2.830J / 6.780J / ESD.63J Control of Manufacturing Processes (SMA 6303)
Spring 2008

Massachusetts Institute of Technology

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Quiz #1
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In all problems, please show your work and explain your reasoning. Statistical tables for the cumulative standard normal distribution, percentage points of the χ^2 distribution, percentage points of the t distribution, percentage points of the F distribution, and factors for constructing variables for control charts (all from Montgomery, 5th Ed.) are provided.

Problem 1 [35%]

Part (a)[*] [10%]

Control charts for $\bar{x}$, R, and s are to be maintained for the threshold voltage of short-channel MOSFETs using sample sizes of $n = 10$. It is known that the process is normally distributed with $\mu = 0.75$ V and $\sigma = 0.10$ V. Find the centerline and $\pm 3\sigma$ control limits for each of these charts.

All units in parts (a) and (b) are in V.

(1) $\bar{x}$ chart: Center $= \mu = \underline{0.75}$

$$\text{UCL} = \mu + \frac{3\sigma}{\sqrt{n}} = \underline{0.845}$$

$$\text{LCL} = \mu - \frac{3\sigma}{\sqrt{n}} = \underline{0.655}$$

(2) R chart: Center $= R = \sigma \cdot d_2 = 3.078 \cdot 0.1 = \underline{0.308}$

$$\text{UCL} = R + 3d_3\sigma = D_4 R = \underline{0.547}$$

$$\text{LCL} = R - 3d_3\sigma = D_3 R = \underline{0.069}$$

(3) s chart: Center $= s = \sigma \cdot c_4 = \underline{0.097}$

$$\text{UCL} = s + 3\sigma\sqrt{1-c_4^2} = B_6\sigma = \underline{0.167}$$

$$\text{LCL} = s - 3\sigma\sqrt{1-c_4^2} = B_5\sigma = \underline{0.028}$$

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[*] May & Spanos, Problem 6.6.

Part (b)[†] **[10%]**

Repeat part (a) assuming that µ and σ are unknown and that we have collected 50 observations of sample size 10. These samples gave a grand average of 0.734 V, and average s_i of 0.125 V, and an average R_i of 0.365 V.

(1) $\bar{x}$ chart: There are two approaches here; in one, we use $\overline{R}$ to estimate variance, and in the other we use $\bar{s}$ to estimate variance. Note that the resulting control limits are very close in the two cases.

Using $\overline{R}$ | Using $\bar{s}$

$\text{Center} = \bar{\bar{x}} = \underline{0.734}$ | $\text{Center} = \bar{\bar{x}} = \underline{0.734}$

$\text{UCL} = \bar{\bar{x}} + A_2\overline{R} = \underline{0.846}$ | $\text{UCL} = \bar{\bar{x}} + 3\dfrac{\bar{s}}{c_4}\dfrac{1}{\sqrt{n}} = \bar{\bar{x}} + A_3\bar{s} = \underline{0.856}$

$\text{LCL} = \bar{\bar{x}} - A_2\overline{R} = \underline{0.622}$ | $\text{LCL} = \bar{\bar{x}} - 3\dfrac{\bar{s}}{c_4}\dfrac{1}{\sqrt{n}} = \bar{\bar{x}} - A_3\bar{s} = \underline{0.612}$

(2) R chart:

$\text{Center} = \overline{R} = \underline{0.365}$

$\text{UCL} = \overline{R} + 3d_3\dfrac{\overline{R}}{d_2} = D_4\overline{R} = \underline{0.649}$

$\text{LCL} = \overline{R} - 3d_3\dfrac{\overline{R}}{d_2} = D_3\overline{R} = \underline{0.081}$

(3) s chart:

$\text{Center} = \bar{s} = \underline{0.125}$

$\text{UCL} = s + 3\dfrac{\bar{s}}{c_4}\sqrt{1-c_4^2} = B_4\bar{s} = \underline{0.215}$

$\text{LCL} = s - 3\dfrac{\bar{s}}{c_4}\sqrt{1-c_4^2} = B_3\bar{s} = \underline{0.036}$

Note that as the sample size gets larger, the s chart is somewhat more effective, as it uses all the data in the sample to estimate the within-sample variance, while the R chart only uses the extremal (max and min) values.

