# 2.737 Mechatronics
# Pre-Lab 0: Controls Review

Assigned: 9/3/14
Due: 9/10/14 in class

Welcome to 2.737! The purpose of this pre-lab homework assignment is to get you thinking about and reviewing controls. This material should be familiar to you from 2.14 or some other equivalent controls course. If you get stuck, you can get help from your fellow students, but make sure that what you turn in is your own work. You can also get help from the staff during office hours which are held in the lab, 1-004.

## Problems

1. **First-Order Response.** For the system

$$H(s) = \frac{10}{0.05s + 1}$$

   **(a)** Sketch the response to a unit step input.
   **(b)** Sketch the system Bode plot.
   **(c)** Verify the step response and Bode plot in MATLAB®.

2. **Second-Order Response.** For the system

$$H(s) = \frac{1}{\frac{s^2}{\omega_n^2} + \frac{2\zeta s}{\omega_n} + 1}$$

   with $\omega_n = 100$ and $\zeta = 0.1$

   **(a)** Sketch the step response to a unit step input.
   **(b)** Sketch the system Bode plot.
   **(c)** Verify the step response and Bode plot in MATLAB.

3. **Doublet Response.** For a system

$$H(s) = \frac{\alpha\tau s + 1}{\tau s + 1}$$

   **(a)** Sketch the Bode plot and show that for $\alpha > 1$ this is a *lead* network and for $0 < \alpha < 1$ this is a *lag* network.
   **(b)** Find an expression for the system response to a unit step input. Sketch representative responses for the cases $\alpha > 1$ (*lead*) and $0 < \alpha < 1$ (*lag*).

4. **Block Diagram.** (This problem is taken from Roberge, J.K., *Operational Amplifiers: Theory and Practice*, Wiley, 1976.) Figure 2.20 shows a block diagram for a linear feedback system. Write a complete, independent set of equations for the relationships implied by this diagram. Solve your set of equations to determine the input-to-output gain of the system.

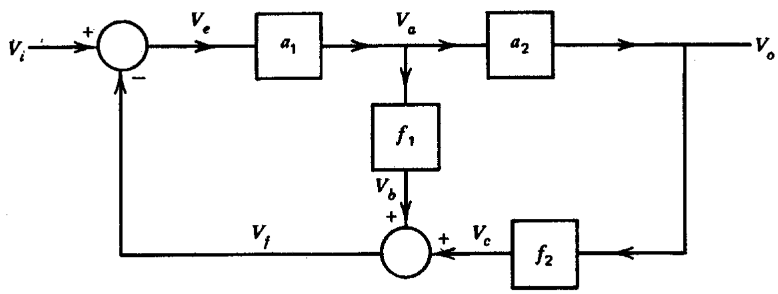
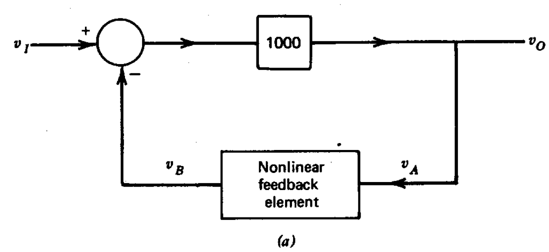



**Figure 2.20** Two-loop feedback system.



5. **Nonlinear Feedback.** (This problem is taken from Roberge, J.K., *Operational Amplifiers: Theory and Practice*, Wiley, 1976.) Plot the closed-loop transfer characteristics for the nonlinear system shown in Fig. 2.22.



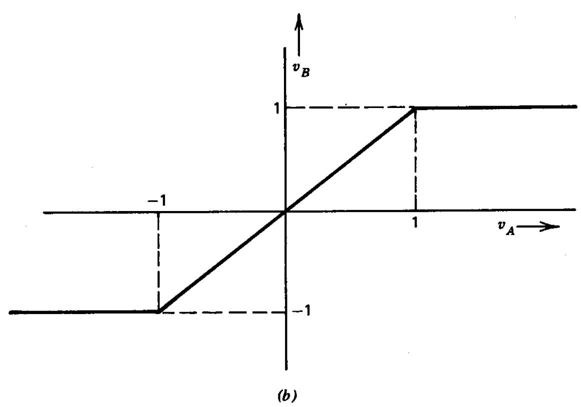




**Figure 2.22** Nonlinear feedback system. (*a*) System. (*b*) Transfer characteristics for nonlinear element.

**6. Servomechanism with Disturbance.** For the servomechanism of Fig. 1.18 let the parameters be

$$\begin{aligned} J_T &= 10^{-5}\ \mathrm{kg \cdot m^2} \\ B &= 10^{-3}\ \mathrm{N \cdot m \cdot s/rad} \\ K_m &= 0.1\ \mathrm{N \cdot m/A} \\ K_t &= 2\ \mathrm{V/rad} \end{aligned}$$

Note that the amplifier with gain $K_a$ is a current-drive which determines the motor current $I_m$ directly. Thus in this model, the motor back emf does not affect the dynamics. The motor torque is given by $T_m = K_m I_m$.

