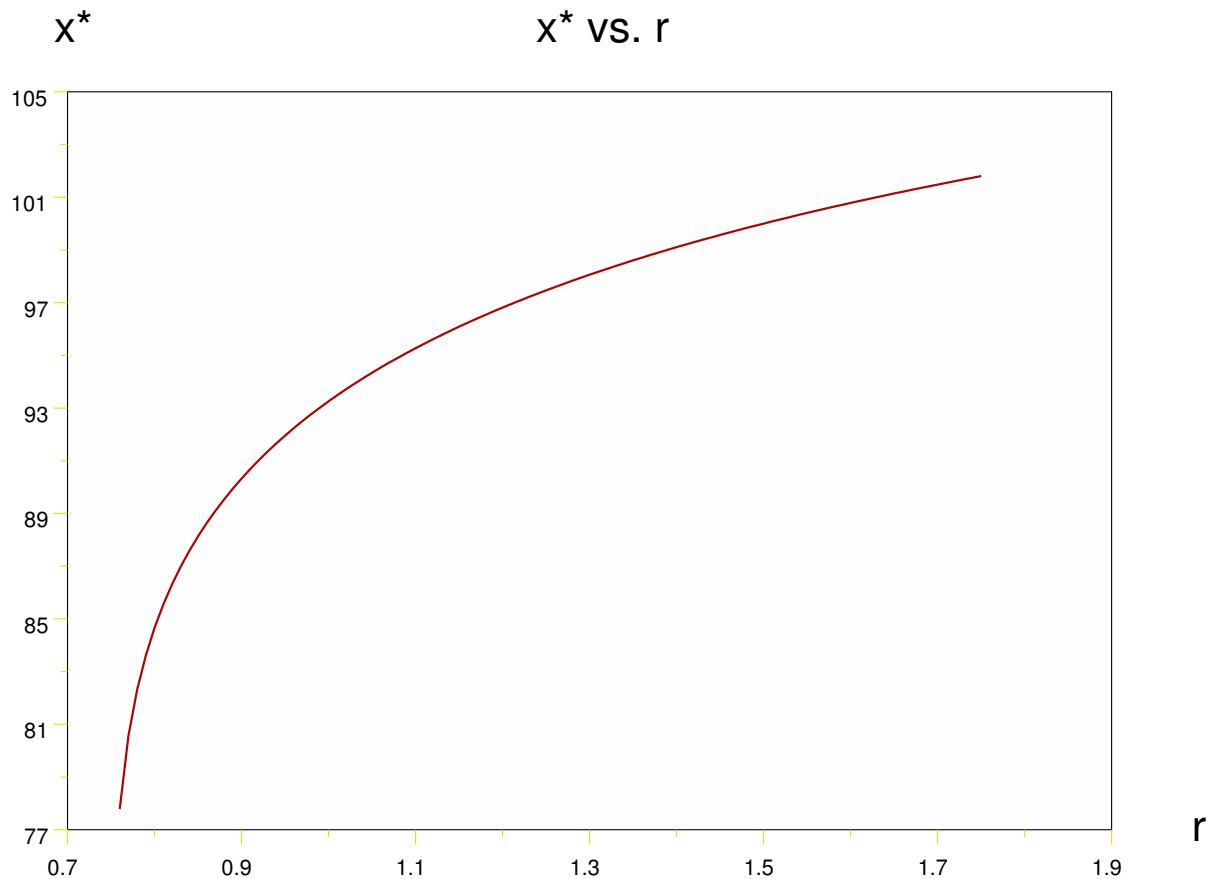
Newsvendor Problem

Example 2: $r = 1, c = .75, s = 0, \mu_w = 100, \sigma_w = 10$



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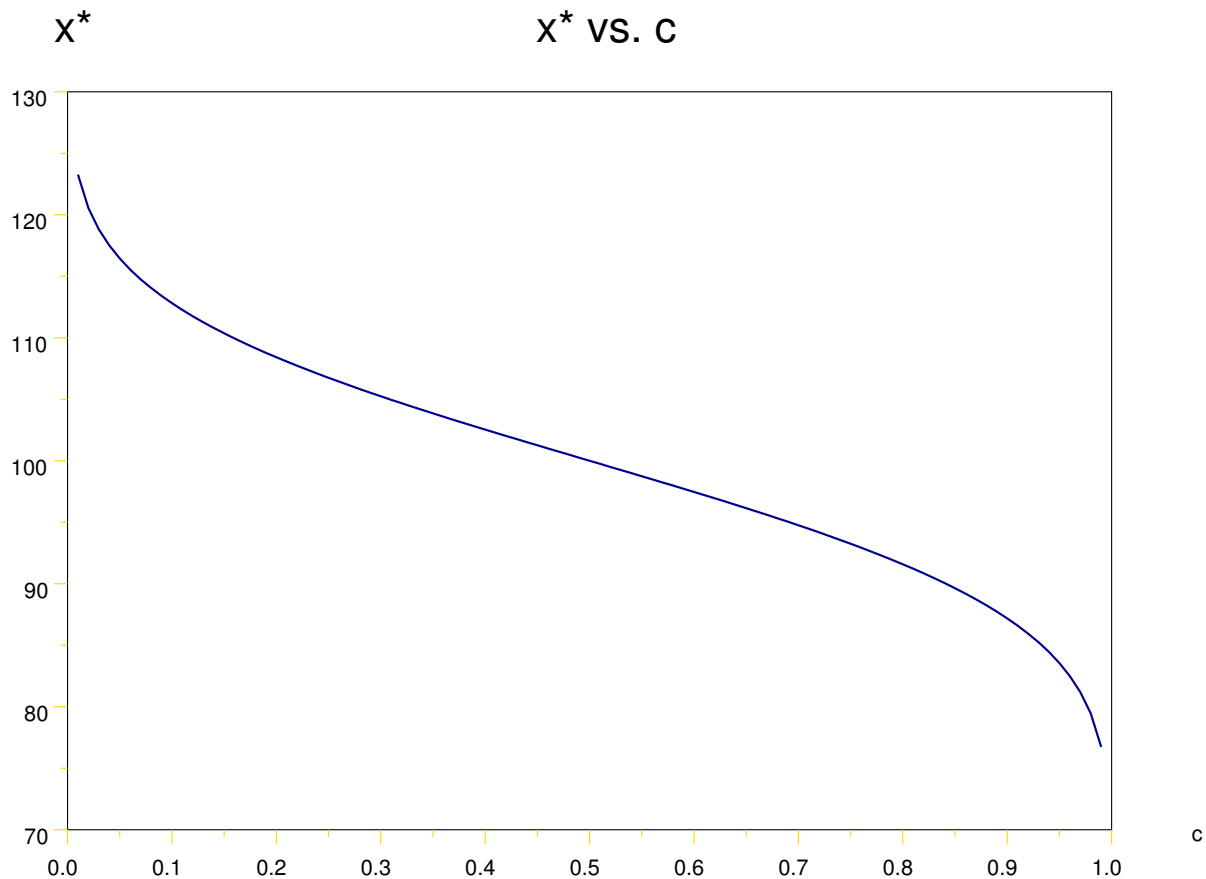

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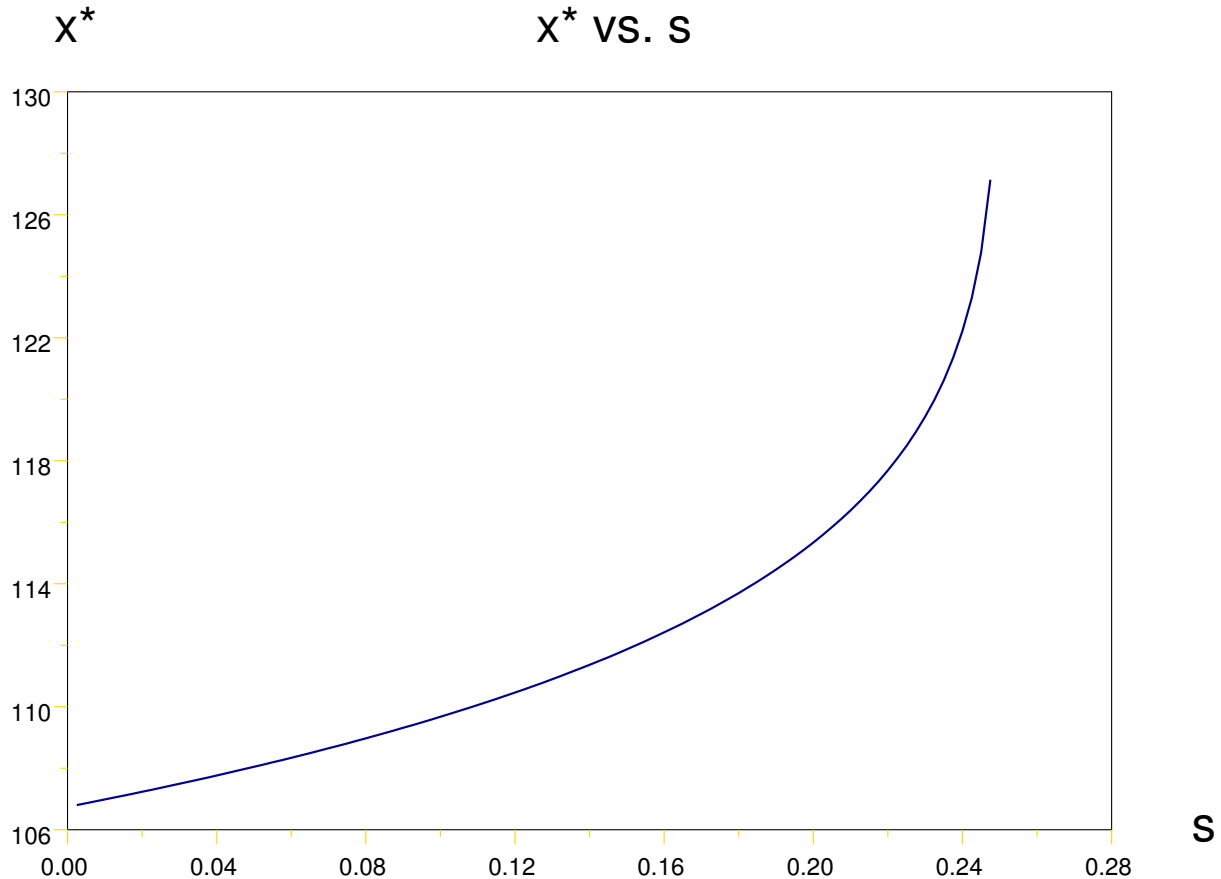

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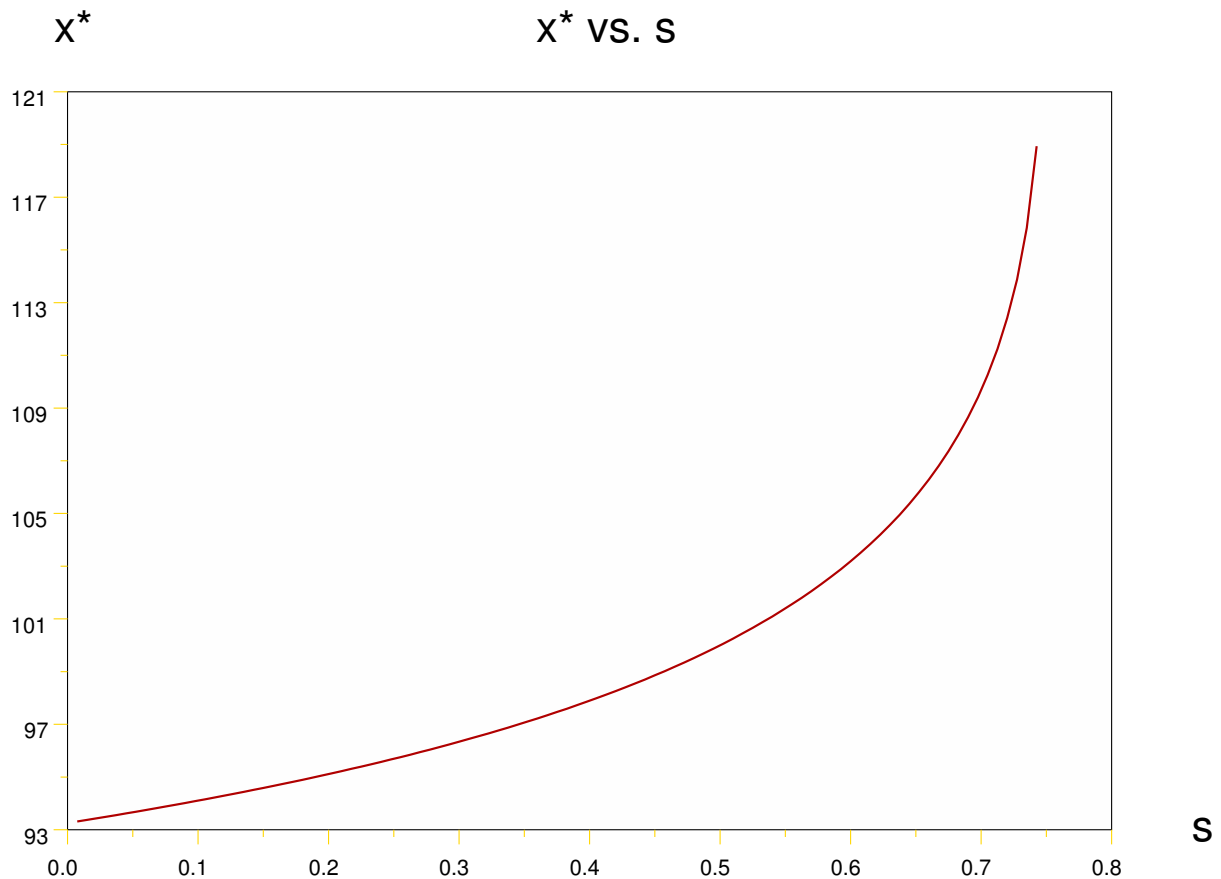

