Efficient Buffer Design Algorithms for Production Line Profit Maximization

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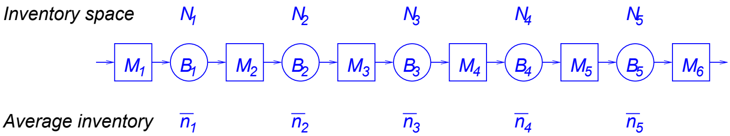
Contents

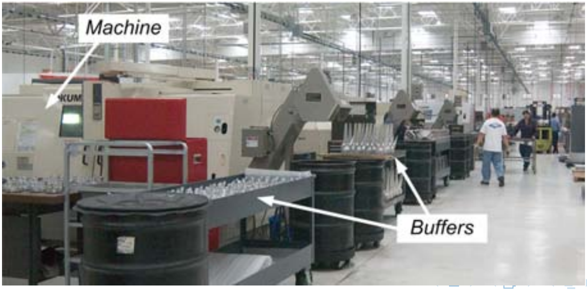
Manufacturing Systems

A manufacturing system is a set of machines, transportation elements, computers, storage buffers, and other items that are used together for manufacturing.

Production Lines

A production line, is organized with machines or groups of machines $(M_1, \cdots, M_k)$ connected in series and separated by buffers $(B_1, \cdots, B_{k-1})$.





Economic importance

Production lines are used in high volume manufacturing, particularly automobile production, in which they make engine blocks, cylinders, connecting rods, etc. Their capital costs range from hundreds of thousands dollars to tens of millions of dollars.

The simplest form of an important phenomenon

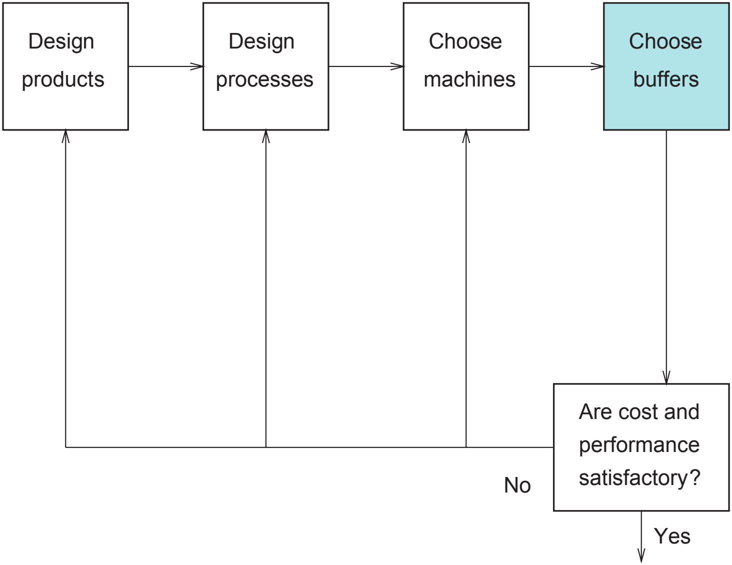
Manufacturing stages interfere with each other and buffers decouple them.

Research goals

- Make factories more efficient and more profitable, including micro- and nano-fabrication factories.

- Develop tools for rapid design of production lines. This is very important for products with short life cycles.

Production line design



Problem description and Assumptions

- Maximize profit for production lines subject to a production rate target constraint.
- Process and machines have already been chosen (3 models).
  - The deterministic processing time model of Gershwin (1987), (1994).
  - The deterministic processing time model of Tolio, Matta, and Gershwin (2002).
  - The continuous production line model of Levantesi, Matta, and Tolio (2003).
- Decision variables: sizes of in-process inventory (buffer) spaces, i.e., $(N_1, \cdots, N_{k-1}) \equiv \mathbf{N}$.
- Cost is due to inventory space and inventory.

Problem description and Assumptions

- The deterministic processing time model of Gershwin (1987).
  - Time required to process a part is the same for all machines and is taken as the time unit.
  - Machine i is parameterized by the probability of failure, $p_i = 1/MTTF_i$, and the probability of repair, $r_i = 1/MTTR_i$, in each time unit.
- The deterministic processing time model of Tolio, Matta, and Gershwin (2002).
  - Processing times of all machines are equal, deterministic, and constant.
  - It allows each machine to have multiple failure modes. Each failure is characterized by a geometrical distribution.
- The continuous production line model of Levantesi, Matta, and Tolio (2003).
  - Machines can have deterministic, yet different, processing speeds.
  - It allows each machine to have multiple failure modes. Each failure is characterized by an exponential distribution.

