MASSACHUSETTS INSTITUTE OF TECHNOLOGY

2.717J/MAS.857J Optical Engineering **Spring '02**

Problem Set #3 Posted Feb. 27, 2002 — Due Monday Mar. 11, 2002

1. The probability of an insect laying k eggs is Poisson with expectation value $\overline{k}$; the probability of an egg developing is p. Assuming mutual independence of the eggs, show that the probability of a total of n survivors is given the Poisson distribution with expectation value $\overline{k}p$.

2. Goodman problem 2-6.

3. Goodman problem 2-9.

4. Goodman problem 2-10.

5. Goodman problem 2-11.

6. Let $X(t)$ be a random process describing the location X of a particle as function of time $t > 0$. The 1$^{\text{st}}$-order statistics of this random process are described by the function

$$p_X(x;t) = \frac{1}{\sqrt{2\pi Dt}} \exp\left\{-\frac{(x-vt)^2}{2Dt}\right\},$$

where v and D are real, positive numbers.

6.a) How do the mean and variance of X behave as time evolves?

6.b) Show that p_X satisfies

$$\frac{\partial p_X}{\partial t} = -v\frac{\partial p_X}{\partial x} + \frac{D}{2}\frac{\partial^2 p_X}{\partial x^2}.$$

This is known as the *Fokker-Planck equation* for this random process.

6.c) Can you describe a physical system which should follow these statistics? What is the physical meaning of v and D in your system? (Hint: the Fokker-Planck equation is also known under a different name; what is then p_X replaced by?).