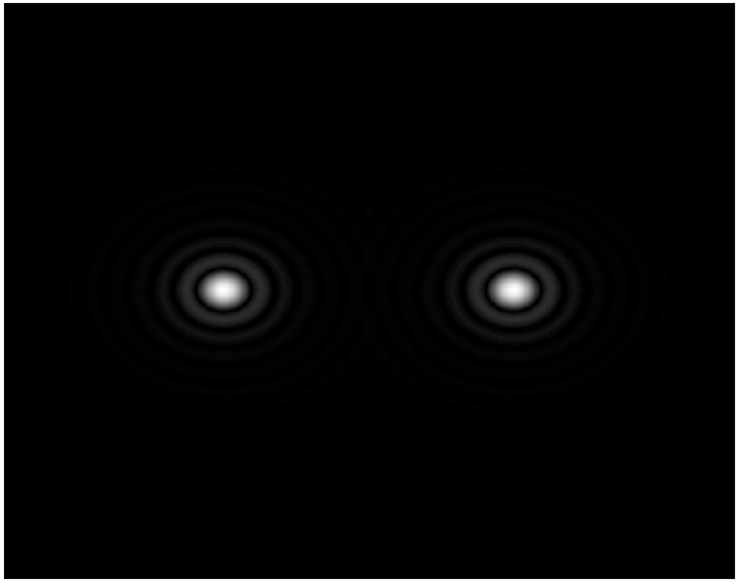
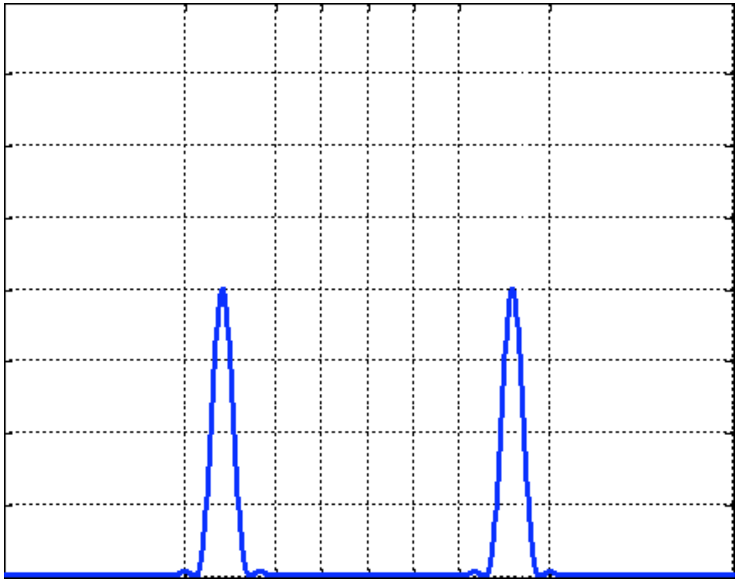
Resolution

[from the New Merriam-Webster Dictionary, 1989 ed.]:

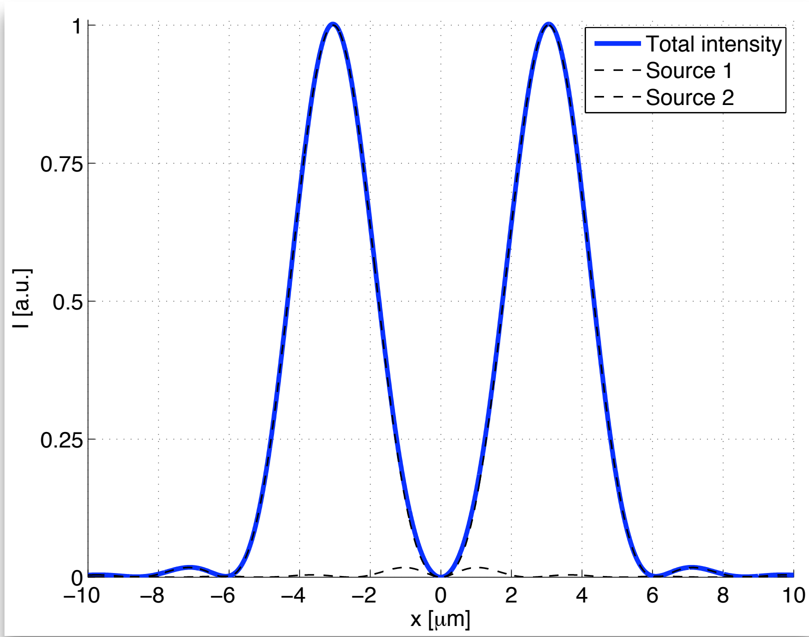
resolve *v* : **1** to break up into constituent parts: ANALYZE; **2** to find an answer to : SOLVE; **3** DETERMINE, DECIDE; **4** to make or pass a formal resolution

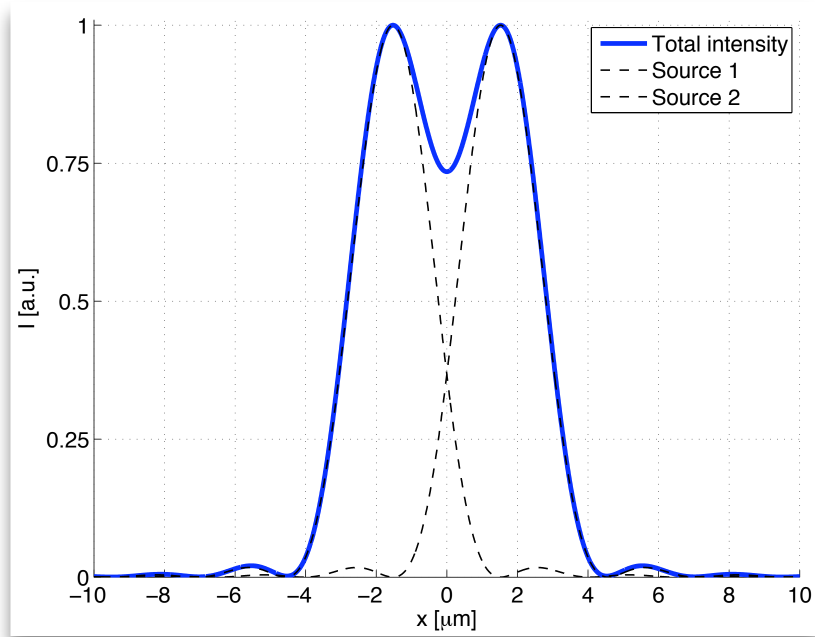
resolution *n* : **1** the act or process of resolving **2** the action of solving, *also* : SOLUTION; **3** the quality of being resolute: FIRMNESS, DETERMINATION; **4** a formal statement expressing the opinion, will or, intent of a body of persons





Rayleigh resolution limit





Two point sources are well resolved if they are spaced such that:

(i) the PSF *diameter*
equals the point source spacing

$$\Delta r = 1.22 \frac{\lambda}{(\mathrm{NA})_{\mathrm{in}}}$$

$$\Delta r' = 1.22 \frac{\lambda}{(\mathrm{NA})_{\mathrm{out}}}$$

(i) the PSF *radius*
equals the point source spacing

$$\Delta r = 0.61 \frac{\lambda}{(\mathrm{NA})_{\mathrm{in}}}$$

$$\Delta r' = 0.61 \frac{\lambda}{(\mathrm{NA})_{\mathrm{out}}}$$

Diffraction limited resolution

Two point objects are "**just resolvable**" (limited by diffraction only) if they are separated by:

Two–dimensional systems (rotationally symmetric PSF)	One–dimensional systems (e.g. slit–like aperture)
Safe definition: (one–lobe spacing) $\Delta r' = 1.22\frac{\lambda}{(\mathrm{NA})}$	$\Delta x' = \frac{\lambda}{(\mathrm{NA})}$
Pushy definition: (1/2–lobe spacing) $\Delta r' = 0.61\frac{\lambda}{(\mathrm{NA})}$	$\Delta x' = 0.5\frac{\lambda}{(\mathrm{NA})}$

You will see different authors giving different definitions. Rayleigh in his original paper (1879) noted the issue of noise and warned that the definition of "just–resolvable" points is system– or application –dependent

Aberrations *further* limit resolution

All our calculations have assumed "geometrically perfect" systems, i.e. we calculated the wave–optics behavior of systems which, in the paraxial geometrical optics approximation would have imaged a point object onto a perfect point image.

The effect of aberrations (calculated with non–paraxial geometrical optics) is to blur the "geometrically perfect" image; including the effects of diffraction causes additional blur.

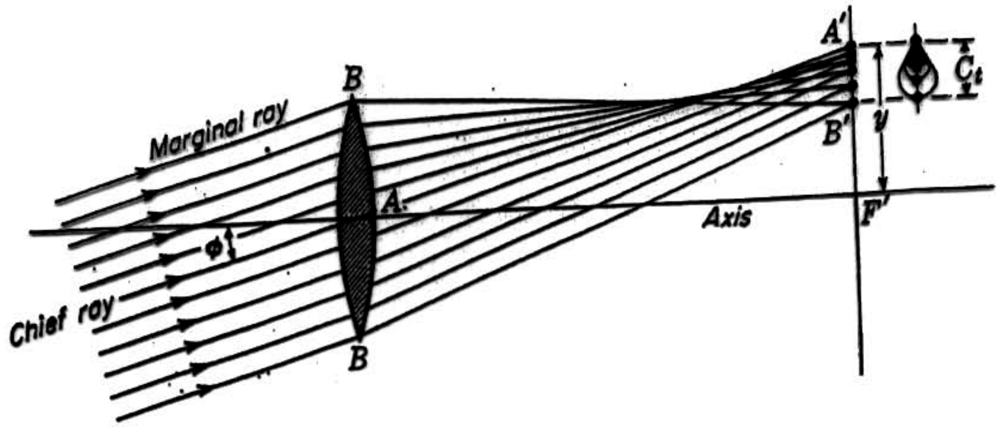
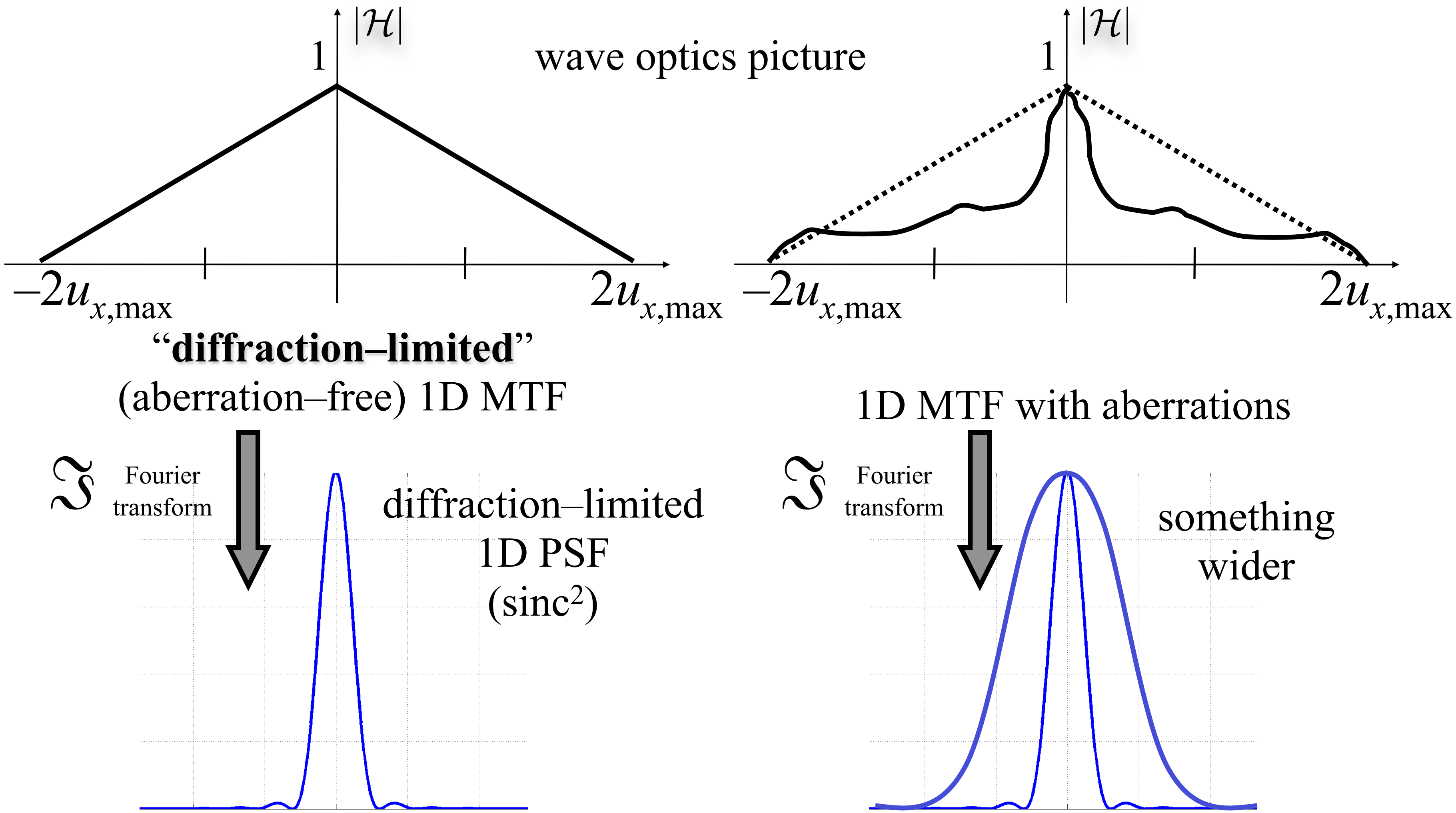


Fig. 9I in Jenkins, Francis A., and Harvey E. White.
Fundamentals of Optics. 4th ed. New York, NY: McGraw-Hill, 1976.
ISBN: 9780070323308.

