Hw #8 Solution

1)(a) $\dot{m}_a = \eta_v \left(\frac{p}{RT}\right) V_D \frac{N}{2} = 0.8 \left(\frac{2.5\times10^5}{287\times343}\right) 1.29\times10^{-2} \times \frac{2000}{120} = 0.381 \text{ kg/s} = \underline{381 \text{ g/s}}$

$\dot{m}_f = \text{bsfc} \times P = \frac{200 \times 294}{3600} = \underline{16.3 \text{ g/s}}$

$A/F = \dot{m}_a/\dot{m}_f = 381/16.3 = \underline{23.3}$

(b) $m_a = \dot{m}_a/(n N/2)$; where n = # of cylinders = 6

$= \frac{381}{6 \times \frac{2000}{120}} = \underline{3.81 \text{ g}}$

A large drop!

$m_f = m_a/(A/F) = \underline{0.163 \text{ g}}$; $V_f = \frac{m_f}{\rho_f} = \left(\frac{0.163}{0.78}\right) = \underline{0.21 \text{ cc}}$

(c) Denote nozzle properties by subscript 2.

$V_2 = \frac{V_f}{n_2 A_2 \tau}$ n_2 = # of nozzle holes = 8

A_2 = hole area

τ = inj duration:

$= \frac{0.21\times10^{-6}}{8 \times \frac{\pi(0.2\times10^{-3})^2}{4} \times 3\times10^{-3}}$ at 2000 rpm → 12°CA/ms

40°CA ⇒ $\tau = 40/12 = 3.3$ ms

$= \underline{253 \text{ m/s}}$

(d) Reynolds no $= \frac{\rho_f V_2 d_2}{\mu} = \frac{253 \times 0.78\times10^3 \times 0.2\times10^{-3}}{5\times10^{-4}} = 7.9\times10^4$ turbulent.

From Moody chart, friction coeff. $f = 0.018$

$\Delta p_2 = \frac{1}{2} \rho_f V_2^2 f\left(\frac{L}{D}\right)_2 = \frac{1}{2} \times 0.78\times10^3 \times (253)^2 \times 0.018 \times 10 = \underline{45 \text{ bar}}$

This is not significant (~3%) compared to the pressure drop of 1350 bar across the nozzle.

(e) Discharge coeff. $C_D = \frac{V_2}{\sqrt{2\Delta p/\rho_f}} = \frac{253}{\sqrt{2(1350\times10^5)/0.78\times10^3}} = \underline{0.43}$

(f) Droplet Diameter $d = (We)_{\text{critical}} \frac{\sigma}{\rho_g V_2^2} = 30 \frac{0.025}{\left(\frac{50\times10^5}{287\times300}\right)(253)^2} = \underline{0.54 \mu\text{m}}$

(g) $N_{\text{drop}} = \left(\frac{V_f}{\pi d^3/6}\right) = \frac{0.21\times10^{-6}}{\pi (0.54\times10^{-6})^3/6} = \underline{2.5\times10^{12} \text{ drops}}$

Droplet spacing $= (V_c/N)^{1/3}$; $V_c = \frac{V_D}{C_{2T}} = \frac{12.9\times10^{-3}}{6(16.5-1)} = 139 \text{ cc}$

Spacing $= \left(\frac{139\times10^{-6}}{2.5\times10^{12}}\right)^{1/3} = 3.8\times10^{-6} \text{ m} = \underline{3.8 \mu\text{m}}$

Thus each drop has to evap. and mix with air of $\left(\frac{3.8}{0.54}\right)^3 = 348$ times its size. A difficult job