Necessity

- Machines are not perfectly reliable and predictable.
- The unreliability has the potential for disrupting the operations of adjacent machines or even machines further away.
- Buffers **decouple** machines, and **mitigate** the effect of a failure of one of the machines on the operation of others.

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Undesirable consequence of buffers: Inventory

- Inventory costs money to create or store.
- The average *lead time* is proportional to the average amount of inventory.
- Inventory in a factory is vulnerable to damage.
- The space and equipment needed for inventory costs money.

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Evaluation

Calculate production rate and average inventory as a function of buffer sizes (and machine reliability).

- *Large state space for long lines*

$$\begin{aligned} \text{state} &= (\alpha_1, \alpha_2, \cdots, \alpha_k, n_1, n_2, \cdots, n_{k-1}) \\ \text{where } \alpha_i &= \text{state of } M_i = \text{operation or repair} \\ n_i &= \text{number of parts in } B_i, 0 \le n_i \le N_i \end{aligned}$$

  Exact numerical solution is impractical due to large state space.

- There is no practical analytical solution to this problem for $k > 2$. For 2-machine lines, there are analytical solutions.

Good approximation is available: decomposition.

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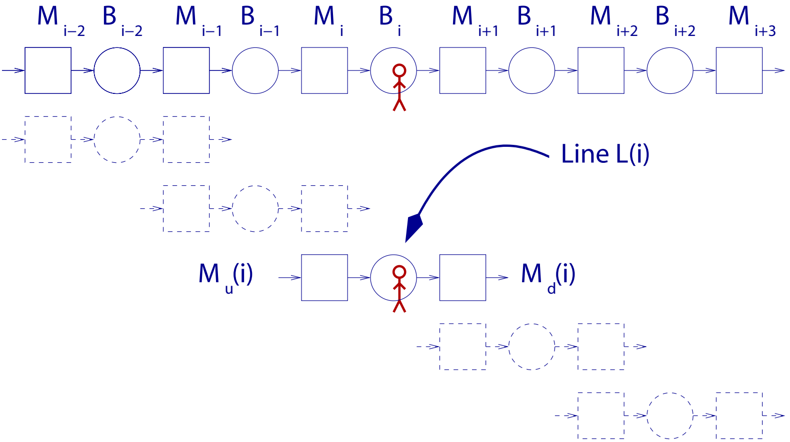
Decomposition



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Optimization[1]

- Determine the optimal set of buffer sizes.
- The cost function is nonlinear.
- The constraints can be nonlinear.

[1] This is my contribution.

There are many studies focusing on maximizing the production rate but few studies concentrating on maximizing the profit.

- Substantial research has been conducted on production line evaluation and optimization (Dallery and Gershwin 1992).
- Buzacott derived the analytic formula for the production rate for two-machine, one-buffer lines in a deterministic processing time model (Buzacott 1967).
- The invention of decomposition methods with unreliable machines and finite buffers (Gershwin 1987) enabled the numerical evaluation of the production rate of lines having more than two machines.
- Diamantidis and Papadopoulos (2004) also presented a dynamic programming algorithm for optimizing buffer allocation based on the aggregation method given by Lim, Meerkov, and Top (1990). But they did not attempt to maximize the profits of lines.
- For other line optimization work, see Chan and Ng (2002), Smith and Cruz (2005), Bautista and Pereira (2007), Jin et al. (2006), and Rajaram and Tian (2009).

Evaluation: simulation methods

- Slow (according to Spinellis and Papadopoulos 2000).
- Statistical.

Optimization: combinatorial or integer programming methods

- Slow (according to Gershwin and Schor 2000).
- Inaccurate (So 1997, Tempelmeier 2003).
- Do not take advantage of special properties of the problem (Shi and Men 2003, Dolgui et al. 2002, Huang et al. 2002).

Schor 1995, Gershwin and Schor 2000

Schor's unconstrained profit maximization problem:

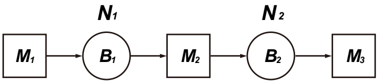
$$\max_{\mathbf{N}} \quad J(\mathbf{N}) = AP(\mathbf{N}) - \sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i(\mathbf{N})$$

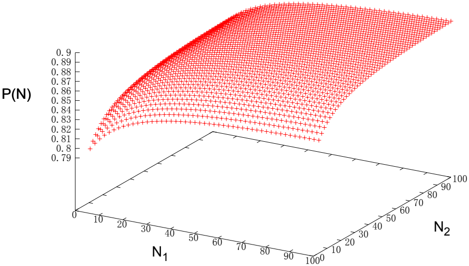
$$\text{s.t.} \quad N_i \geq N_{\min}, \forall i = 1, \cdots, k-1.$$

where
- $P(\mathbf{N})$ = production rate, parts/time unit
- $\hat{P}$ = required production rate, parts/time unit
- A = profit coefficient, \$/part
- $\bar{n}_i(\mathbf{N})$ = average inventory of buffer $i, i = 1, \cdots, k-1$
- b_i = buffer cost coefficient, \$/part/time unit
- c_i = inventory cost coefficient, \$/part/time unit

Assumptions

- $P(\mathbf{N})$ is monotonic and concave.





