Lecture 9

Ultimately, one wants to describe acoustic propagation through a realistic ocean/seabed waveguide, and the simple models discussed, even with perturbations, won't suffice. At that point, one turns to numerical methods. Luckily for those wishing to use such methods, COA is a veritable bible of such methods, written by four leading practitioners. Also, one of the authors (Mike Porter) has, under ONR sponsorship, installed a free library of ocean acoustics codes (OALIB) which are easy to obtain online and use, and are well documented. So, armed with COA and OALIB, one can proceed rather quickly to being a "user" (see the movie "TRON" for more context of that word) of advanced codes. However, being a naïve user of a black-box code is not a good thing (thus the disdain the "programs" on the grid in TRON had for "users"!). So, in order to become savvy users (and perhaps even developers) of numerical codes, it is useful to take a look at their basic workings.

I would state again that the reason we look at numerics in this course was that Prof. Schmidt couldn't teach his COA course last semester, so we "ad libbed" a section into this one as a temporary help to the students. His course provides the full treatment of COA, and is highly recommended.

Since we are looking to solve the modal eigenvalue problem, we first look for the eigenvalues, from which we can construct eigenvectors. There are a number of ways to do this.

One of the most popular numerical methods is finite differences. In this method, we first need to put the (continuous variable) mode equation into "finite difference" (discrete) form, as well as the boundary conditions. This is done by using simple forward, backward, and central difference schemes, as shown quite explicitly in COA section 5.7.1. This results in a tri-diagonal matrix form which can be input simply to standard MATLAB (or other) eigenvalue/eigenvector routines. In a past version of this class, one student noted that the form in COA needed some light recasting to input it to the standard routine, and thus I added some additional notes to do this. The additional notes use "slowness" instead of sound speed, but are otherwise the same as COA.

Another method of finding model eigenvalues is the so called "shooting" method (COA section 5.7.3), which is actually kind of fun, in that it does just what the name implies! One uses a "trial eigenvalue" (a guess), and then numerical shoots a mode function from the surface to the bottom using this guess. If the mode function at the bottom satisfies the boundary condition, then "you have chosen wisely," and your guess is one of the eigenvalues. Of course, one has to have a systematic way of guessing to make this efficient, and to find all the eigenvalues, but that is not so hard.

If we go back to characteristic equations, we can use traditional root finders, such as the secant method or Newton's method. The Pekeris waveguide is a rather good system to try this on, in that the R_{bott} for the transcendental equation is simple, so the algebra is minimal. Different versions of root finders and their "peculiarities" (features, not bugs) are discussed in sectin 5.7.4 of COA.

We note that in the above, where we have found the modes and their eigenvalues, constructing the pressure field for the range independent or adiabatic mode case is straightforward. But what about coupled modes? We treat that next.

Though many ways have been tried (successfully) to solve the coupled mode equations, the most useful way to obtain a solution has been Richard Evan's "stepwise coupled modes," which is discussed in detail in section 5.11 of COA. In this scheme, the range dependent waveguide is divided into a number of constant depth segments in range, so that a slope becomes a staircase. We already know how to solve for the modes in each segment, so that the only remaining task is joining the segments together in range. For our fluid, acoustic case, this is done simply by invoking the continuity of pressure and normal particle velocity across the interface. This solution, though it has been largely supplanted by the PE, does have the attractive feature that one can turn the backscatter "on and off," i.e., look at both 2-way (forward and backscatter) solutions or 1-way (forward) solutions easily, and use the comparison to understand reverberation.

The development of the 2-way solution is presented in COA. This development, while a few pages long, is detailed and straightforward, and should present no trouble to the reader to follow.

When all is done, the 2-way solution presented appears as a large, block matrix system which needs to be solved. Global matrix methods, used successfully by Schmidt for wavenumber integration problems, can cure instability problems (due to exponentials) in such systems and as computation capabilities have increased, even strong range dependent problems are within calculation range.

The full 2-way coupled modes actually allow for a complete "multiple scattering solution between segments, and makes the problem a large, global matrix one. If one still wants backscatter, but can ignore multiple scattering (a good physics question to think on is "where?"), one can go to a much simpler 1-way solution that includes a single-scattering in it as well. This allows fast "marching solutions" to be implemented, and is discussed in section 5.11.2 of COA.

The next (numerical) topic we come to is wavenumber integration, which we saw analytically before, and now look at numerically, following COA section 4.5 through 4.5.4.

While wavenumber integration using the Hankel transform is limited to plane stratified media, it can work easily with elastic media as well as fluid, and so finds use in seismics as well as ocean acoustics.

Two problems that arise in doing the numerical evaluation of the HT integral are: 1) truncation of the infinite integral (we can't get either pressure or wave-number out to infinity!) and 2) aliasing due to discretization.

One other small problem (for me at least) was COA's use of $g(r,z)$ to stand for $p(r,z)$ in the HT equation, Eq. 4.93. (Any other letter than g, grrr!!). Oh well, it's just gnotation, right?!

To kick off the HT discussion, COA section 4.5.1 quickly shows the "Fast Field Program" (FFP) approximation to the HT, which (as previously mentioned in Frisk) makes the HT look like a 1-D Fourier transform, seen in Eq. (4.96). Truncation of the integral to $k_r < \infty$ is discussed in section COA 4.5.2, and it is found that using a k_{max} is viable, but the choice of k_{max} is not trivial, especially for elastic media, where evanescent interface waves like Scholte waves can push k_{max} much higher than one uses in fluid media.

The other bugbear for the FFP evaluation of the HT is aliasing, discussed in COA section 4.5.3. This discussion is simple mathematically, but a bit subtle physically, and might take a re-reading or two.

Finally, in section 4.5.4 of COA, one example of the FFP being used in practice for a Pekeris waveguide is presented, and the solution contrasted to the "exact" HT evaluation.

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