2/14/06 2.58J

LAST TIME:

(1) $\varepsilon'_\lambda \rightarrow \varepsilon_\lambda$, $\varepsilon'_\lambda \rightarrow \varepsilon'$, $\varepsilon'_\lambda \rightarrow \varepsilon$

(2) $\alpha'_\lambda \rightarrow \alpha_\lambda$, $\alpha'_\lambda \rightarrow \alpha'$, $\alpha'_\lambda \rightarrow \alpha$

"WHAT COMES IN" "WHAT GOES OUT"

(3) $\rho''_\lambda \rightarrow \rho'^{\triangle}_\lambda$, $\rho''_\lambda \rightarrow \rho^{\triangle'}_\lambda$

(4) τ''_λ

$$\rho + \alpha + \tau = 1$$

BE CAREFUL

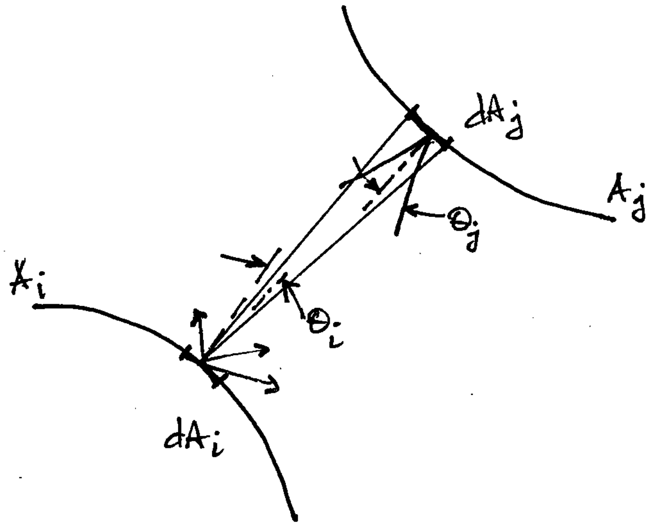
$$\rho'^{\triangle}_\lambda + \alpha'_\lambda + \tau'^{\triangle} = 1$$

$\varepsilon'_\lambda = \alpha'_\lambda$ "KIRCHOFF'S LAW"

$\Longrightarrow$ DIFFUSE - GRAY SURFACE

$$\varepsilon = \alpha$$

2/14 2.58



$$dF_{dA_i - dA_j} = \frac{\cos\theta_i \cos\theta_j}{\pi S^2} dA_j \qquad \text{(NORMALIZED BY AREA } dA_i\text{)}$$

$$dA_i \cdot dF_{dA_i - dA_j} = \frac{\cos\theta_i \cos\theta_j}{\pi S^2} dA_i\, dA_j = dA_j \cdot dF_{dA_j - dA_i} \qquad \text{(RECIPROCITY)}$$

INTEGRATE

$$F_{dA_i - A_j} = \int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi S^2} dA_j$$

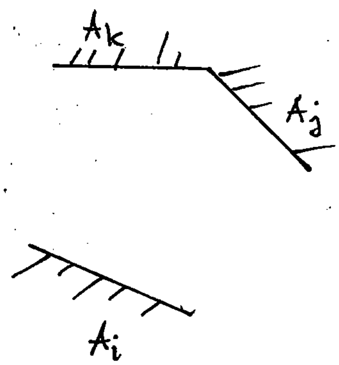
$$\underbrace{F_{A_j - A_i}}_{} = \frac{1}{A_i} \int_{A_i}\int_{A_j} \frac{\cos\theta_i \cos\theta_j}{\pi S^2} dA_i\, dA_j$$

ORDER
IS IMPORTANT

$$\Rightarrow \quad A_i F_{ij} = A_j F_{ji}$$

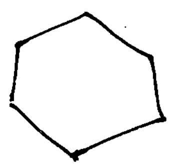
2/14 2.58

$$A_i F_{ij} = A_j F_{ji}$$



$$F_{i-(j+k)} = F_{i-j} + F_{i-k}$$

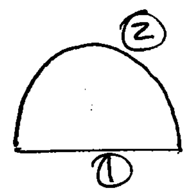
$$F_{(j+k)-i} \neq F_{j-i} + F_{k-i}$$

A_i 

$$\sum_{j=1}^{N} F_{i-j} = 1$$ (NRG BALANCE)