Aberration-limited resolution based on the MTF



Resolution: common misinterpretations

Attempting to resolve object features smaller than the "resolution limit" (e.g. $1.22\lambda/\text{NA}$) is hopeless.

NO: Image quality degradation as object features become smaller than the resolution limit ("exceed the resolution limit") is noise dependent and gradual.

Besides, digital processing of the acquired images (e.g. methods such as the CLEAN algorithm, Wiener filtering, expectation maximization, etc.) can be employed.

Resolution: common misinterpretations

Super-resolution

By engineering the pupil function ("apodizing") to result in a PSF with narrower side–lobe, one can "beat" the resolution limitations imposed by the angular acceptance (NA) of the system.

MAYBE:

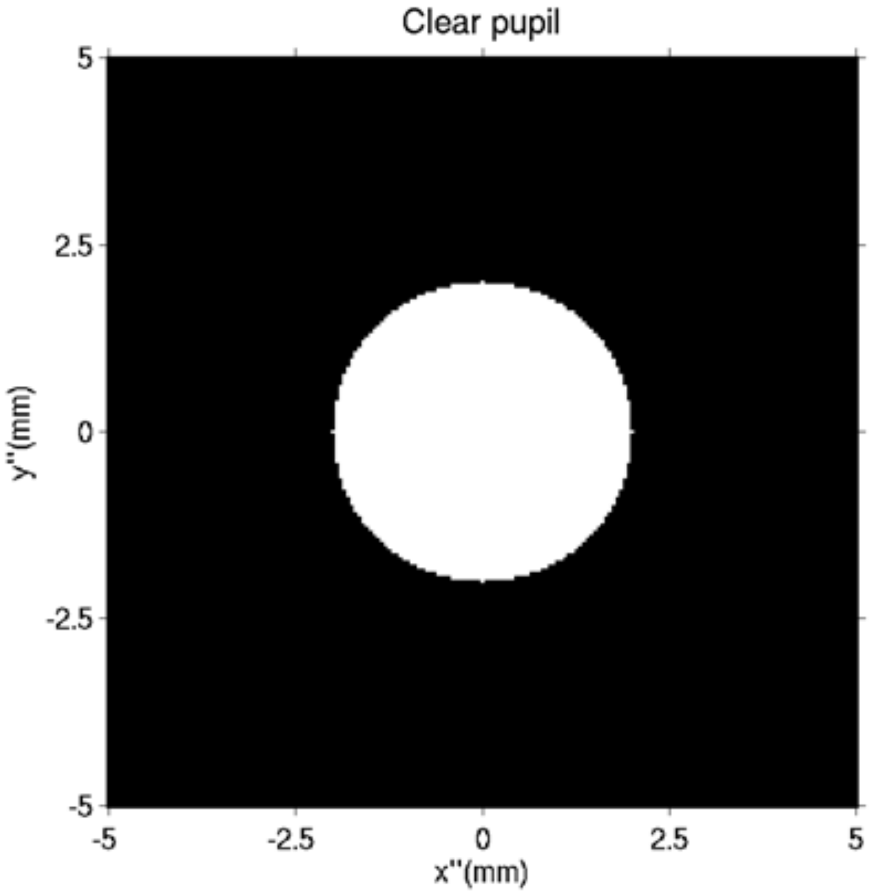
Pupil engineering always results in

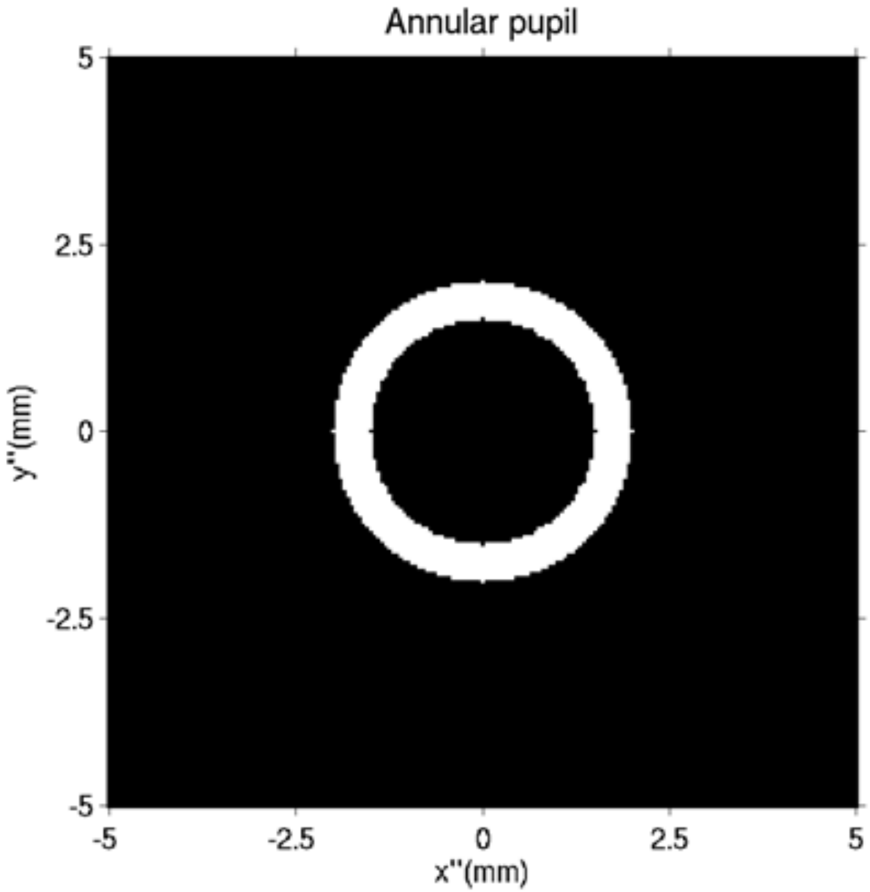
(i) narrower main lobe but accentuated side–lobes

(ii) lower power transmitted through the system

Both effects can be **BAD** on the image

Pupil engineering example: “apodization”