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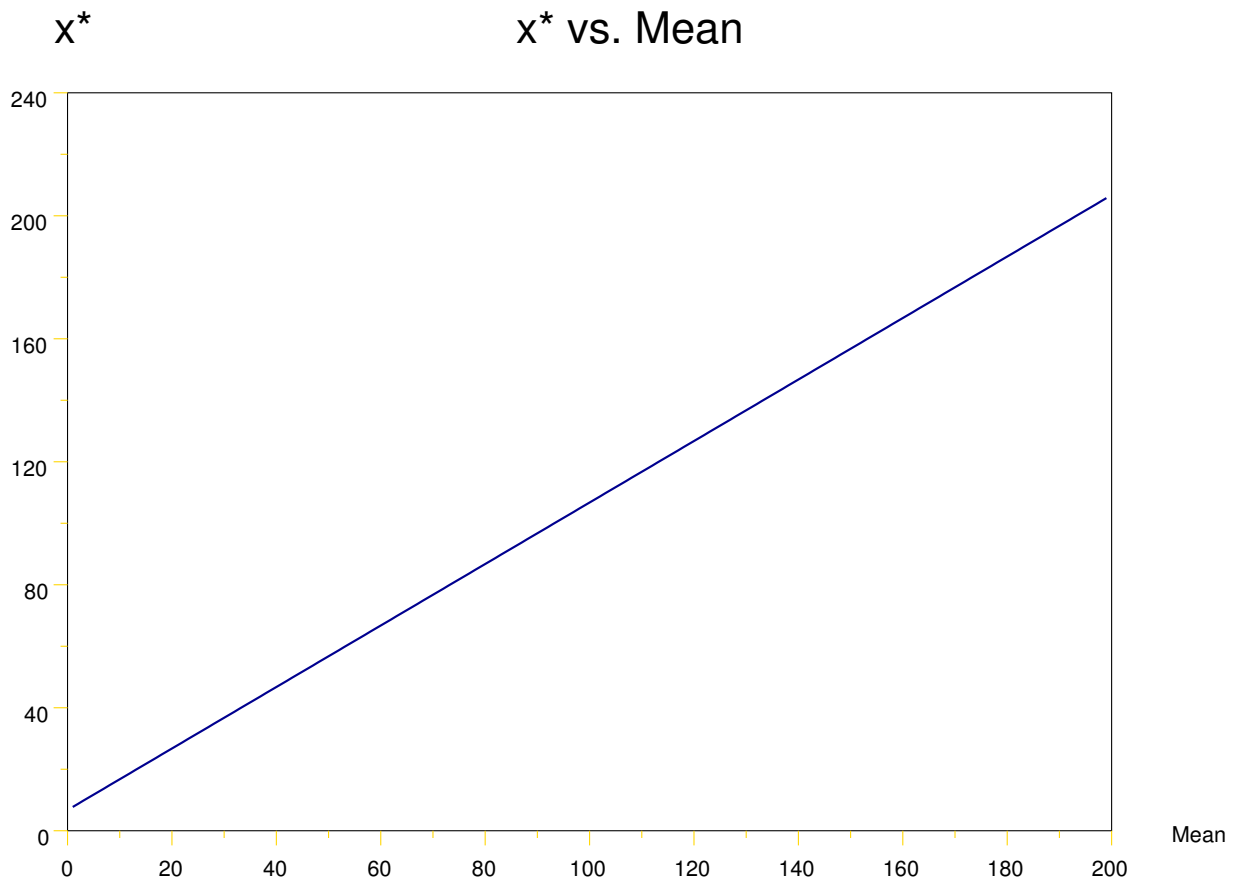

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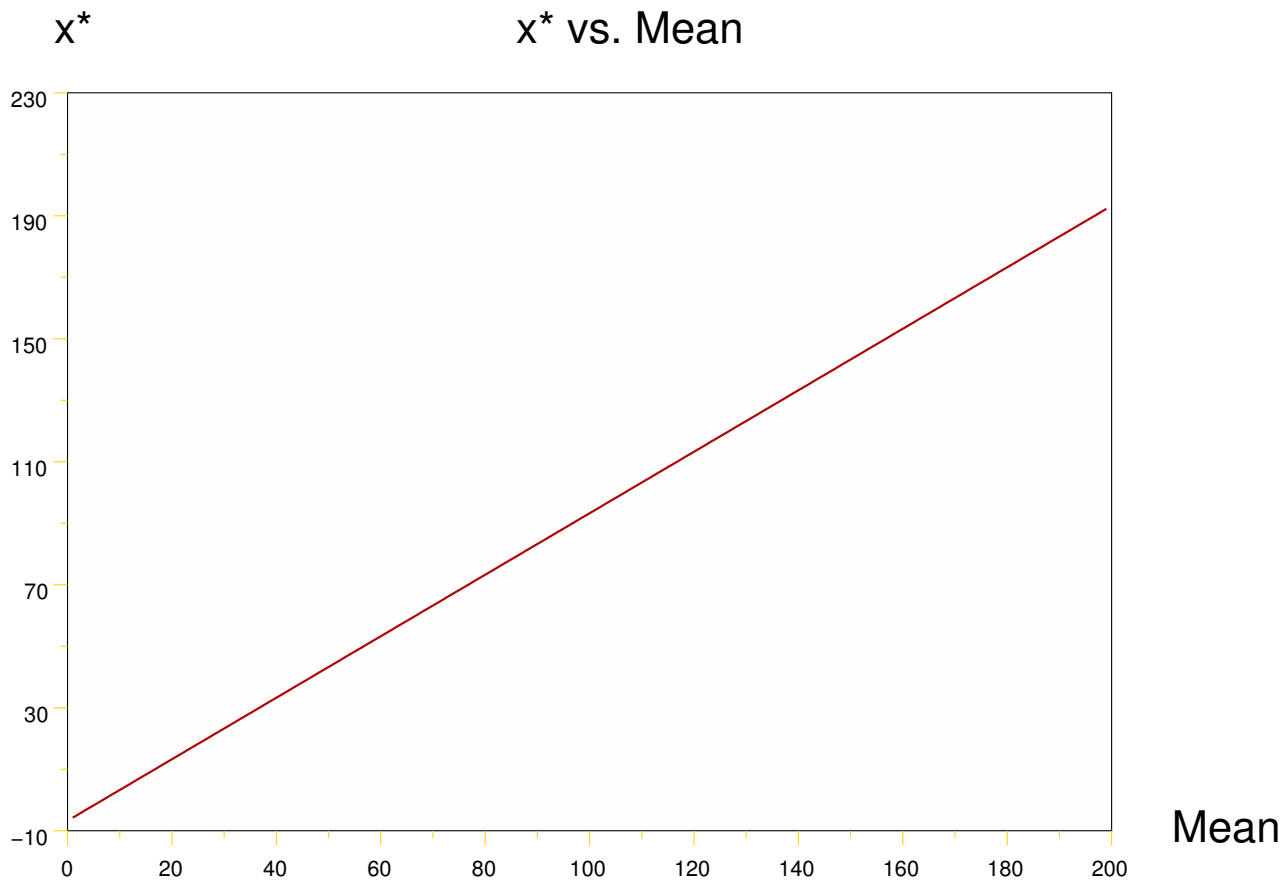

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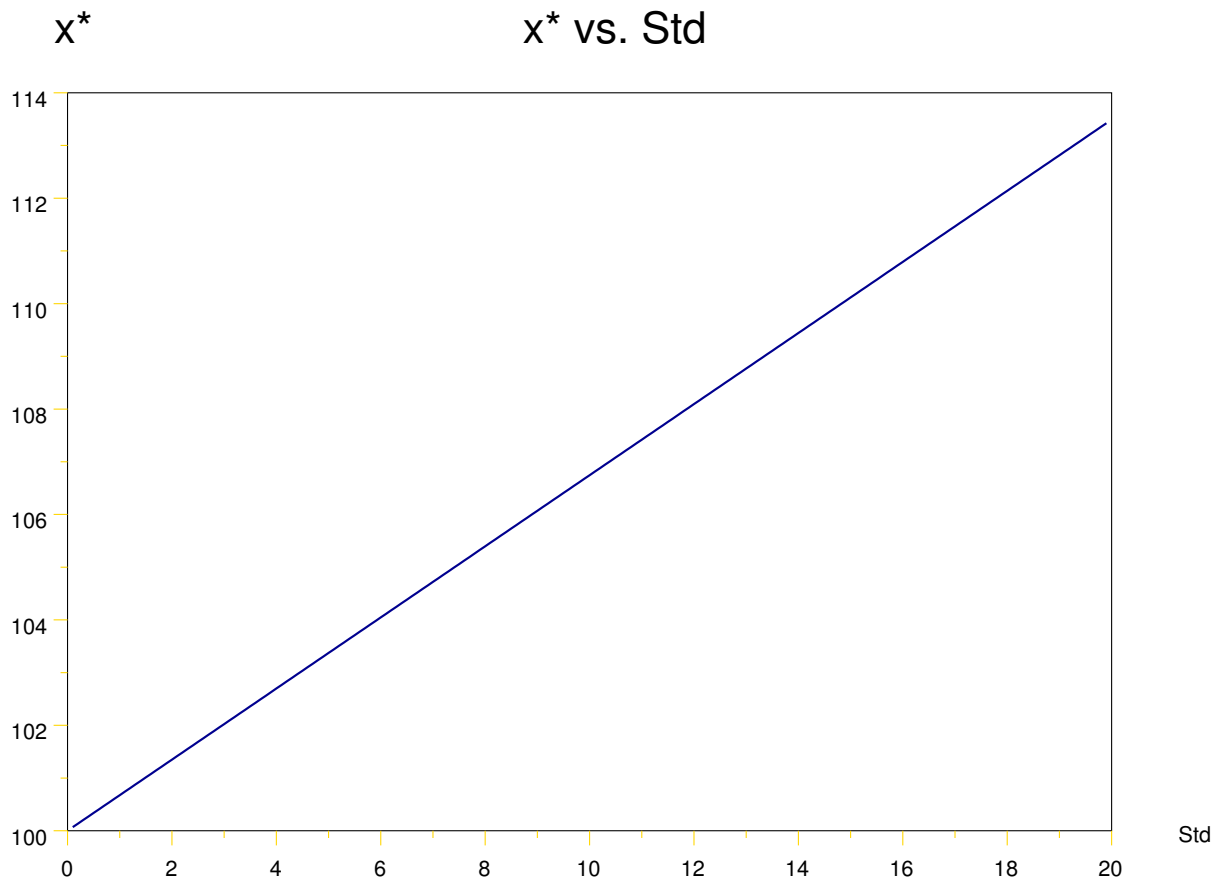

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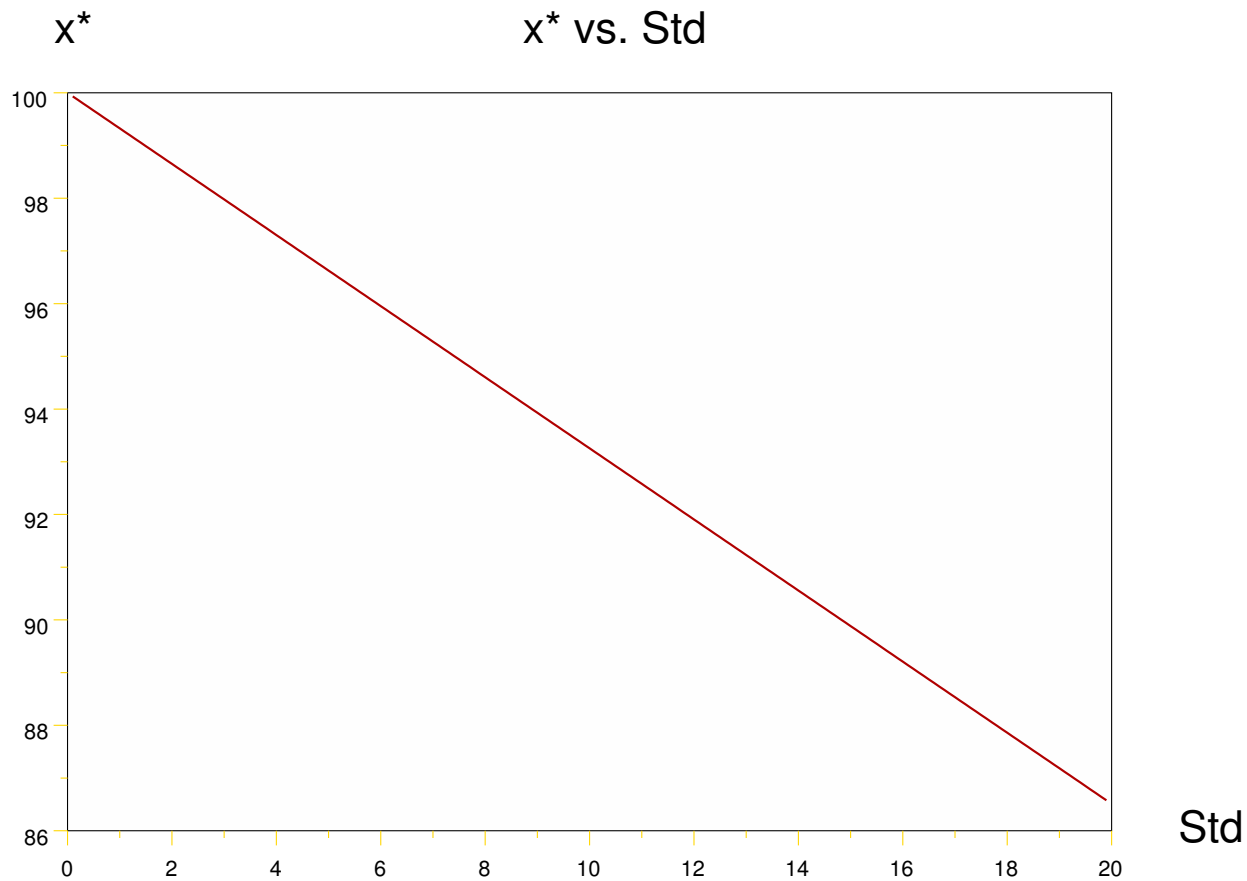

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Newsvendor Problem

Why does x^* look linear in μ_w and σ_w?