[†] May & Spanos, Problem 6.7.

Part (c) [15%]

The specification limits for MOSFET threshold voltage in this process is $x = 0.7 \pm 0.05$ V. What is the Cp and Cpk for this process? Assuming the process as specified in part (a) is in control, what is the fraction of MOSFETs that are nonconforming? If the process in part (a) was recentered such that μ_x is on target at 0.7 V, what would the fraction be of MOSFETs that are nonconforming?

We see that $USL = 0.75$ V and $LSL = 0.65$ V.

(1) $C_p = \dfrac{USL - LSL}{6\sigma} = \dfrac{0.75 - 0.65}{6 \cdot 0.1} = \dfrac{1}{6} = \underline{0.167}$. Note that this is a very poor process capability (and we will see the impact on bad parts in a moment).

(2) $C_{pk} = \min\left(\dfrac{USL - \mu}{3\sigma}, \dfrac{\mu - LSL}{3\sigma}\right) = \underline{0}$ since USL is equal to the actual process mean. This is lower than C_p since the process is not mean centered.

(3) The process is horribly off-center, as noted in (2). All parts above the process mean are bad (above the USL), and an appreciably fraction of parts still fall below LSL because the distribution is so wide compared to the specification limits. Thus

$$\text{Pr(nonconforming, off-center)} = \left[1 - \Phi\left(\frac{USL - \mu}{\sigma}\right)\right] + \left[1 - \Phi\left(\frac{\mu - LSL}{\sigma}\right)\right]$$

$$= \left[1 - \Phi\left(\frac{0.75 - 0.75}{0.1}\right)\right] + \left[1 - \Phi\left(\frac{0.75 - 0.65}{0.1}\right)\right] = [1 - \Phi(0)] + [1 - \Phi(1)] = \underline{0.659}$$

(4) Since the process was so badly off-mean, perhaps by recentering the process we can improve the fraction of good parts. When centered

$$\text{Pr(nonconf, centered)} = 2\left[1 - \Phi\left(\frac{USL - T}{\sigma}\right)\right] = 2\left[1 - \Phi\left(\frac{0.75 - 7}{0.1}\right)\right] = 2[1 - \Phi(0.5)] = \underline{0.617}$$

So we hardly reduce the fraction of bad parts at all? Why? Because the inherent process capability is so poor ($C_p = 0.167$). The real problem is the distribution is too wide compared to the specification limits, not that we are off-center.

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Problem 2 [30%]

Part (a) [10%]

Based on a fairly large historical sample of gate oxide thicknesses consisting of 61 samples, an average oxide thickness of 96.5 Å is observed, with a variance of 6 Å^2. What are your best estimates, and 95% confidence intervals, for the process mean and process variance?

(1) Point estimate for mean: $\hat{\mu} = \bar{x} = \underline{\ 96.5\ \text{Å}}$

(2) 95% confidence interval for mean. Since n is large, the normal is a good approximation:

Using normal distribution (large n): $\mu = \bar{x} \pm z_{\alpha/2} \dfrac{s}{\sqrt{n}} = 96.5 \pm 1.96 \cdot \dfrac{\sqrt{6}}{\sqrt{61}} = 96.5 \pm 0.61\ \text{Å}$

$$\text{or } \underline{95.89 \le \mu \le 97.11}\ \text{Å}$$

For more accuracy, use t distribution: $\mu = \bar{x} \pm t_{\alpha/2,n-1} \dfrac{s}{\sqrt{n}} = 96.5 \pm 2.00 \cdot \dfrac{\sqrt{6}}{\sqrt{61}} = 96.5 \pm 0.63\ \text{Å}$

$$\text{or } \underline{95.87 \le \mu \le 97.13}\ \text{Å}$$

(3) Point estimate for variance: $\hat{\sigma}^2 = s^2 = \underline{\ 6\ \text{Å}^2}$

(4) 95% confidence interval for variance:

$$\dfrac{(n-1)s^2}{X^2_{\alpha/2,n-1}} \le \sigma^2 \le \dfrac{(n-1)s^2}{X^2_{1-\alpha/2,n-1}} \quad \text{so that} \quad \dfrac{(60)6}{83.3} \le \sigma^2 \le \dfrac{(60)6}{40.48} \quad \text{or} \quad \underline{4.32 \le \sigma^2 \le 8.89}\ \text{Å}^2$$

Part (b) [10%]

A change is proposed to increase the gate oxide thickness. Using the modified process, 11 samples are taken, with observed sample mean of 100 Å and sample variance of 20 Å^2. What are your best estimates, and 95% confidence intervals, for the new process mean and process variance?