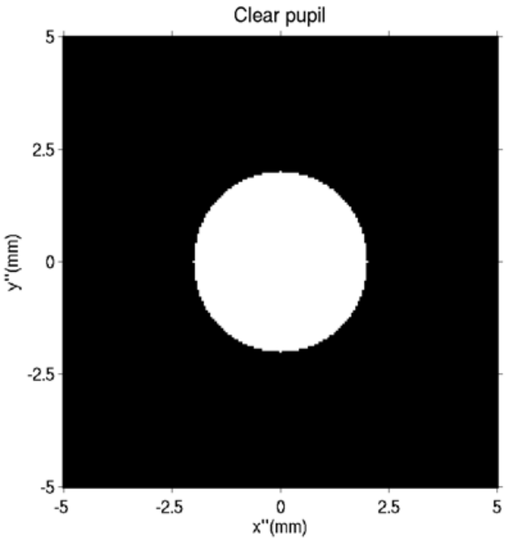
f_1=20cm

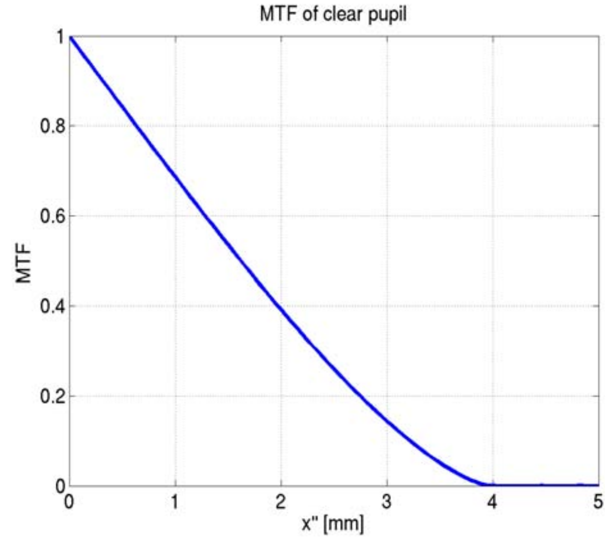
λ=0.5μm

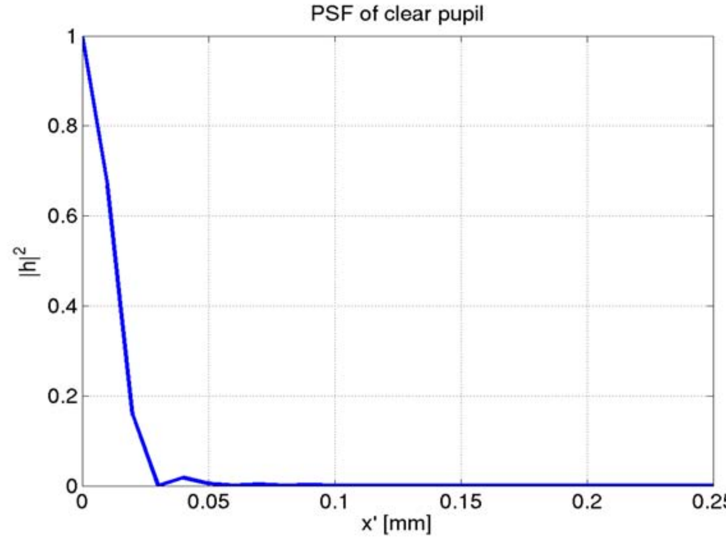
$$g_{\mathrm{PM}}\left(r''\right) = \mathrm{circ}\left(\frac{r''}{R_1}\right) - \mathrm{circ}\left(\frac{r''}{R_2}\right)$$