- x^* is the solution to $F(x^*) = \dfrac{r-c}{r-s}$.
- Assume demand w is $N(\mu_w, \sigma_w)$. Then, for any demand w,

$$F(w) = \Phi\left(\frac{w-\mu_w}{\sigma_w}\right)$$

where Φ is the standard normal cumulative distribution function.

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Newsvendor Problem

Why does x^* look linear in μ_w and σ_w?

- Therefore

$$\Phi\left(\frac{x^* - \mu_w}{\sigma_w}\right) = \frac{r - c}{r - s}$$

- or

$$\frac{x^* - \mu_w}{\sigma_w} = \Phi^{-1}\left(\frac{r - c}{r - s}\right)$$

- or

$$\boxed{x^* = \mu_w + \sigma_w \Phi^{-1}\left(\frac{r - c}{r - s}\right)}$$

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Newsvendor Problem

Why does x^* look linear in μ_w and σ_w?

Is it, really?

For what values of μ_w and σ_w is x^* clearly *nonlinear* in μ_w and σ_w?

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Newsvendor Problem

x^* Increasing or Decreasing in σ_w

- Note that

$$\Phi^{-1}(k) > 0 \text{ if } k > .5$$

and

$$\Phi^{-1}(k) < 0 \text{ if } k < .5$$

- Therefore
  - ⋆ x^* increases with σ_w if $\dfrac{r-c}{r-s} > .5$, and
  - ⋆ x^* decreases with σ_w if $\dfrac{r-c}{r-s} < .5$.
- *Greater potential gain promotes riskier behavior!*

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Newsvendor Problem

A General Strategy

Question: Can we extend this strategy to manage inventory in other settings? In particular, suppose the stock remaining at the end of the day today can be used in the future. Does the result of the newsvendor problem suggest a possible heuristic extension?

Answer: Yes.

"Build enough inventory to satisfy demand $100K\%$ of the time, for some K."

*

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Economic Order Quantity

How much to order and when

- *Economic Order Quantity*
- Motivation: economy of scale in ordering.
- Tradeoff:
  - ⋆ Each time an order is placed, the company incurs a fixed cost in addition to the cost of the goods.
  - ⋆ It costs money to store inventory.

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Economic Order Quantity

Assumptions

- No randomness.
- No delay between ordering and arrival of goods.
- No backlogs.
- Goods are required at an annual rate of λ units per year. Inventory is therefore depleted at the rate of λ units per year.
- If the company orders Q units, it must pay $s + cQ$ for the order. s is the *ordering cost* , c is the unit cost.
- It costs h to store one unit for one year. h is the *holding cost.*

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Economic Order Quantity

Problem

- Find a strategy for ordering materials that will minimize the total cost.
- There are two costs to consider: the ordering cost and the holding cost.

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Economic Order Quantity

Scenario

- At time 0, inventory level is 0.
- Q units are ordered and the inventory level jumps instantaneously to Q.
- Material is depleted at rate λ.
- Since the problem is totally deterministic, we can wait until the inventory goes to zero before we order next. There is no danger that the inventory will go to zero earlier than we expect it to.

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Economic Order Quantity

Scenario

- Because of the very simple assumptions, we can assume that the optimal strategy does not change over time.
- Therefore the policy is to order Q each time the inventory goes to zero. We must determine the optimal Q.

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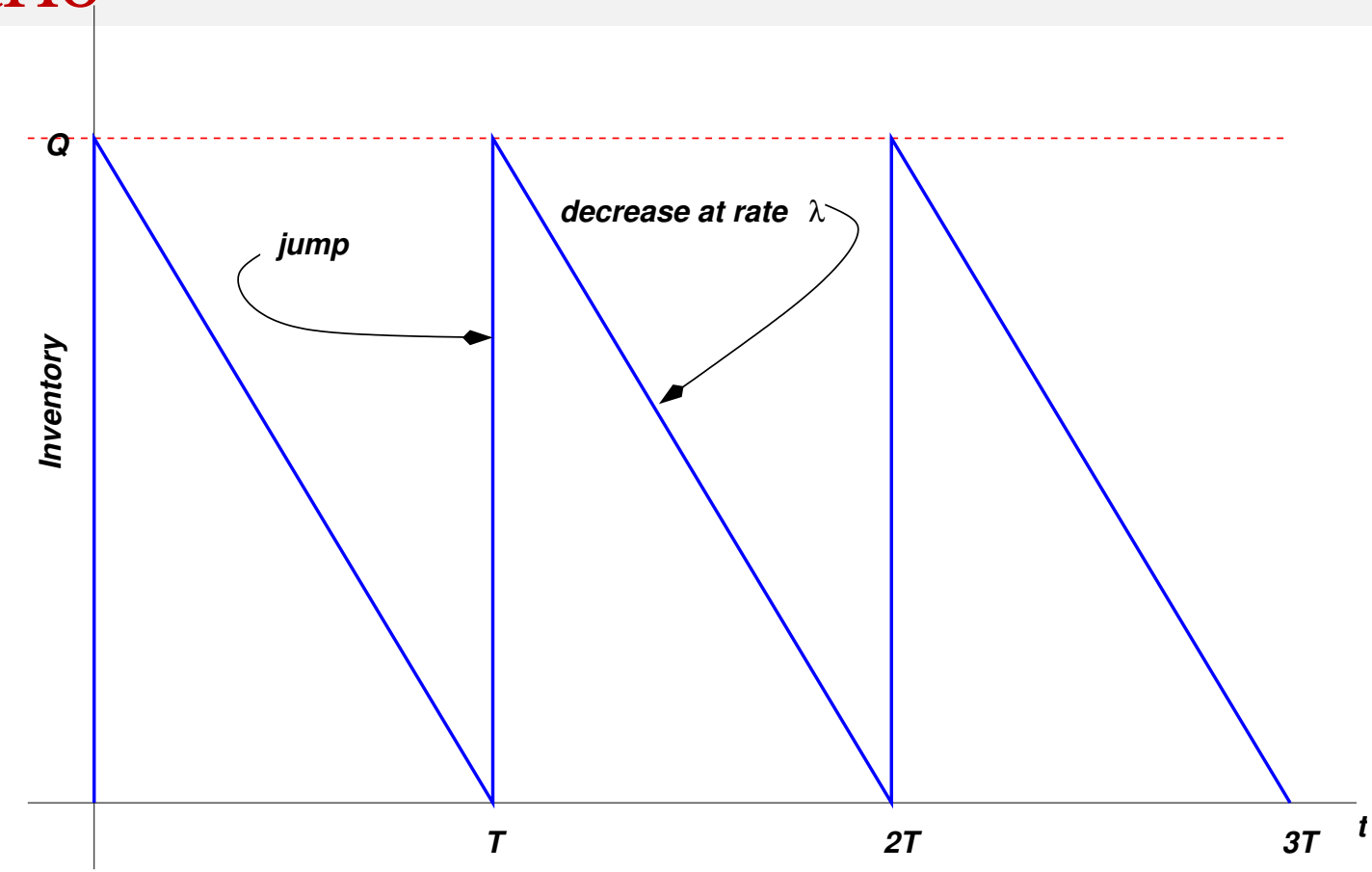
Economic Order Quantity

Scenario



$$T = Q/\lambda \text{ (years)}$$

*

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Economic Order Quantity

Formulation

- The number of orders in a year is $1/T = \lambda/Q$. Therefore, the ordering cost in a year is $s\lambda/Q$.
- The average inventory is $Q/2$. Therefore the average inventory holding cost is $hQ/2$.
- Therefore, we must minimize the annual cost

$$C = \frac{hQ}{2} + \frac{s\lambda}{Q}$$

over Q.

*

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Economic Order Quantity

Formulation

Then

$$\frac{dC}{dQ} = \frac{h}{2} - \frac{s\lambda}{Q^2} = 0$$

or

$$Q^* = \sqrt{\frac{2s\lambda}{h}}$$

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Economic Order Quantity

Examples

In the following graphs, the base case is

- $\lambda = 3000$
- $s = .001$
- $h = 6$

Note that

$$Q^* = 1$$

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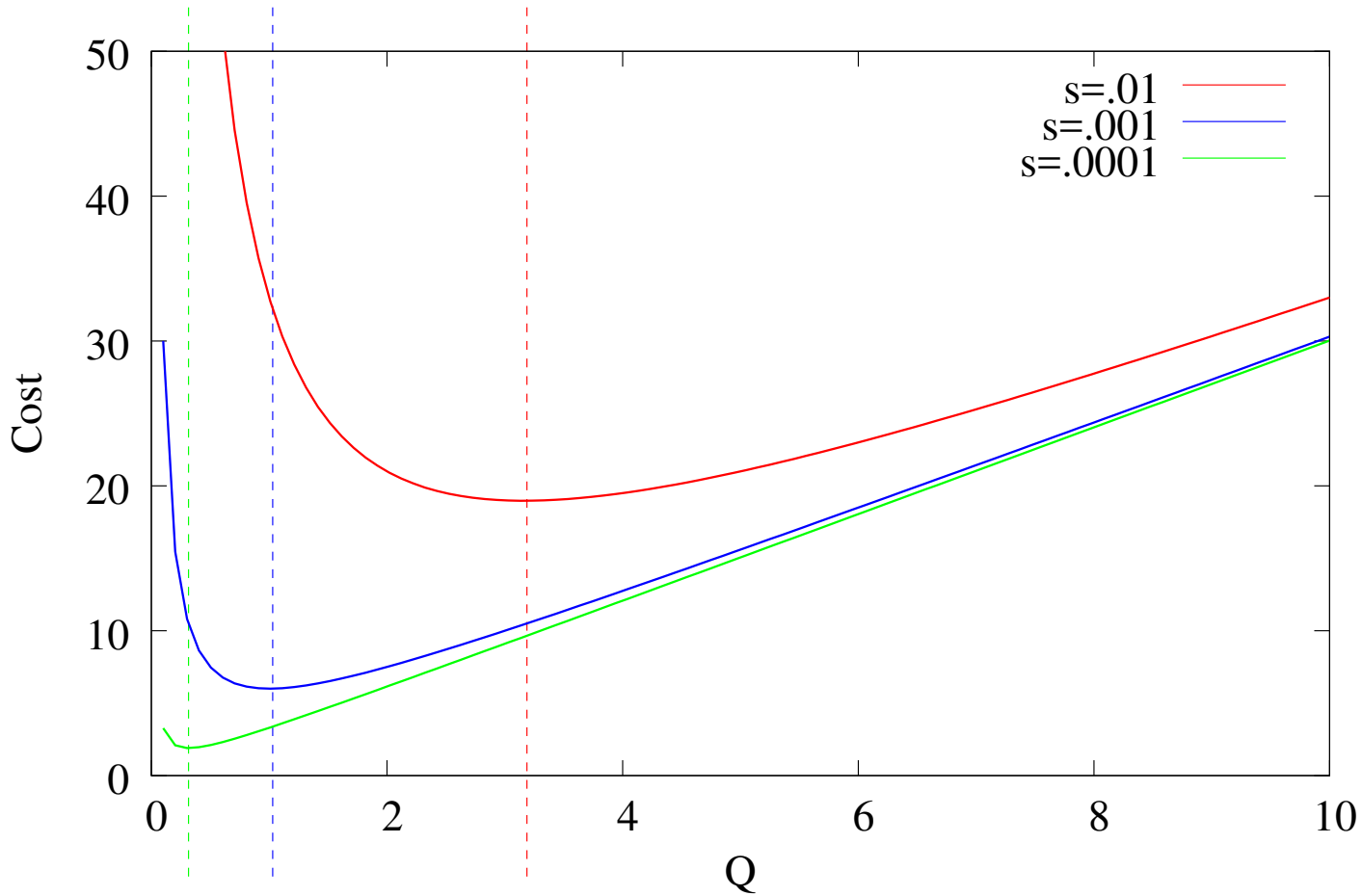
Economic Order Quantity