Ex

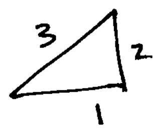
USE DUMMY SURFACE $\Longrightarrow$



$F_{12} = 1$, $F_{11} = 0$

$$F_{21} = \frac{A_1 F_{12}}{A_2} = \frac{A_1}{A_2}$$

$$F_{22} = 1 - F_{21} = 1 - \frac{A_1}{A_2}$$



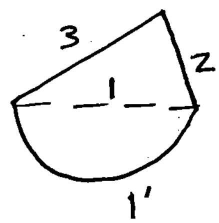
$$F_{12} + F_{13} = 1 \qquad A_1 F_{12} = A_2 F_{21}$$

$$F_{21} + F_{23} = 1 \quad \Longrightarrow \quad A_1 F_{13} = A_3 F_{31}$$

$$F_{31} + F_{32} = 1 \qquad A_2 F_{23} = A_3 F_{32}$$

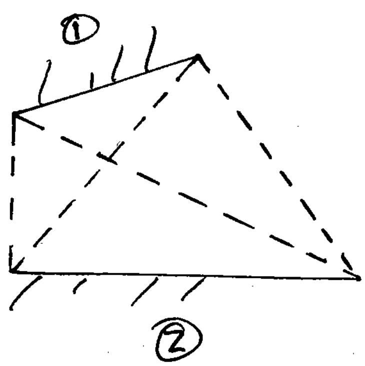
$$F_{12} = \frac{A_1 + A_2 - A_3}{2A_1}$$

WHAT IF



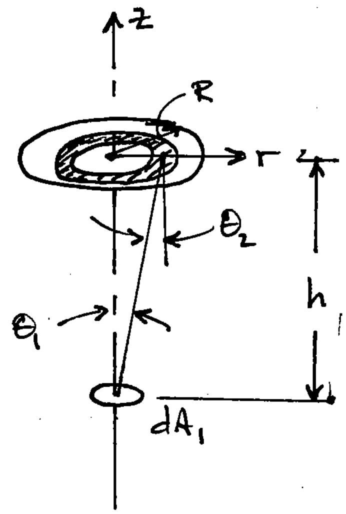
USE DUMMY SURFACE

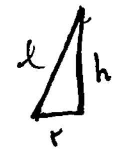
ALSO, HOTTEL'S STRING RULE



2/14 2.58

USE INTEGRATION TO COMPUTE





$l=\sqrt{h^2+r^2}$

NOTE $\theta_1=\theta_2$

$l\cos\theta_1 = h \Rightarrow \cos\theta = \frac{h}{\sqrt{h^2+r^2}}$

$$F_{dA_1-A_2} = \int_{A_2} \frac{\cos\theta_1 \cos\theta_2}{\pi S^2}\, dA_2$$

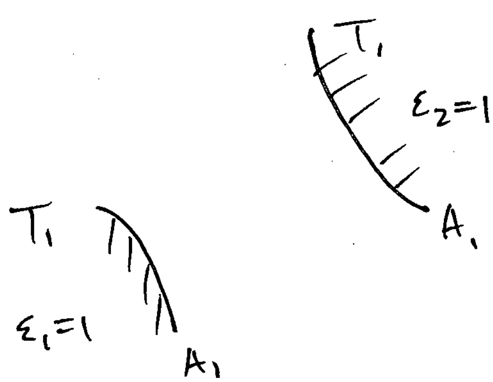
$$\cos^2\theta = \frac{h^2}{h^2+r^2}$$

$$F_{dA_1-A_2} = \int_0^R \frac{1}{\pi(h^2+r^2)} \underbrace{2\pi r\, dr}_{dA_2} \times \left(\frac{h}{\sqrt{h^2+r^2}}\right)^2$$

$$= \frac{\pi h^2}{\pi} \int_0^R \frac{dr^2}{(h^2+r^2)^2} = h^2\left(-\frac{1}{h^2+r^2}\right)\Bigg|_0^R$$

$$= \frac{R^2}{h^2+R^2}$$

BLACK SURFACES



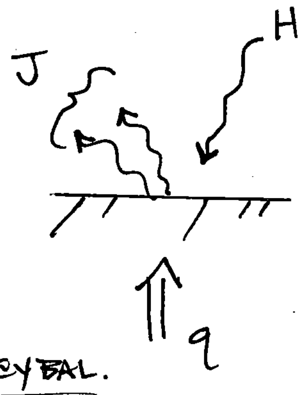
$(A_1 \sigma T_1^4)F_{12} - (A_2 \sigma T_2^4)F_{21} =$ NET HEAT XFER

$A_1 F_{12} \sigma (T_1^4 - T_2^4) =$ "

SANITY CHECK

2/14 2.58

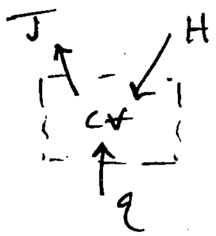
DIFFUSE-GRAY SURFACES



$H \equiv$ IRRADIATION (INCOMING RADIATION)

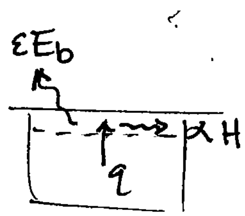
$J \equiv$ RADIOSITY (EMISSION DUE TO BODY TEMP + REFLECTED IRRADIATION)

ENERGY BAL.