Effect of apodization on the MTF and PSF

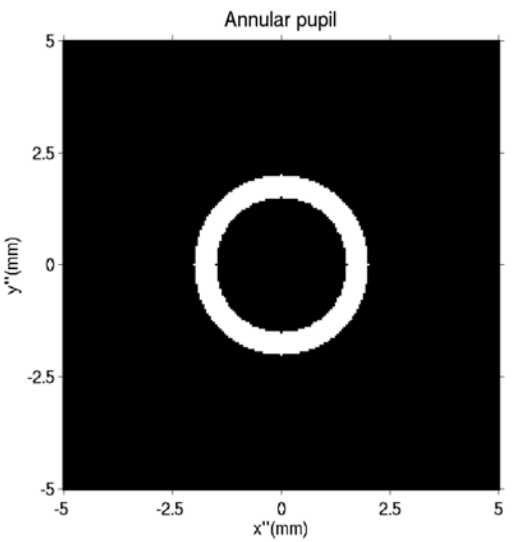
Un-apodized

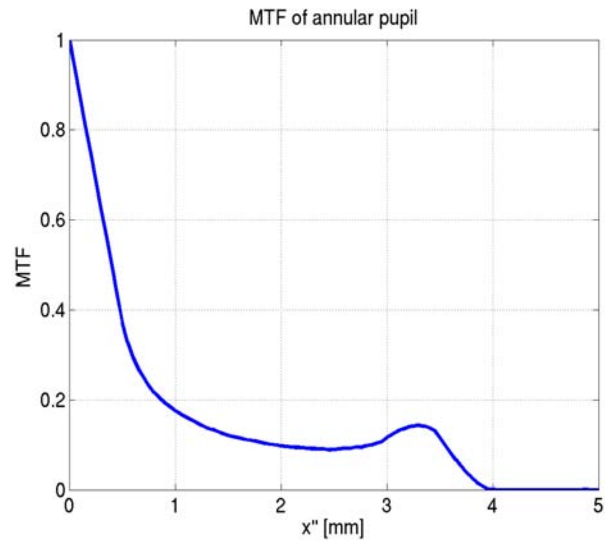


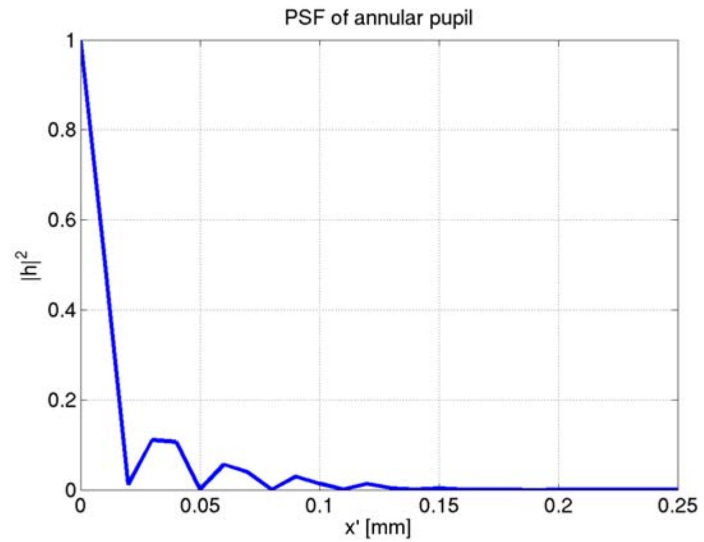




Apodized: annular

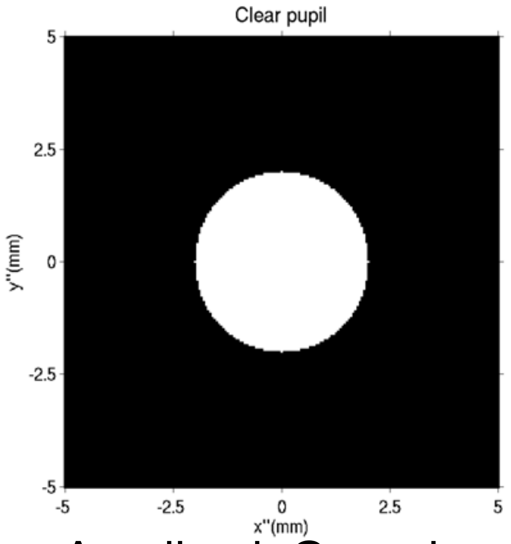


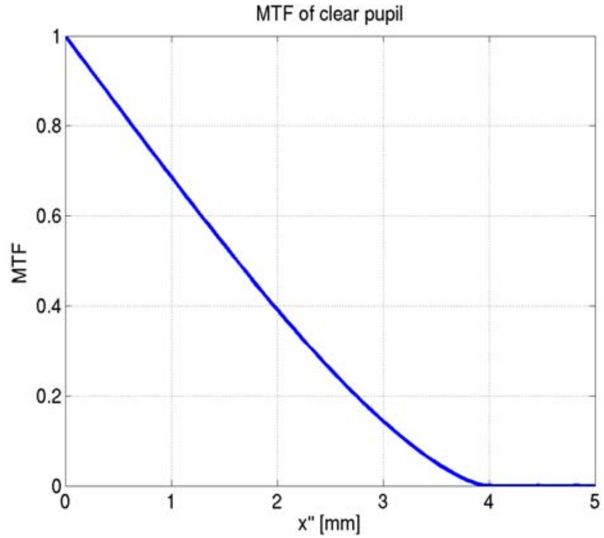


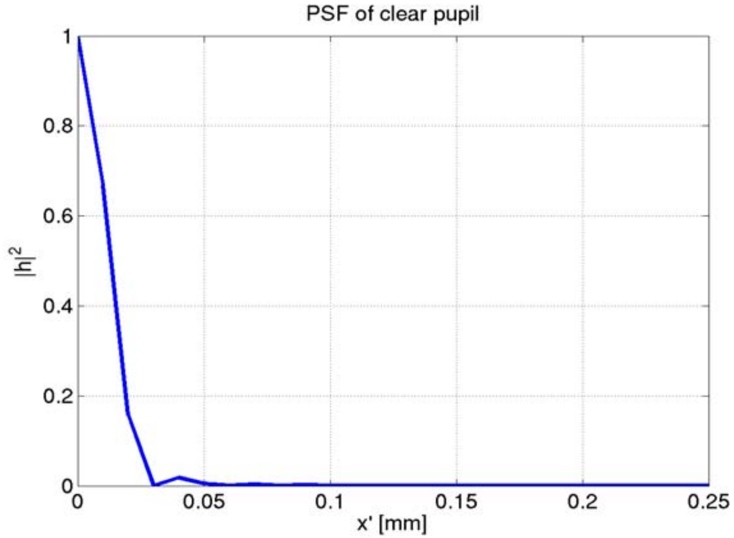


Effect of apodization on the MTF and PSF

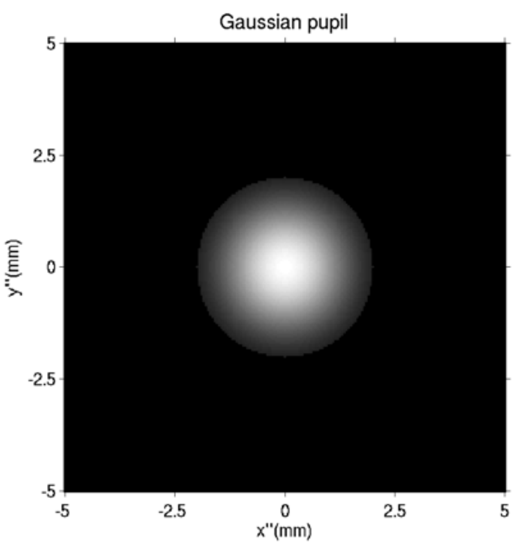
Un-apodized

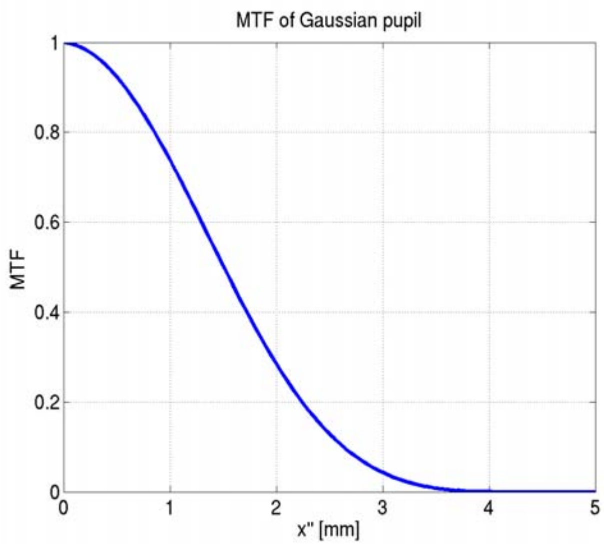


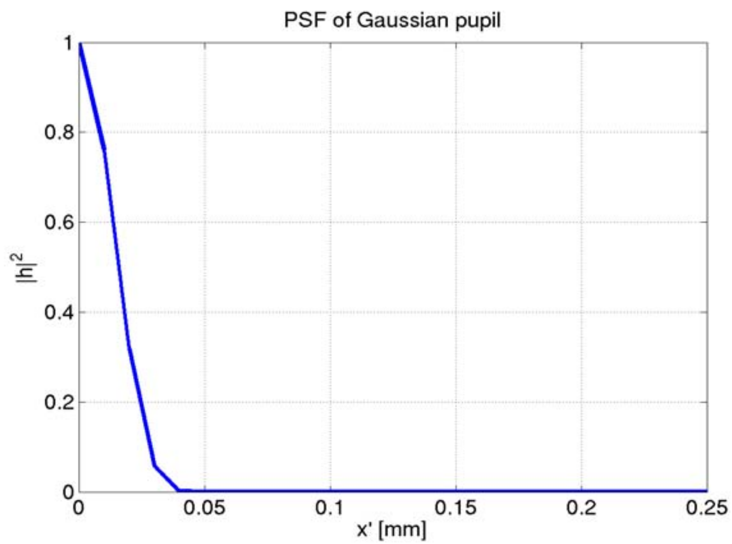




Apodized: Gaussian







Pupil engineering trade-offs

main lobe size ↓ ⇔ sidelobes ↑

and *vice versa*

main lobe size ↑ ⇔ sidelobes ↓

generally, power loss means SNR degradation

- Annular–type pupil functions typically narrow the main lobe of the PSF at the expense of higher side lobes
- Gaussian–type pupil functions typically suppress the side lobes but broaden the main lobe of the PSF
- Compromise? → application dependent
  - for point–like objects (e.g., stars) annular apodizers may be a good idea
  - for low–frequency objects (e.g., diffuse tissue) Gaussian apodizers may image with fewer artifacts
- Caveat: Gaussian amplitude apodizers very difficult to fabricate and introduce energy loss ⇒ binary phase apodizers (lossless by nature) are used instead; typically designed by numerical optimization