Examples



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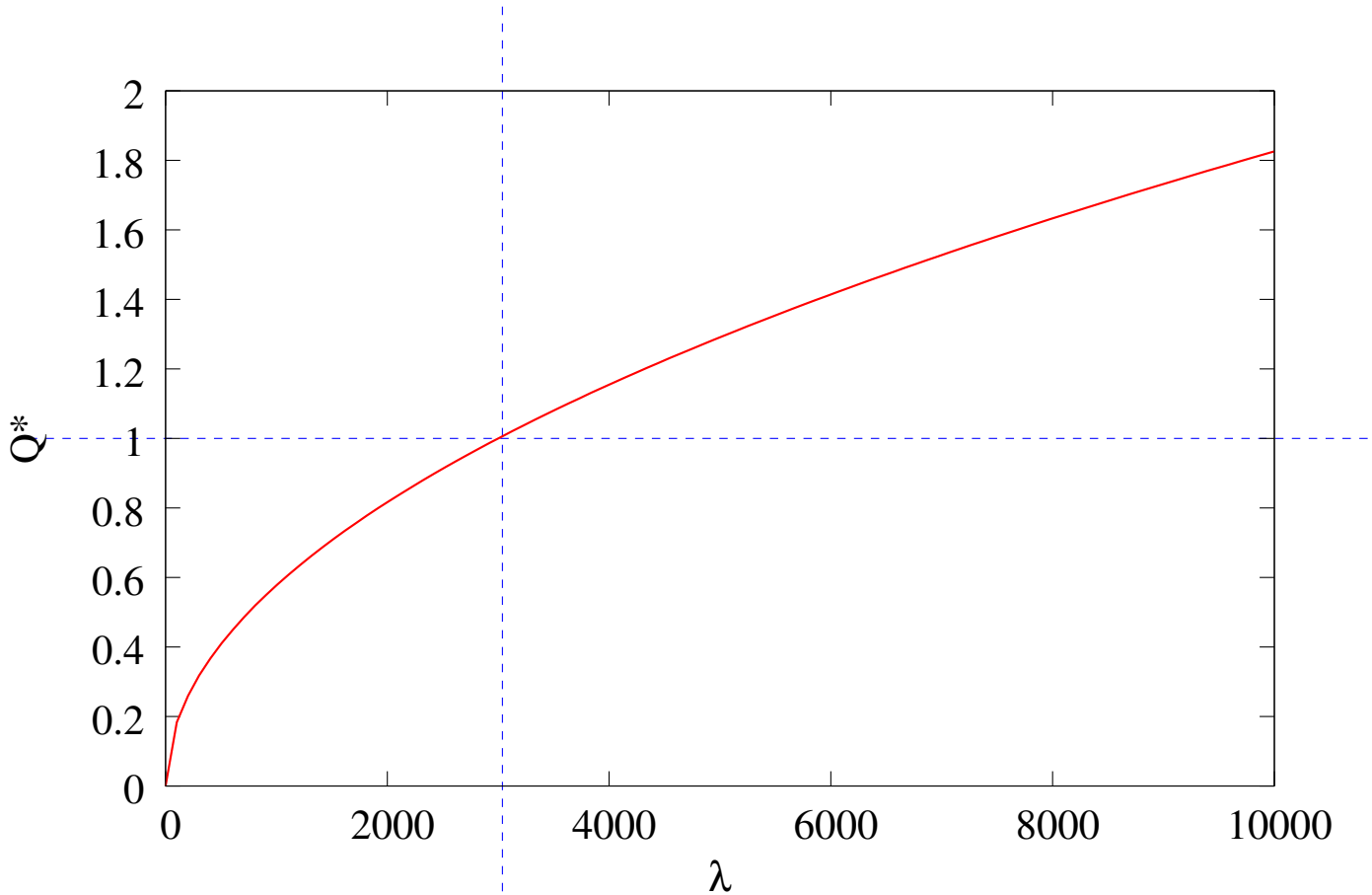
Economic Order Quantity

Examples



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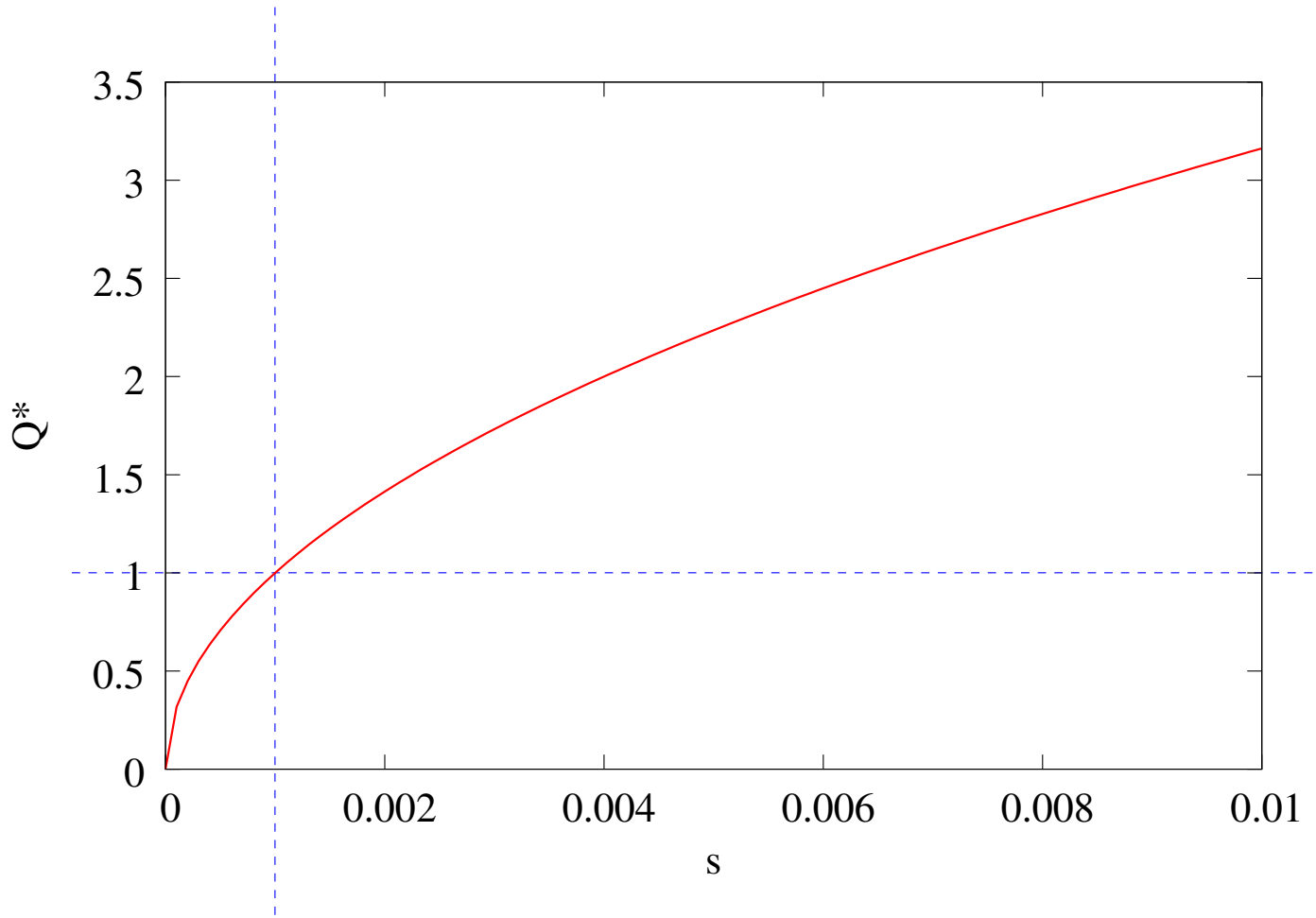
Economic Order Quantity

Examples



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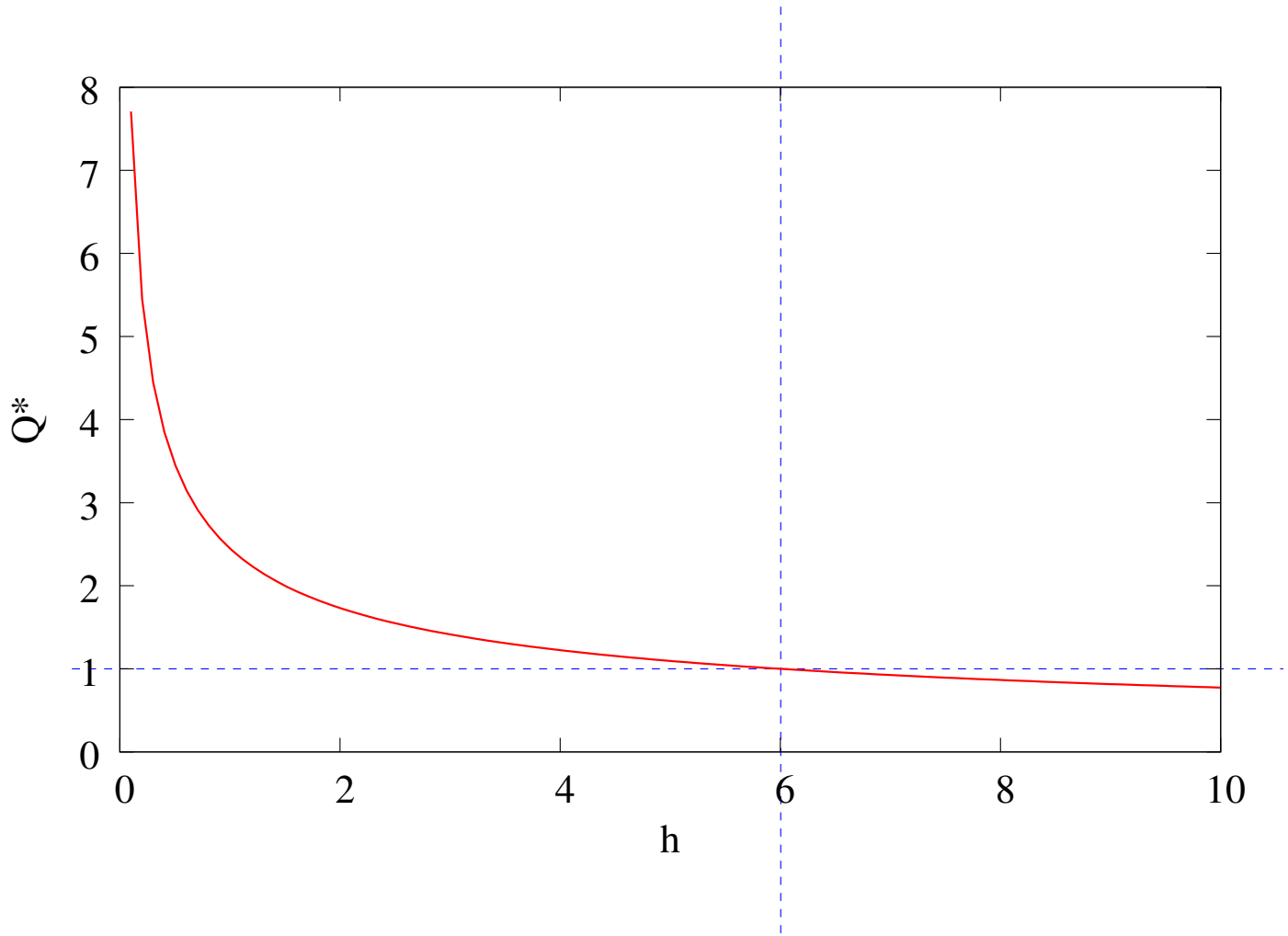
Economic Order Quantity

Examples



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More Issues

- Random delivery times and amounts.
- Order lead time.
- Vanishing inventory.
- *Combinations of these issues and random demand, ordering/setup costs.*

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Base Stock Policy

Make-to-Stock Queue

Usual assumptions:

- Demand is random.
- Inventory is held (unlike newsvendor problem).
- No ordering cost, batching, etc.

Policy

- Try to keep inventory at a fixed level S.

Issue

- Stockout: a demand occurs when there is no stock with which to satisfy it.

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Base Stock Policy

Issues

Issues

- What fraction of the time will there be stockout, and how much demand occurs during stockout periods?
- How much inventory will there be, on the average?
- How much backlog will there be, on the average?

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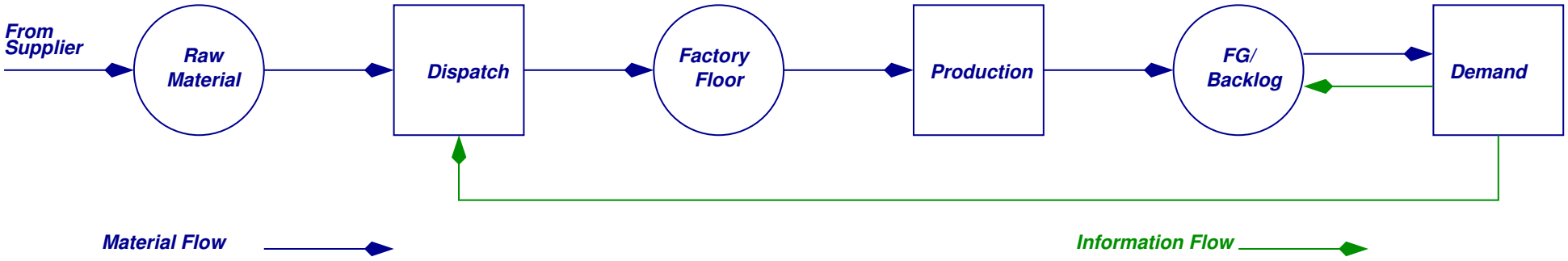
Base Stock Policy

Make-to-Stock Queue



- When an order arrives (generated by "Demand"),
  - ⋆ an item is requested from finished goods inventory (stock),
  - ⋆ an item is delivered to the customer from the finished goods inventory (FG) *or*, if FG is empty, backlog is increased by 1, and
  - ⋆ a signal is sent to "Dispatch" to move one item from raw material inventory to factory floor inventory.

