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Introduction to Manufacturing Systems
Assignment 1[1]

Problem 1 (10 points)
A white die and a red die are tossed. Both dice are perfectly balanced. Let w and r denote the numbers appearing of the top face of the white and red die, respectively.

1. What is the probability that $w = r$?
2. What is the probability that $w < r$?
3. What is the probability that $w = 3$ given that $w < r$?
4. Are the events $w = 3$ and $w < r$ dependent or independent?
5. Are the events $w = 3$ and $w \neq r$ dependent or independent?

Problem 2 (10 points)
Urn A contains 4 white balls and 8 black balls. Urn B contains 9 white balls and 3 black balls. A die will be tossed. If the die comes up a 1 or 2, a ball is selected from Urn A. If the die comes up $3-6$, a ball is selected from Urn B.

1. Draw a probability tree that depicts this experiment. That is, draw a flow chart to illustrate all possible sequences of events, and the probability of each sequence.
2. Find the probability that a white ball will be drawn.
3. If a white ball is drawn, find the probability that it was taken from Urn A.

Problem 3 (6 points)
In a city, 87% of the households have TVs, 45% have stereos, and 38% have both. Is the event that a randomly selected household has a TV independent of the event that a randomly selected household has a stereo? What is the probability that a randomly selected household has at least one of these appliances?

[1] Problems 1-12 excerpted from *The Science of Decision Making*, Eric V. Denardo, John Wiley.

Problem 4 (6 points)
Suppose that $P(A) = P(A|B)$. Demonstrate that $P(B) = P(B|A)$.

Problem 5 (6 points)
Two balanced dice are rolled. What is the probability that the sum of their pips equals 7?

Problem 6 (6 points)
Suppose that X is equally likely to take the values $1, 2, 3, 4, 5$ and 6. Specify the probability distribution of X. Compute $E(X)$, $Var(X)$, and $StDev(x)$.

Problem 7 (6 points)
Suppose that X and Y are independent random variables, each of which is equally likely to take the values $1, 2, 3, 4, 5$ and 6. Specify the probability distribution of the random variable $T = X + Y$.

Problem 8 (10 points)
The random variable W takes the values $1.5, 3, 6$ and 7 with probabilities $0.2, 0.25, 0.4$ and 0.15, respectively.

1. Compute the expectation of W and W^2. Compute the variance of W and its standard deviation.
2. Define the cumulative distribution function $F(t)$ of t by $F(t) = P(W \leq t)$. Plot the function $F(t)$ versus t.
3. Shade the region consisting of each point (x, y) in which $x \geq 0$, and $F(X) \leq y \leq 1$. Compute the area of the shaded region. Does it equal $E(W)$?

Problem 9 (10 points)
Let X be binomial with parameters $n = 10$ and $p = 0.5$. Compute:

1. $P(X = 5)$.
2. $P(3 \leq X \leq 6)$.
3. $P(X \leq 3)$.

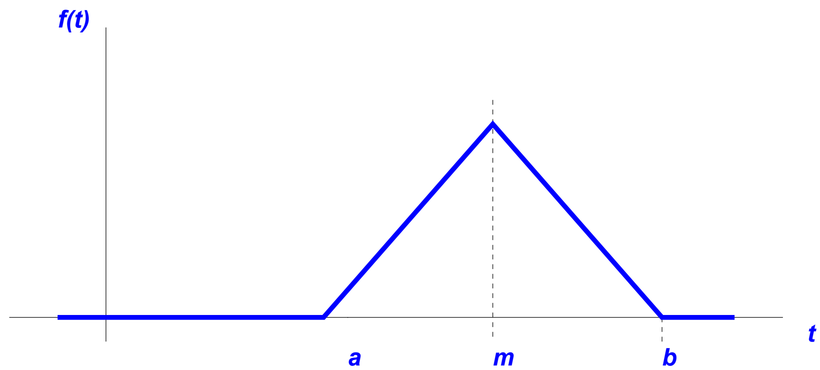

Figure 1: Triangular density function

Problem 10 (10 points)
Let T have the symmetric triangular density, with parameters a, b and $m = (a+b)/2$. See Figure 1.

1. For each number t between a and m, the probability $P(T \leq t)$ equals the area of a triangle. What is this triangle's height? What is the area?

2. For each number t between m and b, the probability $P(T \geq t)$ equals the area of a triangle. Compute this probability.

3. For each value of t, specify the CDF $F(t) = P(T \leq t)$ of this random variable.

Problem 11 (10 points)
The annual income of graduates from college **H** in their first year of employment is normally distributed with mean of $45,000 and standard deviation of $10,000. A graduate is picked at random. Compute the probability that this graduate earns:

1. Less than $35,000.

2. Between $35,000 and $45,000.

3. Between \$45,000 and \$60,000.

4. Exactly \$46,000.

5. Greater than \$65,000.

Problem 12 (10 points)
The random variable X and Y are independent, and their distributions are normal, with $E(X) = 1000$, $E(Y) = 900$, $StDev(X) = 300$, and $StDev(Y) = 400$.

1. What can you say about the random variable $(X - Y)$?

2. Compute $P(X < Y)$.

Problem 13 (10 points)

1. Random variables X and Y_1 are independent. Their probability distributions are given by:

   $$P(X=1) = 1/2 \qquad P(X=2) = 1/2$$

   $$P(Y_1=1) = 1/4 \qquad P(Y_1=2) = 1/4 \qquad P(Y_1=3) = 1/4 \qquad P(Y_1=4) = 1/4$$

   Calculate Cov(X,Y_1).

2. Random variables X and Y_2 are given by:

   $$P(X=1) = 1/2 \qquad P(X=2) = 1/2 \qquad Y_2 = 2X$$

   Calculate Cov(X,Y_2).

3. Random variables X and Y_3 are given by:

   $$P(X=1) = 1/2 \qquad P(X=2) = 1/2 \qquad Y_3 = 6 - 2X$$

   Calculate Cov(X,Y_3).