Resolution: common misinterpretations

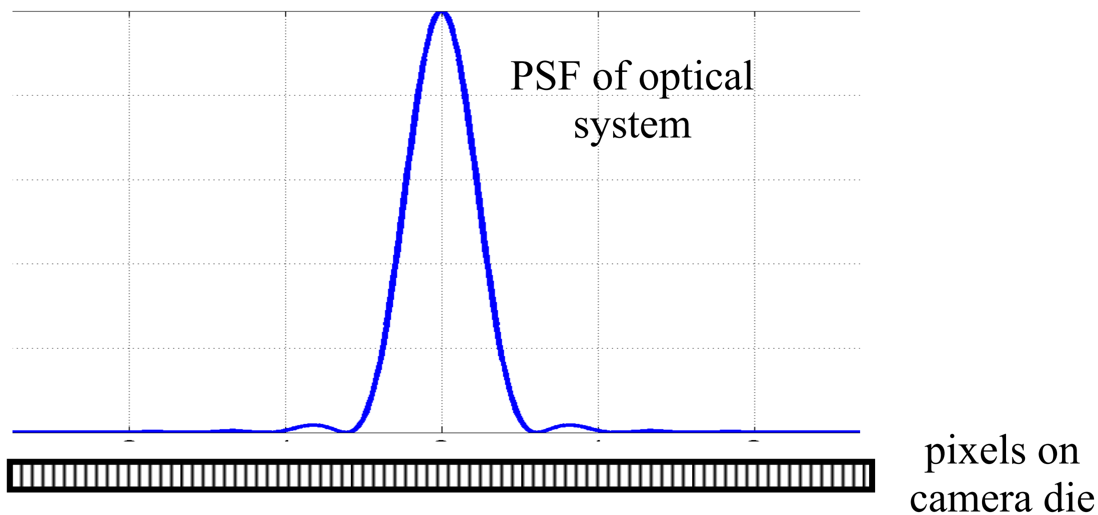
"This super cool digital camera has resolution of 8 Mega pixels (8 million pixels)."

NO:

This is the most common and worst misuse of the term "resolution." They are actually referring to the **space–bandwidth product (SBP)** of the camera

Are resolution and number of pixels related?

Answer depends on the magnification and
PSF of the optical system attached to the camera



Pixels significantly smaller than the system PSF
are somewhat underutilized (the effective SBP is reduced)

Some more misstatements

- It is pointless to attempt to resolve beyond the Rayleigh criterion (however defined)
  - NO: difficulty increases gradually as feature size shrinks, and difficulty is noise dependent
- Apodization can be used to beat the resolution limit imposed by the numerical aperture
  - NO: watch sidelobe growth and power efficiency loss
- The resolution of my camera is $N \times M$ pixels
  - NO: the maximum possible SBP of your system may be $N \times M$ pixels but you can easily underutilize it (i.e., achieve SBP that is less than $N \times M$) by using a suboptimal optical system

So, what *is* resolution?

- Our ability to resolve two point objects (in general, two distinct features in a more general object) based on the image
  - however, this may be difficult to quantify
- Resolution is *related* to the NA but *not exclusively* limited by it
- Resolution, as it relates to NA: it's true that
  - **resolution improves as NA increases**
- Other factors affecting resolution: caveats to the previous statement are
  - **aberrations / apodization** (i.e., the exact shape of the PSF)
  - **NOISE!**
- Is there an easy answer?
  - No ……

but when in doubt quote $0.61\lambda/(\mathrm{NA})$ or $1.22\lambda/(\mathrm{NA})$ as an *estimate* (not as an exact limit).

Today

- Two more applications of the Transfer Function
  - defocus and Depth of Focus / Depth of Field (DoF)
  - image reconstruction:
    - deconvolution and its problems
    - Tikhonov-regularized inverse filters

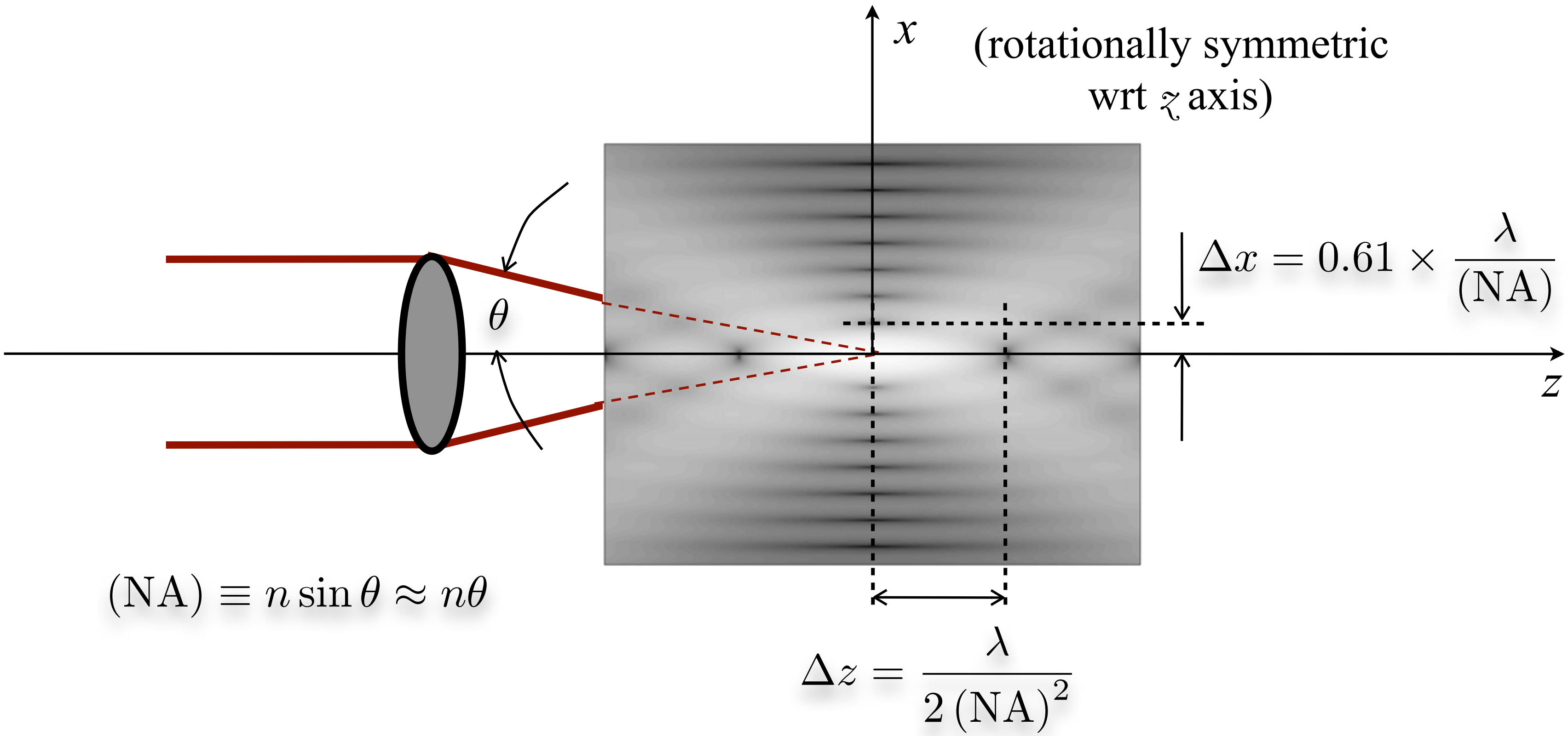
Wednesday

- Polarization
- The intensity distribution near the focus of high-NA imaging systems
- Utilizing the short depth of field of high-NA imaging:
  confocal microscopy and related 3D imaging systems