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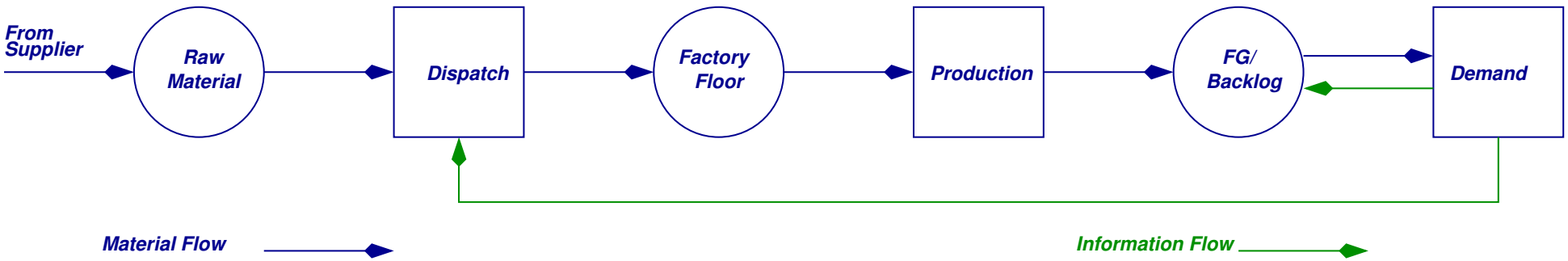
Base Stock Policy

Make-to-Stock Queue



- There is some mechanism for ordering raw material from suppliers.
- We do not consider it here.
- *We assume that the raw material buffer is never empty.*

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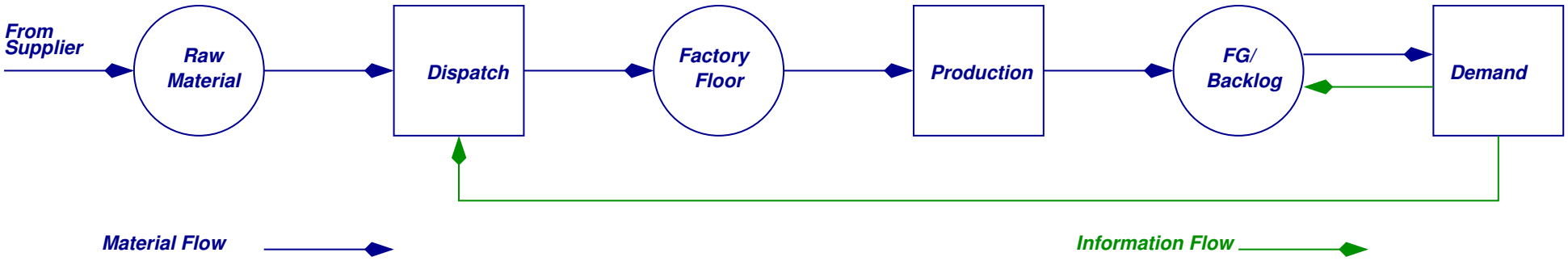
Base Stock Policy

Make-to-Stock Queue



- Whenever the factory floor buffer is not empty and the production line is available, the production line takes a raw part from the factory floor buffer and starts to work on it.
- When the production line completes work on a part, it puts the finished part in the finished goods buffer *or,* if there is backlog, it sends the part to the customer and backlog is reduced by 1.

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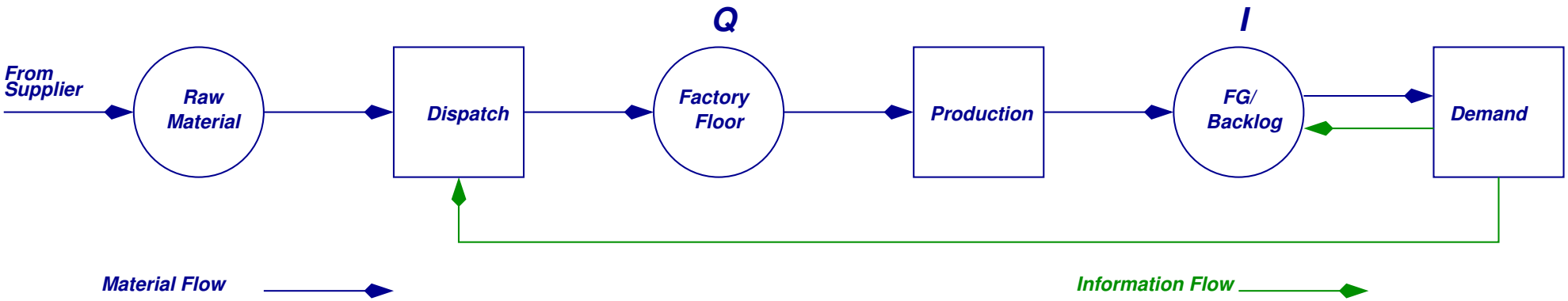
Base Stock Policy

Make-to-Stock Queue



- $Q(t) =$ factory floor inventory at time t.
- $I(t) =$
  - ⋆ the number of items in FG, if this is non-negative and there is no backlog; *or*
  - ⋆ –(the amount of backlog), if backlog is non-negative and FG is empty.
- There are *no lost sales.* $I(t)$ is not bounded from below. It can take on any negative value.

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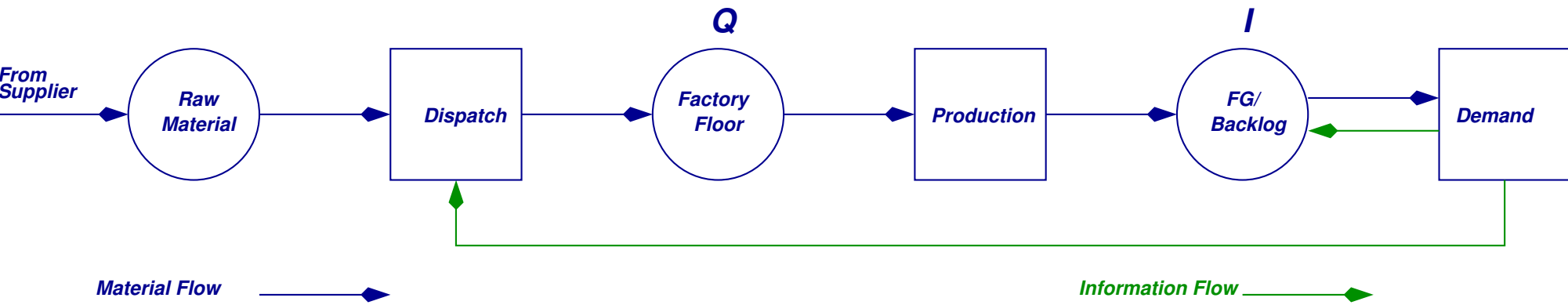
Base Stock Policy

Make-to-Stock Queue



- Assume $Q(0) = 0$ and $I(0) = S$. Then $Q(0) + I(0) = S$.
- At every time t when a demand arrives,
  - ⋆ $I(t)$ decreases by 1 and $Q(t)$ increases by 1 so $Q(t) + I(t)$ does not change.
- At every time t when a part is produced,
  - ⋆ $I(t)$ increases by 1 and $Q(t)$ decreases by 1 so $Q(t) + I(t)$ does not change.
- Therefore $\boxed{Q(t) + I(t) = S \quad \text{for all } t.}$

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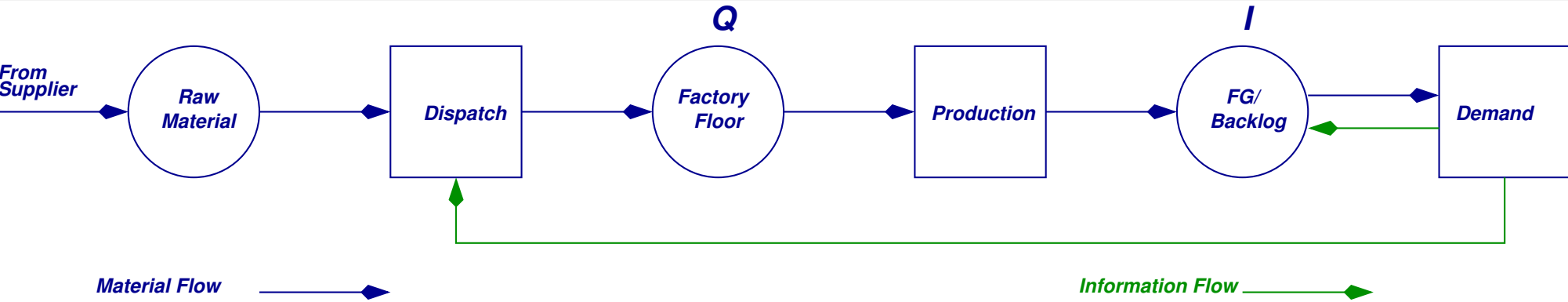
Base Stock Policy

Make-to-Stock Queue



- Also, $Q(t) \geq 0$ and $I(t) \leq S$.
- Assume the factory floor buffer is infinite.
- Assume demands arrive according to a Poisson process with rate parameter λ.
- Assume the production process time is exponentially distributed with rate parameter μ.
- *Then $Q(t)$ behaves like the state of an M/M/1 queue.*

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Base Stock Policy

Make-to-Stock Queue

- Assume $\lambda < \mu$.

Therefore, in steady state,

$$\text{prob}(Q = q) = (1 - \rho)\rho^q, q \geq 0$$

where $\rho = \lambda/\mu < 1$.

- *What fraction of the time will there be stockout?*

  That is, what is prob$(I \leq 0)$?

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Base Stock Policy

Make-to-Stock Queue

$$\text{prob}(I \leq 0) = \text{prob}(Q \geq S) = \sum_{q=S}^{\infty} (1-\rho)\rho^q$$

$$= (1-\rho)\sum_{q=S}^{\infty} \rho^q = (1-\rho)\rho^S \sum_{q=0}^{\infty} \rho^q = (1-\rho)\rho^S \frac{1}{1-\rho}$$

so

$$\text{prob}(I \leq 0) = \rho^S$$

- *How much demand occurs during stockout periods?*

$$\rho^S \lambda$$

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Base Stock Policy

Make-to-Stock Queue

- *How much inventory will there be, on the average?* (It depends on what you mean by "inventory".)
  - $\star$ $$EQ = \frac{\rho}{1-\rho}$$
- $EI = S - EQ$ is *not* the expected finished goods inventory. It is the expected inventory/backlog.
- We want to know

$$E(I^+) = E\begin{cases} I \text{ if } I > 0 \\ \\ 0 \qquad\qquad \text{otherwise} \end{cases} = E(I|I>0)\text{prob}(I>0)$$

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Base Stock Policy

Make-to-Stock Queue $_S$

$$E(I|I>0) = \sum_{i=1}^{S} i \text{ prob}(I=i|I>0)$$

$$= \sum_{i=1}^{S} \frac{i \text{ prob}(I=i \cap I>0)}{\text{prob}(I>0)} = \frac{\sum_{i=1}^{S} i \text{ prob}(I=i)}{\sum_{j=1}^{S} \text{prob}(I=j)}$$

$$= \frac{\sum_{q=0}^{S-1} (S-q) \text{ prob}(Q=q)}{\sum_{s=0}^{S-1} \text{prob}(Q=s)}$$

where, because $Q = S - I$, we substitute $q = S - i$ and $s = S - j$.

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Base Stock Policy

Make-to-Stock Queue

$$= \frac{\sum_{q=0}^{S-1} S \text{ prob}(Q=q) - \sum_{q=0}^{S-1} q \text{ prob}(Q=q)}{\sum_{q'=0}^{S-1} \text{prob}(Q=q')}$$

$$= S - \frac{\sum_{q=0}^{S-1} q \text{ prob}(Q=q)}{\sum_{q'=0}^{S-1} \text{prob}(Q=q')} = S - \frac{(1-\rho)\sum_{q=0}^{S-1} q\rho^q}{(1-\rho)\sum_{q=0}^{S-1} \rho^q} = S - \frac{\sum_{q=0}^{S-1} q\rho^q}{\sum_{q=0}^{S-1} \rho^q}$$

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Base Stock Policy

Make-to-Stock Queue

Therefore

$$E(I|I>0) = S - \frac{\sum_{q=0}^{S-1} q\rho^q}{\sum_{q=0}^{S-1} \rho^q}$$

and

$$E(I^+) = S(1-\rho)\sum_{q=0}^{S-1}\rho^q - (1-\rho)\sum_{q=0}^{S-1} q\rho^q$$

$$= S\ \text{prob}(Q<S) - (1-\rho)\sum_{q=0}^{S-1} q\rho^q$$

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Base Stock Policy

Make-to-Stock Queue

- *How much backlog will there be, on the average?*
- This time, we are looking for $-E(I^-)$, where

$$E(I^-) = E\begin{cases} I & \text{if } I \leq 0 \\ & \\ 0 & \text{otherwise} \end{cases} = E(I|I \leq 0)\text{prob}(I \leq 0$$

$$E(I|I \leq 0) = \sum_{i=0}^{-\infty} i \text{ prob}(I = i|I \leq 0)$$

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Base Stock Policy

Make-to-Stock Queue

$$= \sum_{i=0}^{-\infty} i \; \frac{\text{prob}(I = i \cap I < 0)}{\text{prob}(I < 0)} = \frac{\sum_{i=0}^{-\infty} i \; \text{prob}(I = i)}{\sum_{j=0}^{-\infty} \text{prob}(I = j)}$$

$$= \frac{\sum_{q=S}^{\infty} (S - q) \; \text{prob}(Q = q)}{\sum_{s=S}^{\infty} \text{prob}(Q = s)}$$

where, because $Q = S - I$, we substitute $q = S - i$ and $s = S - j$.