- N_i can be treated as continuous variables (Schor 1995, Gershwin and Schor 2000).
- $P(\mathbf{N})$ and $J(\mathbf{N})$ can be treated as continuously differentiable functions (Schor 1995, Gershwin and Schor 2000).
- The decomposition is a good approximation.

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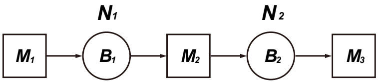

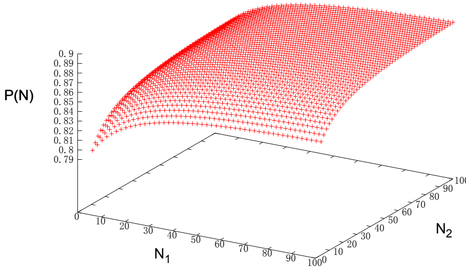

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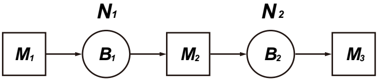

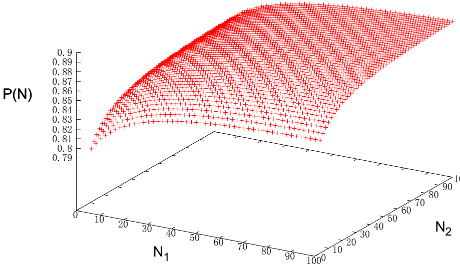

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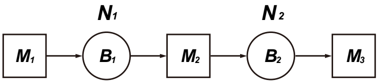

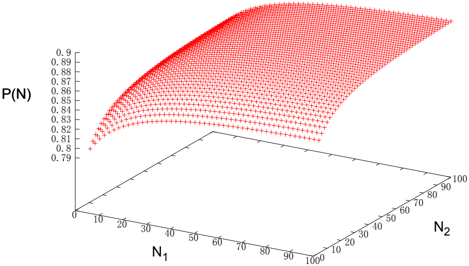

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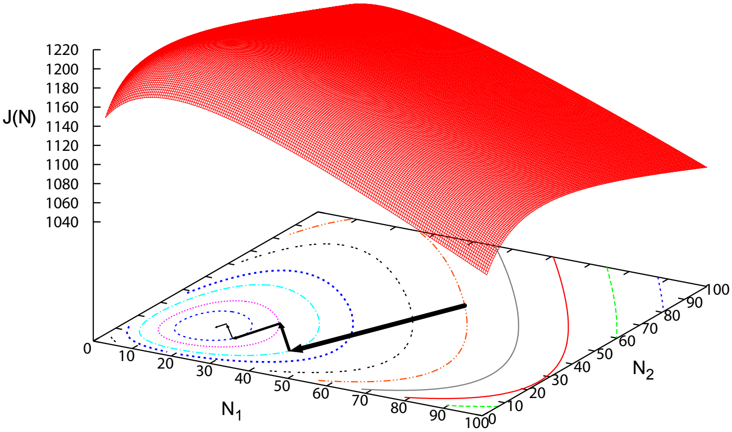

Figure 1: $J(\mathbf{N})$ vs. N_1 and N_2

We calculate the gradient direction to move in $(N_1, \cdots, N_{k-1})$ space. A line search is then conducted in that direction until a maximum is encountered. This becomes the next guess. A new direction is chosen and the process continues until no further improvement can be achieved. There is no analytical expression to compute profits of lines having more than two machines. Consequently, to determine the search direction, we compute the gradient, **g**, according to a forward difference formula, which is

$$g_i = \frac{J(N_1, \cdots, N_i + \delta N_i, \cdots, N_{k-1}) - J(N_1, \cdots, N_i, \cdots, N_{k-1})}{\delta N_i}$$

where g_i is the gradient component of buffer B_i, J is the profit of the line, and δN_i is the increment of buffer B_i.

Topics that have been finished/are in process

- Profit maximization for production lines with a production rate constraint.
- Profit maximization for production lines with both time window constraint and production rate constraint.
- Evaluation and profit maximization for lines with an arbitrary single loop structure.

Topics that might be considered in the future

- Systems with quality control.
- Systems with set-up cost for buffers.
- etc.

2.852 Manufacturing Systems Analysis
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