This system can be represented by the block diagram as shown in Figure 1.19. (Note: There should not be gaps near the summing junction and output in Figure 1.19.)

**(a)** Choose gain $K_a$ to yield a system with a step-response peak overshoot of 20% for an input $V_i$ and output $\theta$ . What is the resulting damping ratio $\zeta$ and natural frequency, $\omega_n$?

**(b)** Calculate the disturbance transfer function $\frac{\Theta(s)}{T_d(s)}$. How is the disturbance rejection affected by varying $K_a$?

**(c)** Sketch a root locus for this system as $K_a$ varies between 0 and $\infty$. How is stability affected with increasing $K_a$?

**(d)** Verify your root locus in MATLAB.

**(e)** For the value of $K_a$ found in part (a), determine an analytical expression for the loop return ratio transfer function. Make a sketch of the Bode plot for this return ratio. What are the loop crossover frequency and phase margin? Be sure to indicate these values on the Bode plot.

**(f)** Conduct a Nyquist analysis and make a sketch of the Nyquist plot for this loop. Is the loop stable? Why or why not? Show how the crossover frequency and phase margin are represented on this loop.

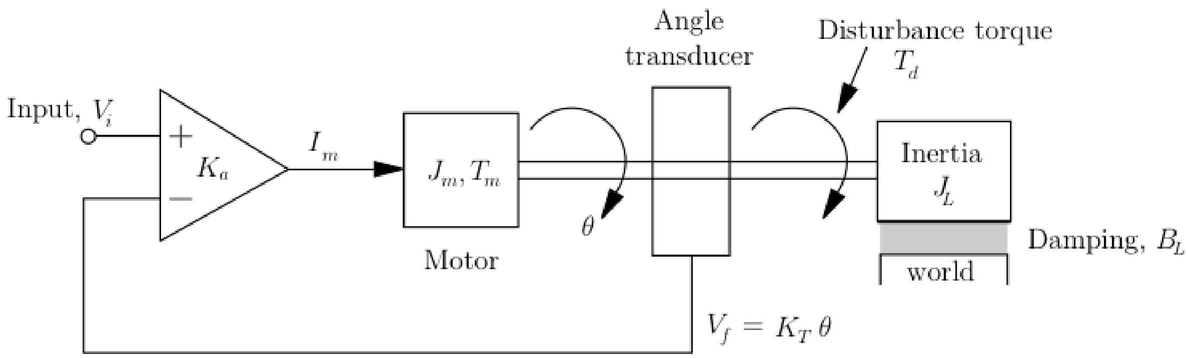



Figure 1.18: Servomechanism with an added disturbance torque.