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Base Stock Policy

Make-to-Stock Queue

$$= \frac{\sum_{q=S}^{\infty} S \text{ prob}(Q = q) - \sum_{q=S}^{\infty} q \text{ prob}(Q = q)}{\sum_{q'=S}^{\infty} \text{prob}(Q = q')}$$

$$= S - \frac{\sum_{q=S}^{\infty} q \text{ prob}(Q = q)}{\sum_{q'=S}^{\infty} \text{prob}(Q = q')} = S - \frac{EQ - \sum_{q=0}^{S-1} q \text{ prob}(Q = q)}{1 - \sum_{q'=0}^{S-1} \text{prob}(Q = q')}$$

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Base Stock Policy

Make-to-Stock Queue

$$= S - \frac{EQ - (1-\rho)\sum_{q=0}^{S-1} q\rho^q}{1 - (1-\rho)\sum_{q=0}^{S-1} \rho^q}$$

so

$$E(I^-) = E(I|I \le 0)\text{prob}(I \le 0)$$

Note: $E(I^+) + E(I^-) + EQ = S.$

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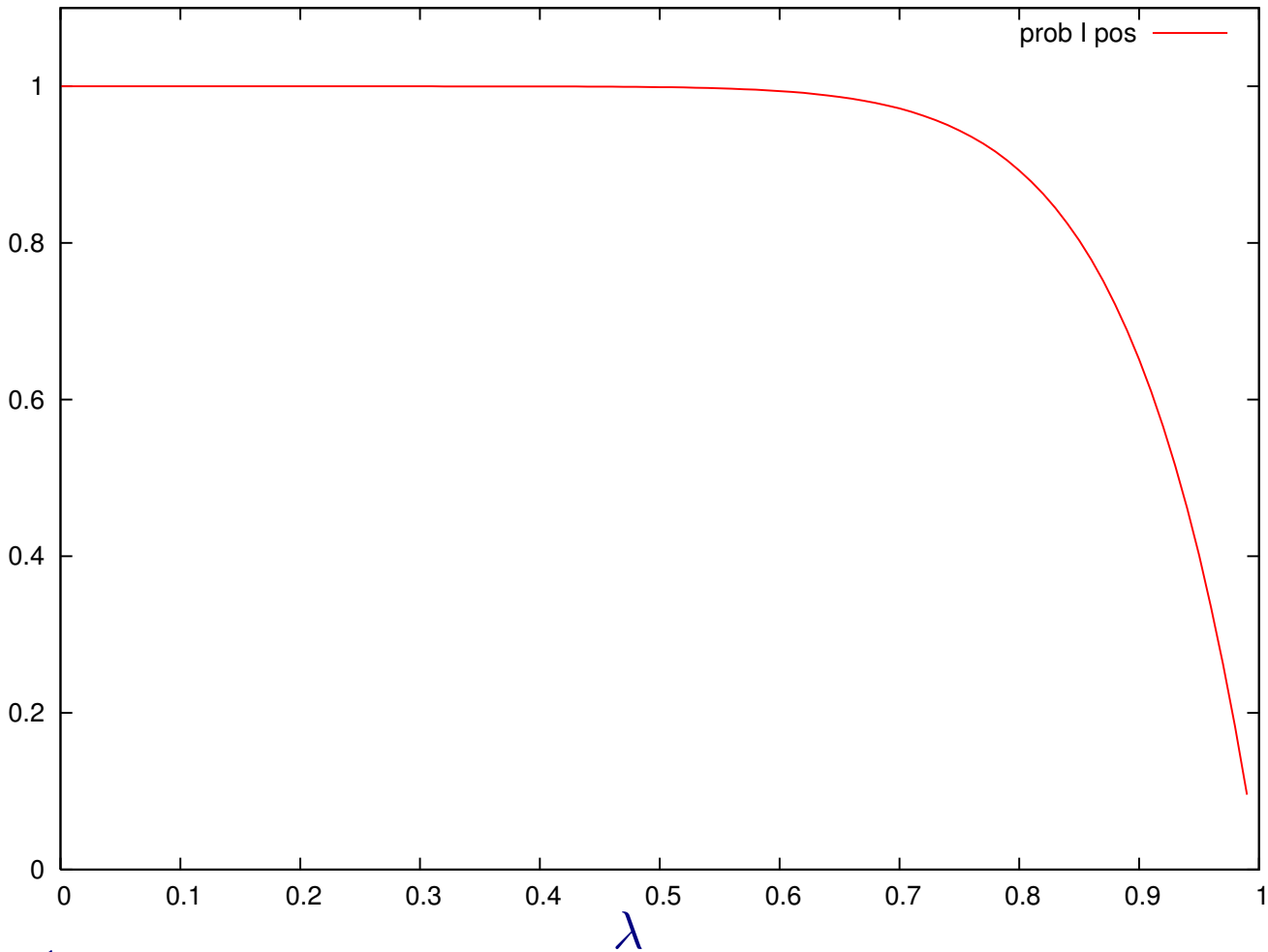
Base Stock Policy

Make-to-Stock Queue



$S = 10, \mu = 1$

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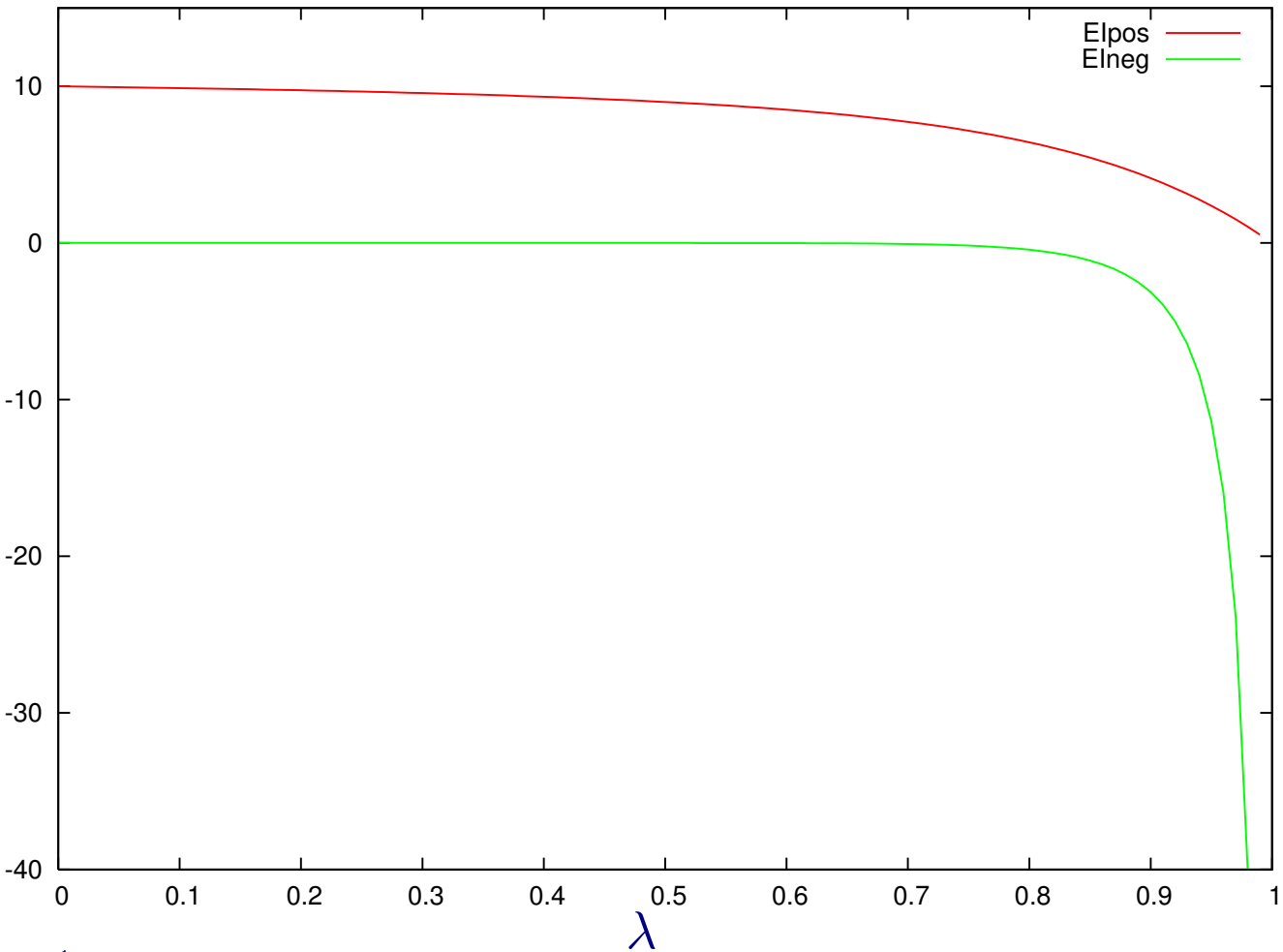
Base Stock Policy

Make-to-Stock Queue



$S = 10, \mu = 1$

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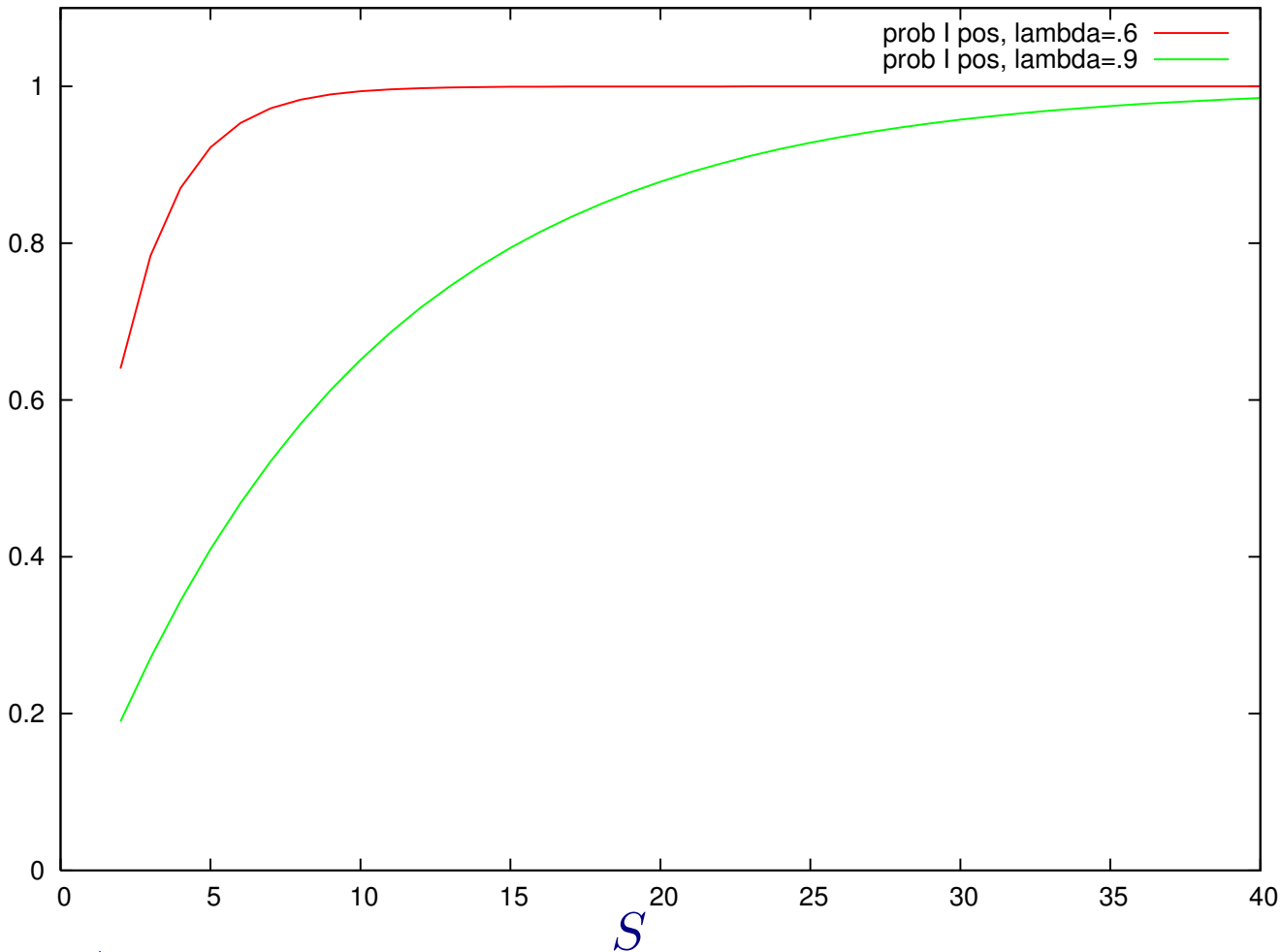
Base Stock Policy