Defocus in wide field imaging

Image removed due to copyright restrictions.
Please see:
http://www.imdb.com/media/rm4216035584/tt0137523

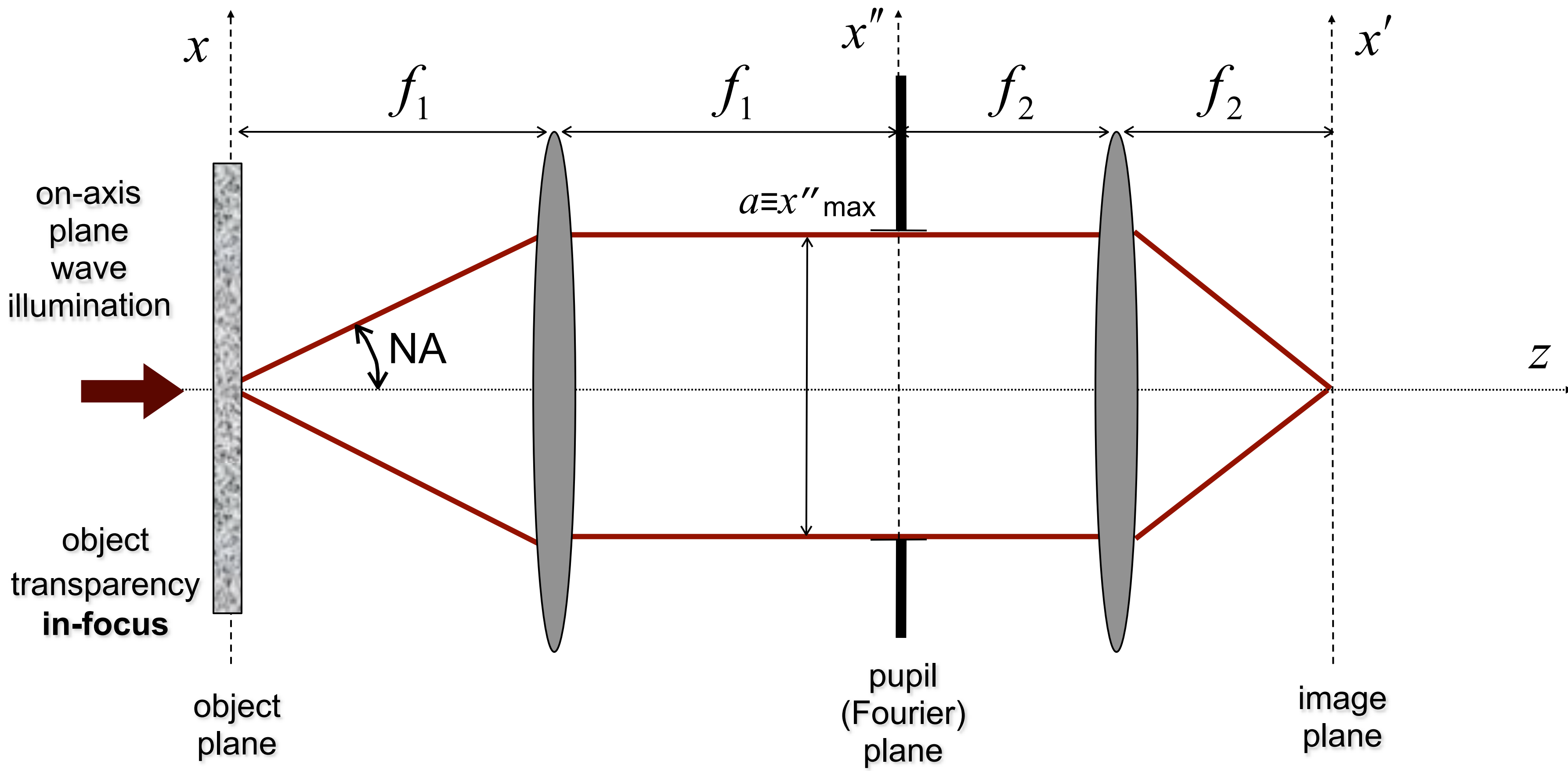
Paraxial intensity distribution near focus



Δx: Rayleigh resolution criterion [Lecture 23]
Δz: Depth of Focus / Depth of Field (DoF) [today's topic]

Note: at very high numerical apertures, the scalar approximation is no longer good; the vectorial nature of the electromagnetic field becomes important.