(1) Point estimate for mean: $\hat{\mu} = \bar{x} = \underline{\ 100\ \text{Å}}$

(2) 95% confidence interval for mean: Now we need to use the t distribution for sure, since n is

small: $\mu = \bar{x} \pm t_{\alpha/2,n-1} \dfrac{s}{\sqrt{n}} = 96.5 \pm 2.228 \cdot \dfrac{\sqrt{20}}{\sqrt{11}} = 100 \pm 3.00\ \text{Å} \quad \text{or} \quad \underline{97 \le \mu \le 103}\ \text{Å}$

(3) Point estimate for variance: $\hat{\sigma}^2 = s^2 = \underline{20\ \text{Å}^2}$

(4) 95% confidence interval for variance:

$$\dfrac{(n-1)s^2}{X^2_{\alpha/2,n-1}} \le \sigma^2 \le \dfrac{(n-1)s^2}{X^2_{1-\alpha/2,n-1}} \quad \text{so that} \quad \dfrac{(10)20}{20.48} \le \sigma^2 \le \dfrac{(10)20}{3.25} \quad \text{or} \quad \underline{9.76 \le \sigma^2 \le 61.6}\ \text{Å}^2$$

Part (c) [10%]

To 95% confidence, has the modification resulted in a process with a different mean than in part (a)? To 95% confidence, has the modification resulted in a process with a different variance than in part (b)?

(1) No – because the 95% confidence intervals for the old and new means overlap, we don't have enough evidence to be 95% confident or better that the means are different.

One can also attempt a direct hypothesis test on the difference in means. This is more work than needed, given we already determined confidence intervals. However, if a hypothesis test is used, we should be careful *not* to assume the variances are the same (we have strong evidence they are not), and so we would then need to use the somewhat complicated procedure described in Montgomery, pp. 120-121.

(2) Yes – here the 95% confidence intervals for the new variance and old do not overlap. We are 95% confident or better that the chance has resulted in a process with larger variance.

Here also, one can formulate a hypothesis test as an alternative approach. Given that one variance appears to be so much larger than the other, we're best off using a one sided F-test (that is to say, the one sided test will require even larger ratio to declare a larger variance, and thus the one sided test is more conservative). So in this case, we would need to observe an $F > F_0$, where $F_0 = F_{0.05, n_b - 1, n_a - 1} = F_{0.05, 10, 60} = 1.99$. In our case, we observe $F = \dfrac{s_b^2}{s_a^2} = \dfrac{20}{6} = 3.33$ giving us better than 95% confidence that the second variance is larger than the first.

Problem 3 [35%]

You are in charge of a process manufacturing parts whose key output parameter, x, is known to be normally distributed. When the process is in control, the mean value of x is known to be μ and its standard deviation σ.

Unfortunately, the supplier of an important input material for this process is rather unreliable, and every so often they inadvertently supply a large batch of material whose properties cause the mean of x to shift to $\mu + k\sigma$, with the standard deviation of x remaining unchanged.

When this mean shift occurs, you want to be able to identify it as quickly as possible. Your metrology staff tells you that they can currently measure at most a fraction f of parts produced, although you are free to choose how this measurement budget is apportioned: *i.e.* you can dictate any n, where samples of n parts are taken at intervals of n/f parts.

Part (a) [3%]

Write down, in terms of µ, σ and *n*, expressions for the upper and lower control limits for an $\bar{x}$ control chart monitoring this process.

Answer: $\mu \pm 3\sigma/(n^{0.5})$

Part (b) [5%]

Write down an expression for the probability, *β*, that the mean, $\bar{x}$, of any given sample of size *n* from an out-of-control process with mean output *x* = µ + *k*σ, will *not* fall outside the control limits.