Make-to-Stock Queue



$\lambda = .6, .9, \mu = 1$

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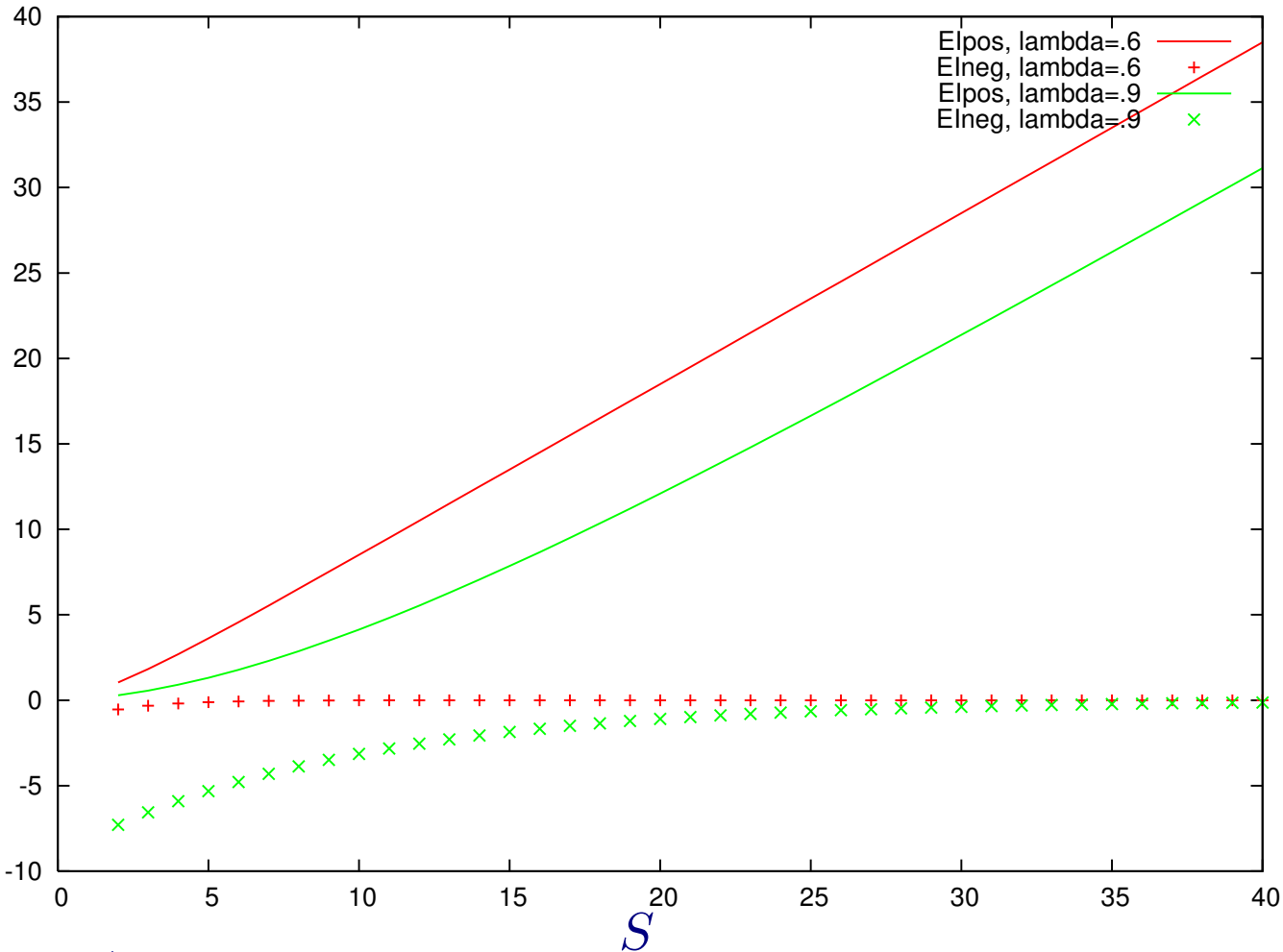
Base Stock Policy

Make-to-Stock Queue



$\lambda = .6, .9, \mu = 1$

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Base Stock Policy

Optimization of Stock

We have analyzed a policy but we haven't completely specified it.

How do we select S?

One possible way: choose S to minimize a function that penalizes expected costs due to finished goods inventory ($E(I^+)$) and backlog ($E(I^-)$).

That is, minimize

$$EC(I) = E(hI^+ - bI^-) = hE(I^+) - bE(I^-),$$

where h is the cost of holding one unit of inventory for one time unit and b is the cost of having one unit of backlog for one time unit.

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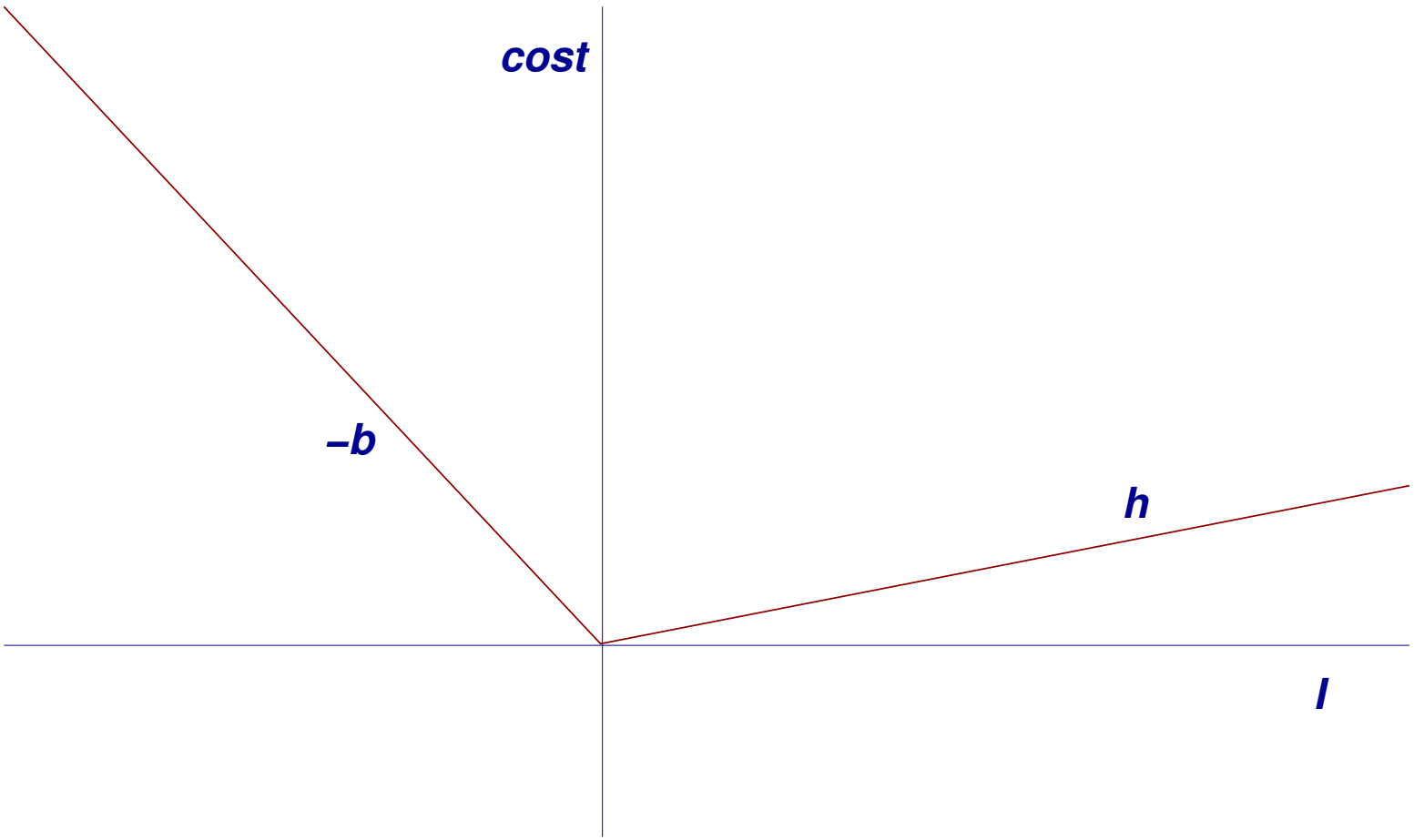
Base Stock Policy

Optimization of Stock



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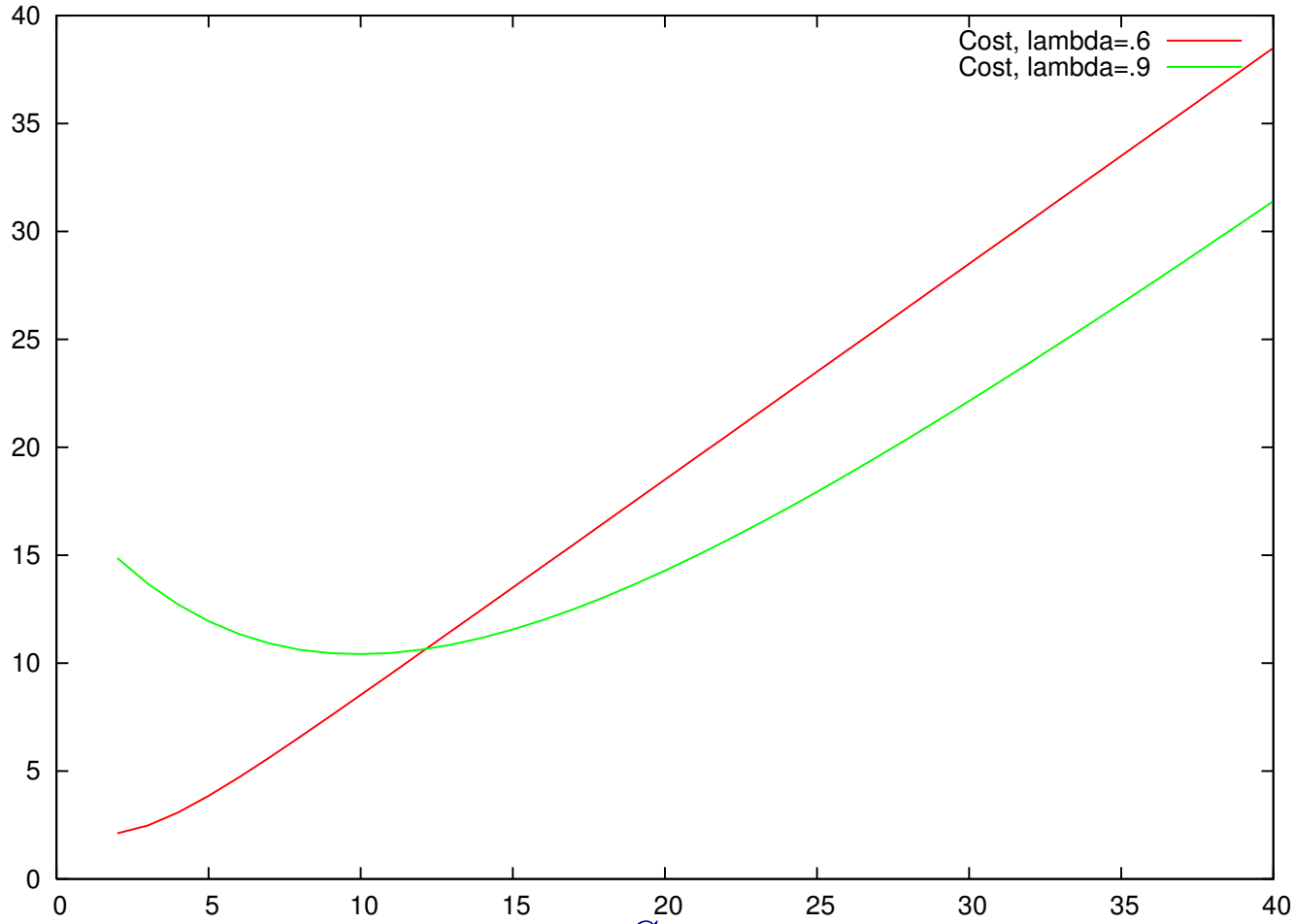
Base Stock Policy

Optimization of Stock



$\lambda = .6, .9, \mu = 1, h = 1, b = 2,$ Expected cost$= hE(I^+) - bE(I^-)$

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Base Stock Policy

Optimization of Stock

Problem: Find S^* to minimize $hE(I^+) - bE(I^-)$.