4F system with in-focus input

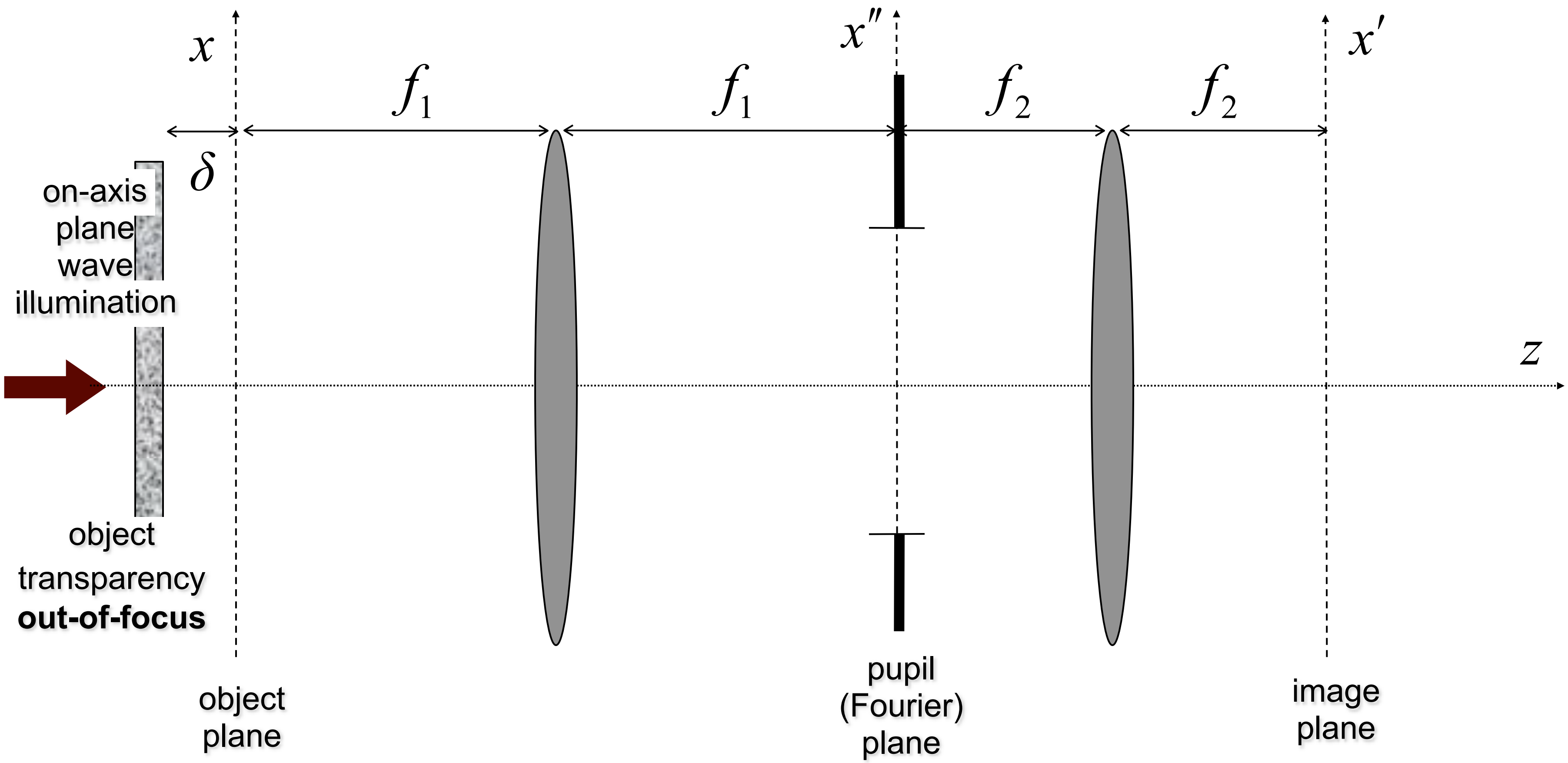


$$g_{\mathrm{t}}(x) \qquad G_{\mathrm{t}}\left(\frac{x''}{\lambda f_1}\right) \qquad \approx g_{\mathrm{t}}\left(-\frac{f_1}{f_2}x'\right)$$

Numerical Aperture $\quad (\mathrm{NA}) \equiv \dfrac{x''_{\max}}{f_1}$ in the paraxial approximation

neglecting the low-pass filtering due to the finite pupil mask

4F system with out-of-focus input



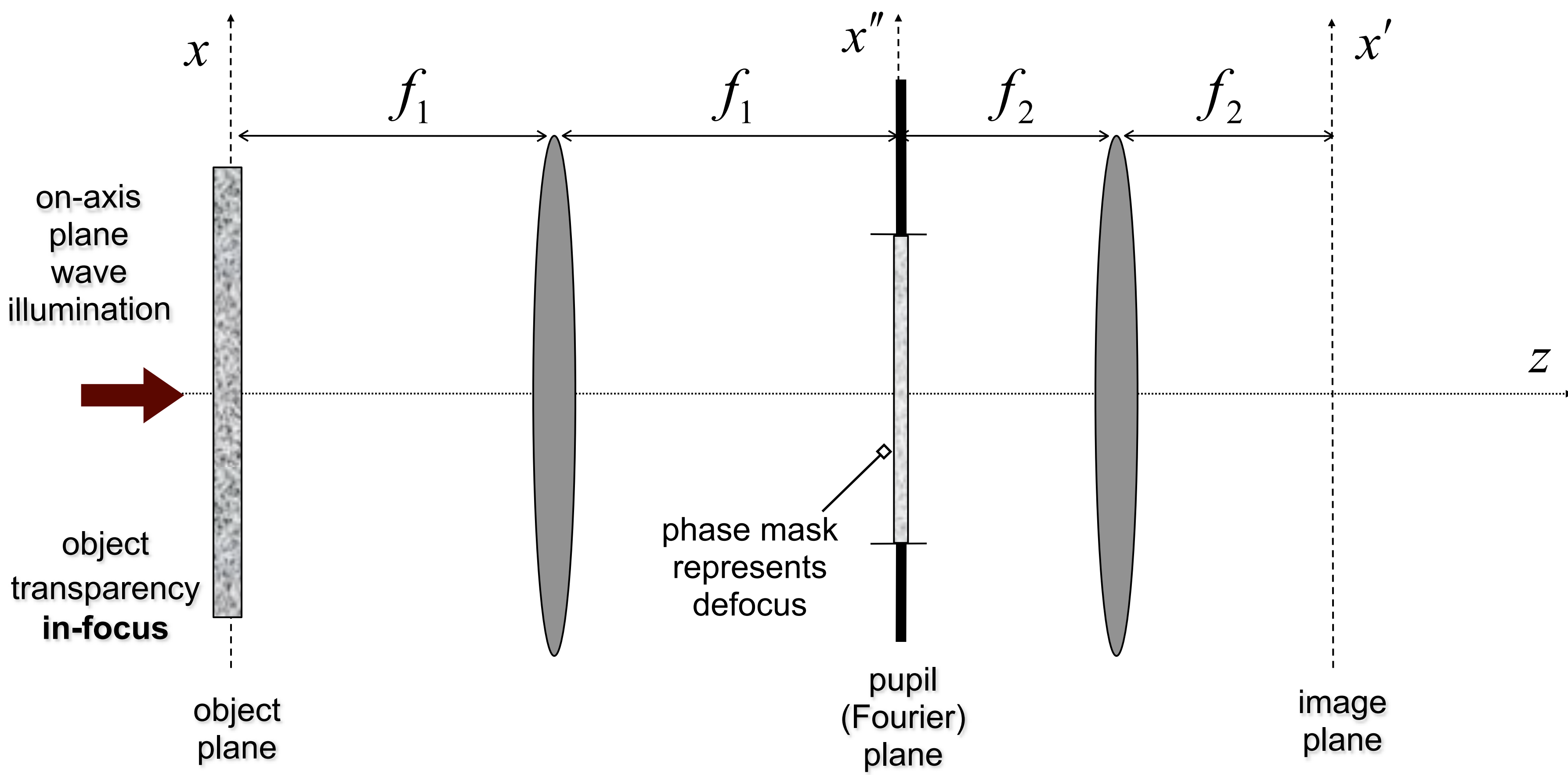
$$g_t(x) \circledast \exp\left\{ i\pi \frac{x^2}{\lambda\delta} \right\}$$

object *convolved* with the propagation kernel

$$G_t\left(\frac{x''}{\lambda f_1}\right) \times \exp\left\{ -i\pi\lambda\delta \, \frac{x''^2}{\lambda^2 f_1^2} \right\}$$

object spectrum *multiplied* by the Fourier transform of the propagation kernel

Equivalent optical system



$g_{\mathrm{t}}(x)$

$$G_{\mathrm{t}}\left(\frac{x''}{\lambda f_1}\right) \times \exp\left\{-i\pi\lambda\delta\,\frac{x''^2}{\lambda^2 f_1^2}\right\}$$

object spectrum *multiplied* by
the complex transmissivity of an "equivalent" phase mask

defocus ATF

$$\exp\left\{-i\pi\lambda\delta u^2\right\}$$

$$g_{\mathrm{PM,defocus}} \equiv \exp\left\{-i\pi\lambda\delta\,\frac{x''^2}{\lambda^2 f_1^2}\right\}$$

2.71 / 2.710 Optics
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