Answer: $\beta = \Phi\!\left(3 - k\sqrt{n}\right) - \Phi\!\left(-3 - k\sqrt{n}\right)$

Part (c) [10%]

Derive, in terms of *n*, *f*, and *β*, an expression for the average number of parts, *P*, that will be produced after a mean shift occurs and before a sample falls outside the range (LCL, UCL).

Answer: $P = n/[f(1-\beta)]$

We *could* use the expression that you have just derived to choose a value for *n* that minimizes *P* for a given *f* (do *not* attempt to choose *n* now). Suggest one other consideration that would feature in the selection of sample size and sample frequency.

Answer: The need to make the inter-sample interval longer than the correlation time of the output, so that samples are plausibly independent.

Problem 3, cont'd.

Part (d) [10%]

Now suppose that you are told that the quality loss function for the process is defined as follows:
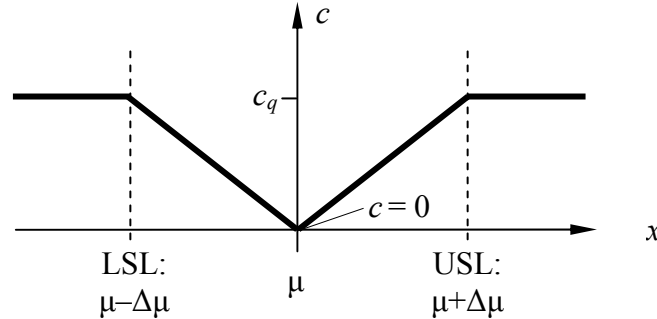
Here, c is the cost penalty associated with producing a part with a particular x. Write down an expression for the expected cost penalty $E_k[c]$ for a part produced when the process is $x \sim N(\mu + k\sigma, \sigma^2)$. (Hint: your answer should be expressed as a sum of integrals; do not attempt to evaluate the integral.)

Note: the probability density function of $x \sim N(\mu, \sigma^2)$ is given by $\dfrac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$.

Answer:

$$E_k[c] = \frac{c_q}{\sigma\sqrt{2\pi}}\left\{ \int_{-\infty}^{\mu-\Delta\mu}\exp\left[-\frac{1}{2}\left(\frac{x-\mu-k\sigma}{\sigma}\right)^2\right]dx + \int_{\mu-\Delta\mu}^{\mu+\Delta\mu}\frac{|x-\mu|}{\Delta\mu}\exp\left[-\frac{1}{2}\left(\frac{x-\mu-k\sigma}{\sigma}\right)^2\right]dx + \int_{\mu+\Delta\mu}^{\infty}\exp\left[-\frac{1}{2}\left(\frac{x-\mu-k\sigma}{\sigma}\right)^2\right]dx \right\}$$

Problem 3, cont'd.

Part (e) [7%]

With the above information in mind, you now want to investigate whether you should install more metrology equipment so that the fraction of parts measured, f, can be increased. Suppose that measurement costs can be completely described by an incremental cost c_m per part measured.

Write down, and briefly explain, a cost expression that should be minimized when selecting an optimal value for f. Your answer should be given in terms of the following parameters:

n sample size

c_m cost of measuring one part

f fraction of all parts produced that are measured

$E_k[c]$ expected cost penalty for a part produced when the output is $x \sim N(\mu + k\sigma, \sigma^2)$

$E_0[c]$ expected cost penalty for a part produced when the output is $x \sim N(\mu, \sigma^2)$

β your answer to part (b)

D the frequency (in parts^{-1}), on average, with which the material supplier delivers a defective batch of material (e.g. if $d = 10^{-5}$ parts^{-1}, the supplier would deliver a defective batch for every 10^5 parts produced by the process. Hopefully, not much of the batch will be used before the problem is noticed!

Answer: Cost expression: $fc_m + \left(E_k[c] - E_0[c]\right)\dfrac{nd}{f(1-\beta)}$

We need to minimize the sum of measurement costs and the *extra* quality-related cost penalty that occurs when processing with defective material. The more frequently the suppliers deliver the defective material, or the lower the measurement cost per part, the larger the optimal f.

What other factors should be considered when choosing f?

Answer: The defective material input may not be the only cause of mean shifts that need to be detected. We may also need to choose f to allow us to detect particular *standard deviation* changes. Increasing f may slow down the production line, even if more measurement equipment were to be installed: the metrology team may have other constraints.