Solution: $S^* = \lfloor \tilde{S} \rfloor$ or $\lfloor \tilde{S} \rfloor + 1$, where

$$\rho^{\tilde{S}+1} = \frac{h}{h+b}$$

Note: $\rho^{\tilde{S}+1} = \text{prob}(Q \geq \tilde{S}+1) = \text{prob}(I < 0)$

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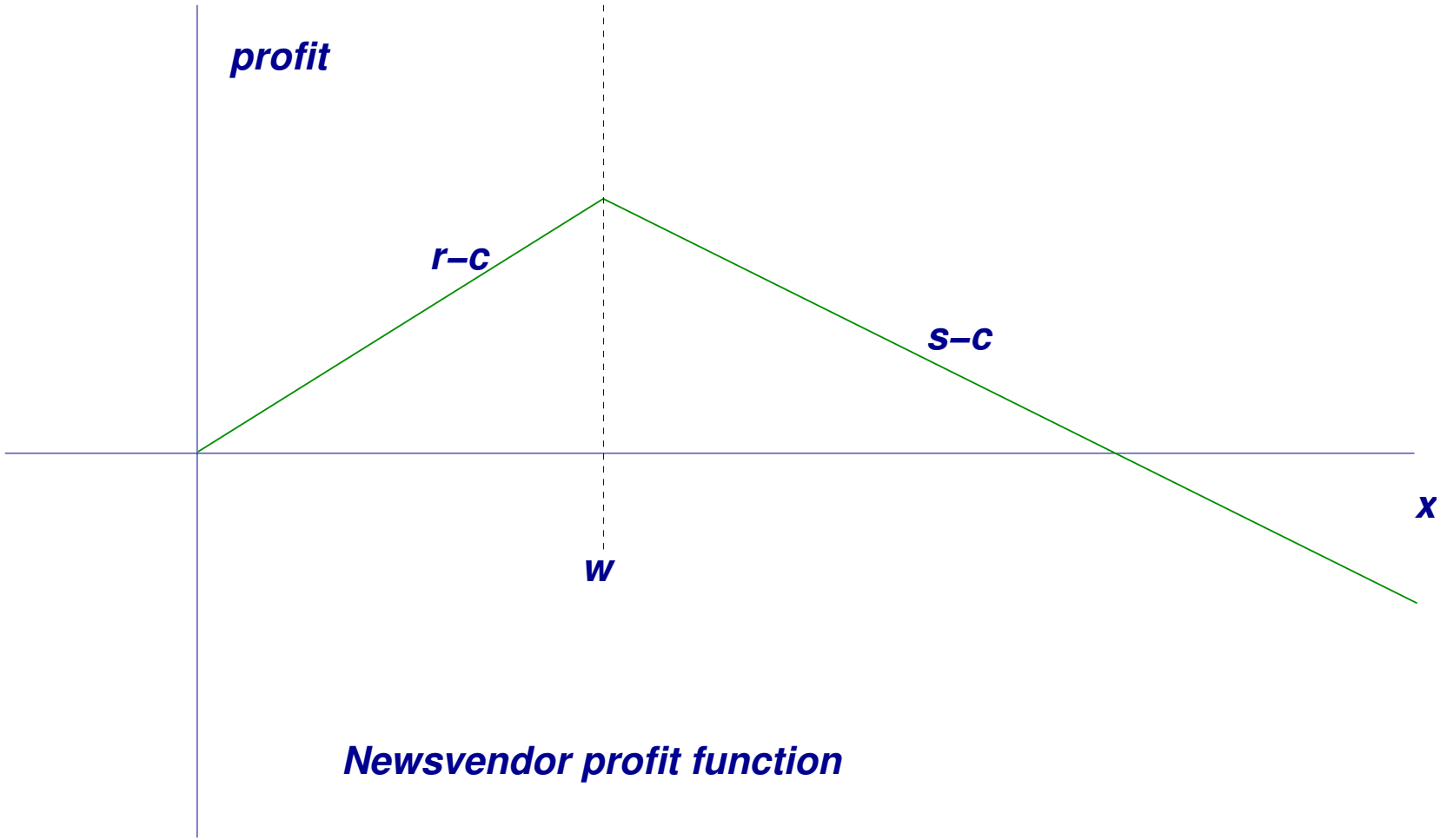
Base Stock Policy

Optimization of Stock



Newsvendor profit function

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Base Stock Policy

Optimization of Stock

Questions:

- How does the problem change if we consider factory floor inventory?

  How does the solution change?

- How does the problem change if we consider finished goods inventory *space* ?

  How does the solution change?

- How else could we have selected S?

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Supplier Lead Time

Issues

Issue: Supplies are not delivered instantaneously. The time between order and delivery, the lead time $L > 0$.

- If everything were deterministic, this would not be a problem. Just order earlier.

Issue: ... but demand is random.

- *Problem:* The demand between when the order arrives and and when the goods are delivered might be too large.

Other issues: Lead time could be random, production quantity could be random, etc.

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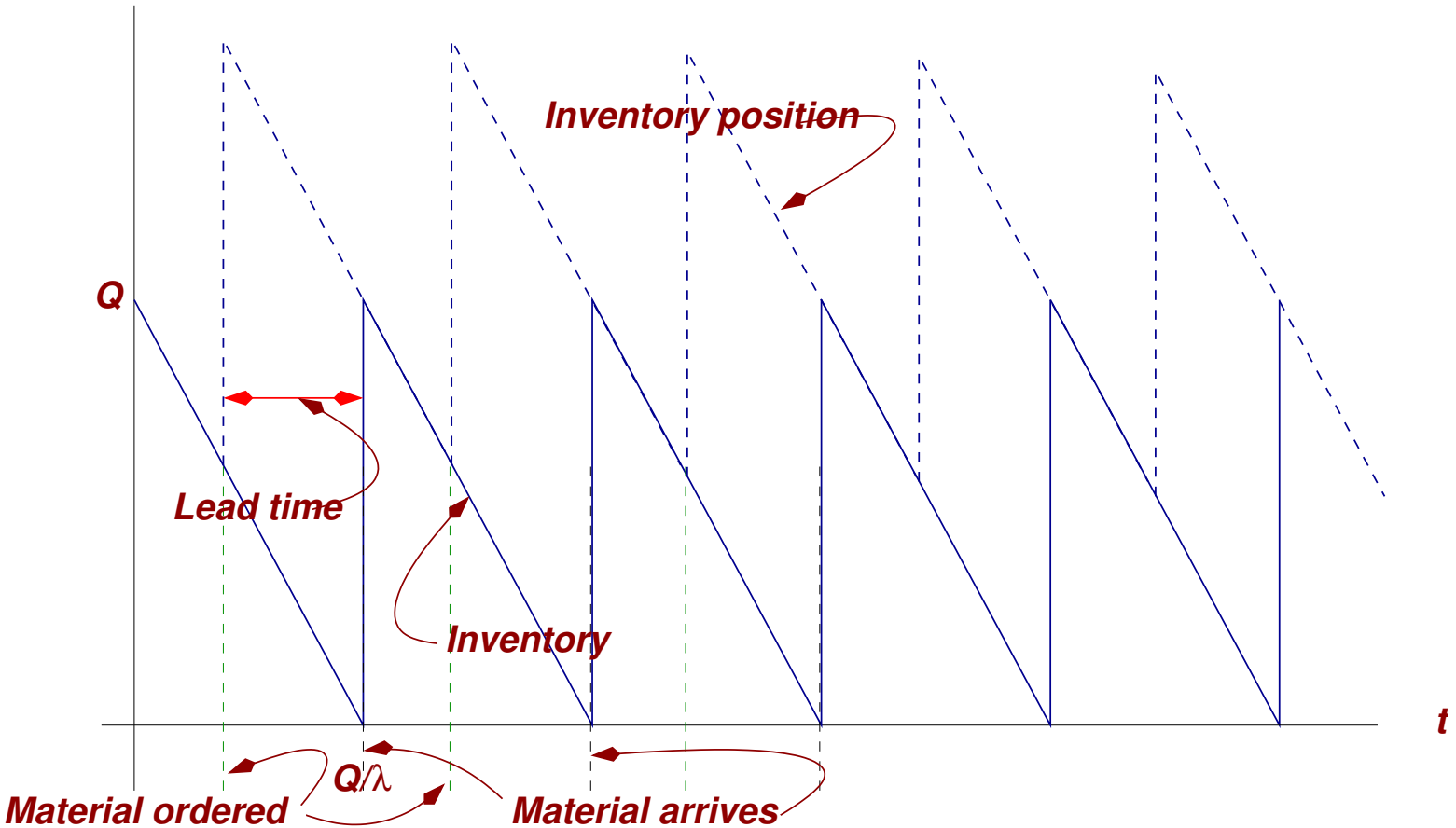
Supplier Lead Time

Inventory Position



The *current* order must satisfy demand until the *next* order arrives. *

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Supplier Lead Time *and* Random Demand

Q, R Policy

- Fixed ordering cost, like in EOQ problem
- Random demand, like in newsvendor or base stock problems.
- Make to stock — no advance ordering.
- Policy: when the inventory level goes down to R, buy a quantity Q.
- Hard to get optimal R and Q; use EOQ and base stock ideas.

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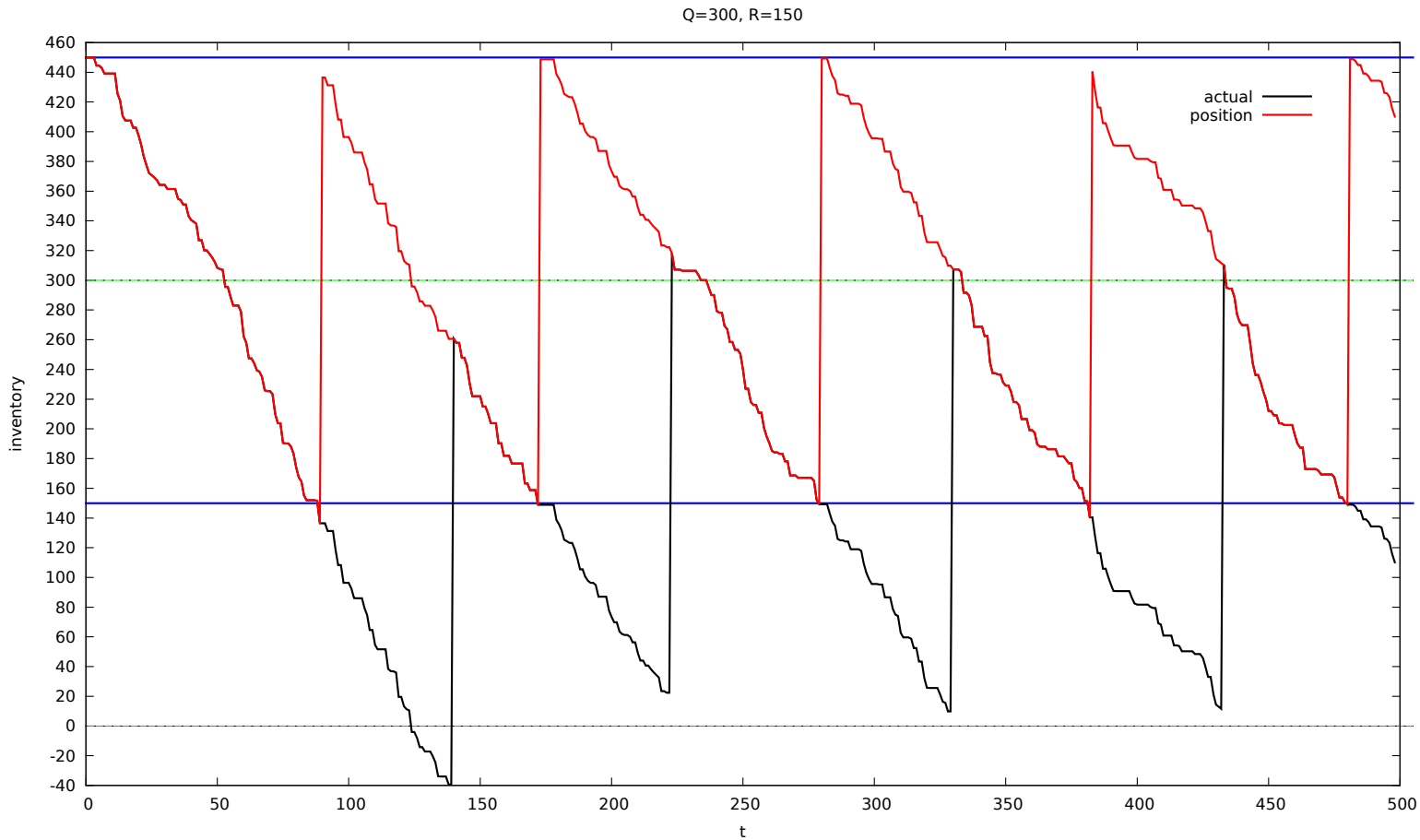
Random Demand

Q, R Policy



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Random Demand

Optimization of Policy

- The *expected* time between orders is $T = Q/\lambda$.
- The expected number of orders in a year is λ/Q. Therefore, the expected ordering cost in a year is $s\lambda/Q$.
- The average inventory is *approximately* $R + Q/2$. Therefore the average inventory holding cost is approximately $h(R + Q/2)$.
- Therefore, we choose R and Q to minimize the approximate expected annual cost

$$C = h\left(R + \frac{Q}{2}\right) + \frac{s\lambda}{Q}$$

such that

$$\text{prob}\{\text{total demand during a period of length } L > R\} \leq K'$$

for some K'.

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Miscellaneous Additional Inventory Issues

- Advance ordering: customers are willing to order large quantities in advance.
  - ⋆ How much advance ordering should factory allow?
  - ⋆ Optimization problem must be reformulated.
- s, S policy, also called *order-up-to* policy: when inventory reaches s, order enough to bring it up to S.
- Inventory quality: order enough raw material to account for scrapping due to
  - ⋆ ... production quality problems; or
  - ⋆ ... raw material (supplier) quality problems.

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Miscellaneous Additional Inventory Issues

- Raw material stockout due to
  - ⋆ ... late or erroneous deliveries from suppliers; or
  - ⋆ ... larger demand than expected.
- Demand lead time vs. production lead time
- Make-to-stock/make-to-order boundary

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2.854 / 2.853 Introduction To Manufacturing Systems

Fall 2016

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