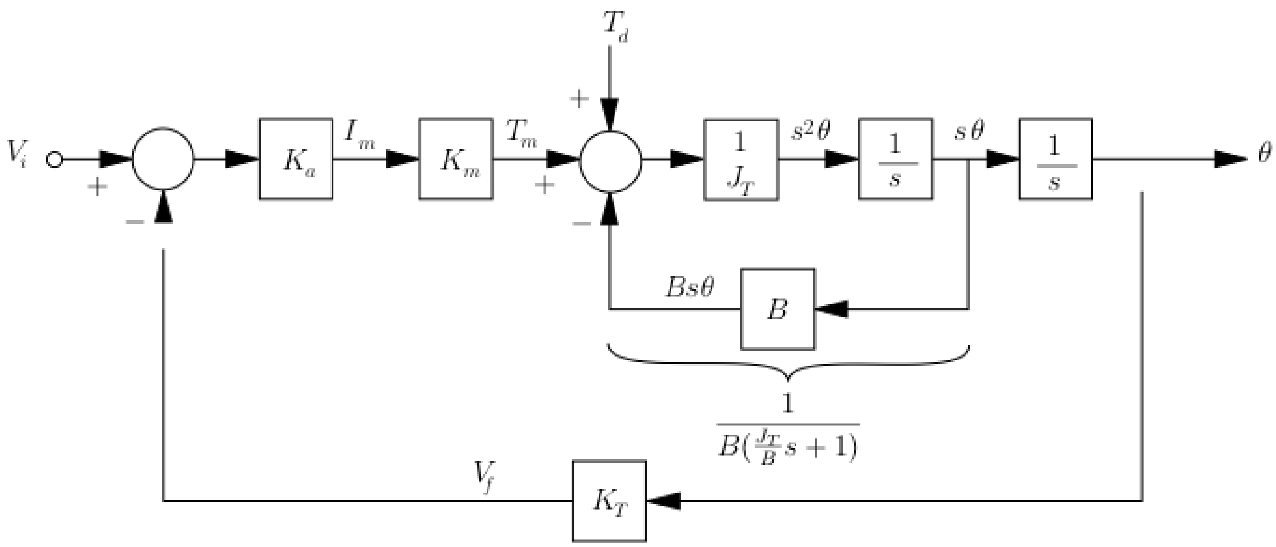
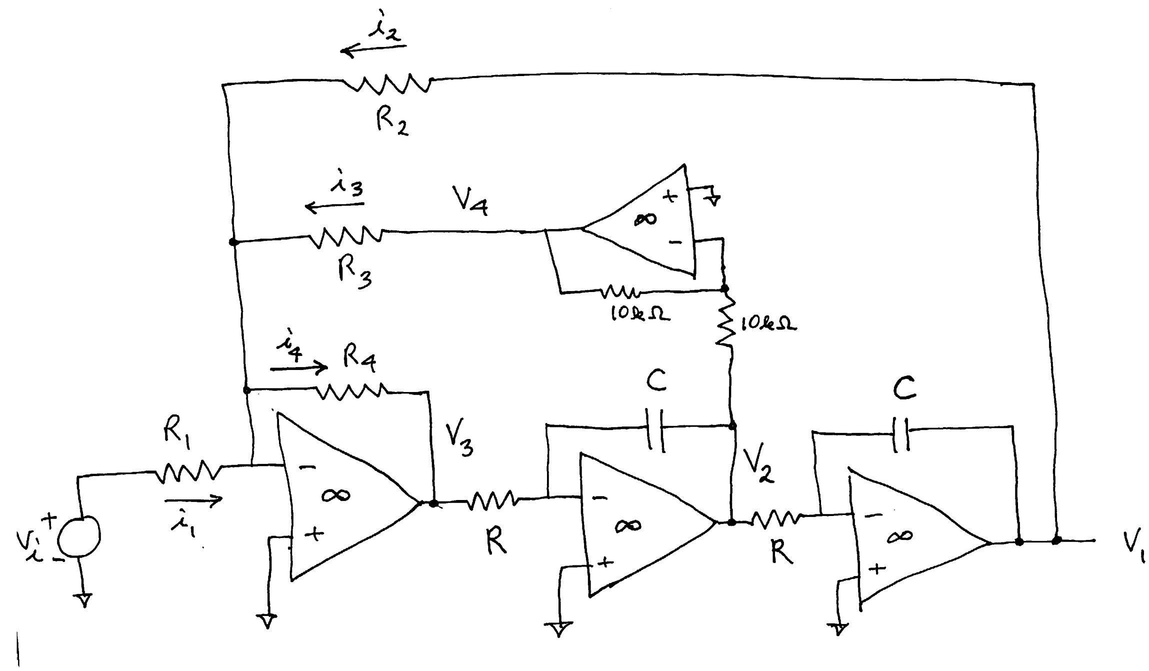



Figure 1.19: Block diagram representation of servomechanism with external disturbance.

7. Op amp circuit The circuit shown below purports to be an analog computer simulation of a mass/spring/damper system. In this circuit, all amplifiers are ideal, with infinite gain.




**a)** Draw a block diagram which represents the circuit, and which includes the variables $V_i$, $i_1$, $i_2$, $i_3$, $i_4$, $V_1$, $V_2$, $V_3$, and $V_4$. Expressions in the blocks should be in terms of the given system parameters.

**b)** Show that the system can be described by the differential equation

$$a\ddot{V}_1 + b\dot{V}_1 + cV_1 = dV_i$$

where $a$, $b$, $c$, and $d$ are scalar constants. Give expressions for these constants in terms of the system parameters. Hint: $V_2 \propto \dot{V}_1$, and $V_3 \propto \dot{V}_2 \propto \ddot{V}_1$