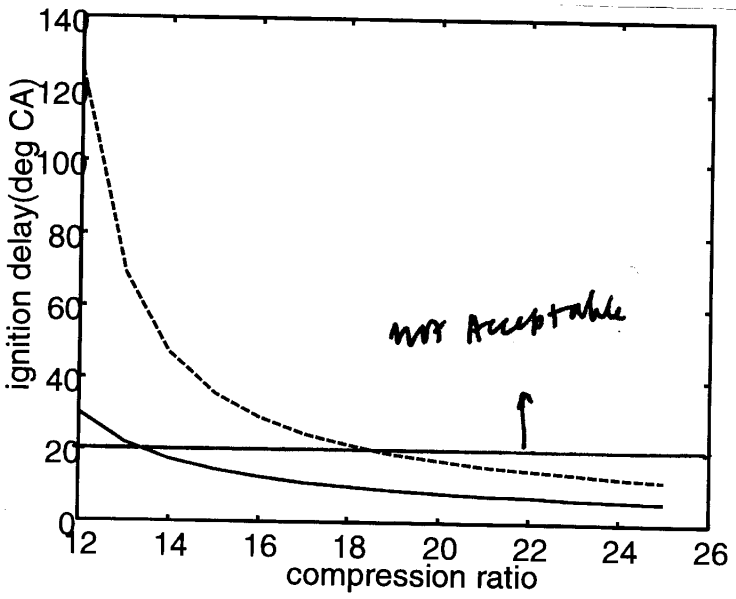
2/ Ignition delay:

$$\tau_i(CA) = \left(0.36 + 0.22\,\bar{S}_p\right) \exp\left\{E_a\left[\left(\frac{1}{\bar{R}T}\right) - \frac{1}{17190}\right] + \left(\frac{21.2}{P - 12.4}\right)^{0.63}\right\}$$

$$E_a = 618840/(CN + 25)$$

$$T = T_i (CR)^{n-1} \quad ; \quad P = P_i (CR)^n$$

Results:



CR	ign delay (a)	ign delay (b) in °CA
12.00	30.34	125.73
13.00	22.15	69.04
14.00	17.56	47.10
15.00	14.64	35.88
16.00	12.64	29.18
17.00	11.17	24.76
18.00	10.05	21.63
19.00	9.17	19.30
20.00	8.46	17.50
21.00	7.87	16.07
22.00	7.38	14.90
23.00	6.96	13.93
24.00	6.59	13.11
25.00	6.28	12.40

To have $\tau_{id} \leq 20°CA$

For the larger truck engine, the minimum compression ratio is 14.

For the small passenger car engine, the minimum is 19.

8-3) Mass Diffusivity $D = 1.8 \times 10^{-5} \left(\frac{p_0}{p}\right)\left(\frac{T}{T_0}\right)^{1.81}$ (m^2/s)

Diffusion Time $\tau_{diff} = d^2/D$

Conditions	(a)	(b)	(c)	
p (bar)	100	70	30	
T (K)	2500	2000	1400	
D (m^2/s)	8.36×10^{-6}	7.97×10^{-6}	9.75×10^{-6}	
τ_{diff} - 10 nm	1.2×10^{-11} s	1.25×10^{-11} s	1.03×10^{-11} s	All small values compared to oxidation time
τ_{diff} - 100 nm	1.2×10^{-9} s	1.25×10^{-9}	1.03×10^{-9}	
τ_{diff} - 1000 nm	1.2×10^{-7} s	1.25×10^{-7}	1.03×10^{-7}	

Oxidation Time

mass conservation $\rho(4\pi r^2)\frac{dr}{dt} = -(4\pi r^2)w$

$$\Rightarrow \quad \frac{dr}{dt} = -\frac{w}{\rho}$$

integrating $r - r_0 = -\frac{w}{\rho}t$; time to oxidize particle of radius r_0

is $t_0 = \frac{r_0 w}{\rho}$ — The time is proportional to r_0.

For the Nagle-Strickland Constable formula, $p_{O_2} = p \cdot x_{O_2}$ then the numerical values are: (for 100 nm = 0.1 μm diameter particle)

oxidation time (s) for 0.1 micron soot

T(k)	p(bar)	time@xo2=0.1%	1%	10%
2.5000e+003	1.0000e+002	1.5798e-002	1.9327e-003	6.9704e-004
2.0000e+003	7.0000e+001	8.8868e-003	4.9975e-003	4.6617e-003
1.4000e+003	3.0000e+001	2.5396e-001	1.9351e-001	1.8747e-001

More the sensitivity to temperature. Sufficient O_2 has to be mixed in early (at high T) to oxidize the soot.

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