$$q = J - H$$

ALTERNATIVE APPROACH



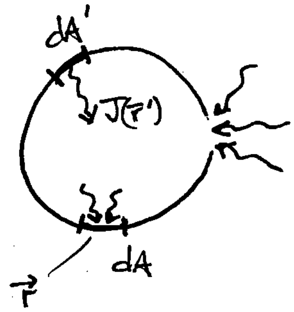
$$q = \varepsilon E_b - \alpha H$$

COMBINE TO ELIMINATE H

$$\boxed{q = \frac{\varepsilon(E_b - J)}{1-\varepsilon}}$$

2/14 2.58

$\bar{r} \equiv$ LOCAL VALUES



$$q''(\bar{r}) = \varepsilon(\bar{r})\,E_b(\bar{r}) - \alpha(\bar{r})\,H(\bar{r}) = J(\bar{r}) - H(\bar{r})$$

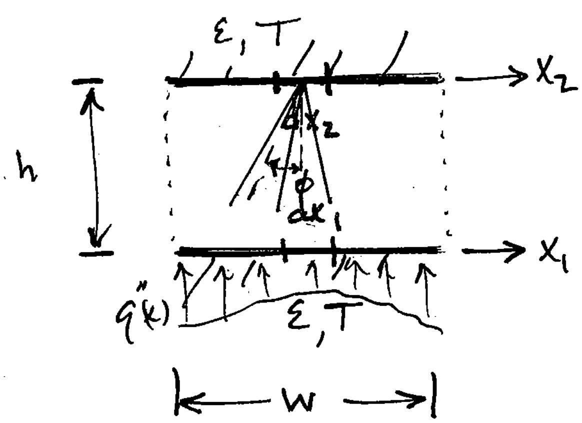
$$dA \cdot H(\bar{r}) = \int_{A'} \underbrace{J(\bar{r}')\,dA'\,dF_{dA'-dA}}_{dA \cdot \cancel{\ } dF_{dA-dA'}} + \underbrace{H_o(\bar{r})\,dA}$$

H_o INCOMING FROM OPEN; INCLUDE SHAPE FACTOR

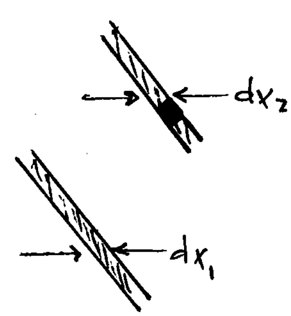
$$H(\bar{r}) = \int_{A'} J(\bar{r}')\,dF_{dA-dA'} + H_o(\bar{r})$$

$$q(\bar{r}) = J(\bar{r}) - \left[\int_{A'} J(\bar{r}')\,dF_{dA-dA'} + H_o(\bar{r})\right]$$

Ex LET $T_{BACKGROUND} = 0 \Rightarrow H_o = 0$



WANT TO MAINTAIN SURFACES AT CONSTANT TEMPERATURE, BUT SUPPLIED $q'' = q''(x)$

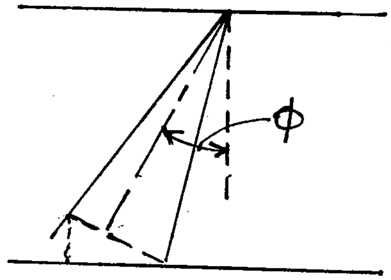


$$J(x_2) = \varepsilon\sigma T^4 + (1-\varepsilon)\int_0^W J_1(x_1)\,dF_{dA_2-dA_1}$$

FROM TEXT (pp 768) $\quad dF_{dA_2-dA_1} = \dfrac{\cos\phi\,d\phi}{2}$

$$\cos\phi = \frac{1}{\sqrt{h^2 + (x_2-x_1)^2}}$$

2.58 2/14



$$d\phi = \frac{dx_1 \cos\phi}{\sqrt{h^2+(x_2-x_1)^2}}$$

$$J_2(x_2) = \varepsilon\sigma T^4 + \frac{(1-\varepsilon)h^2}{2}\int_0^W \frac{J_1(x_1)}{\left[h^2+(x_2-x_1)^2\right]^{3/2}}\,dx_1$$

FROM SYMMETRY $J_1(x_1) = J_2(x_1)$

$$J_2(x_2) = \underbrace{\varepsilon\sigma T^4}_{\substack{\text{=0 IN}\\ \text{FREDHOLM } \int \text{ OF } 1^{st} \text{ KIND}}} + \frac{(1-\varepsilon)h^2}{2}\int_0^W \frac{J_2(x_1)}{\left[h^2+(x_2-x_1)^2\right]^{3/2}}\,dx_1$$

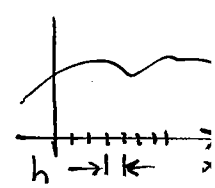
(FREDHOLM INTEGRAL OF 2^{nd} KIND

"KERNEL" – A FUNCTION OF x_1, x_2, WRITTEN AS $K(x_1, x_2)$

SOLUTION TECHNIQUES

TRAPEZOIDAL RULE

$$\int_a^b f(x)\,dx = \sum_{i=1}^{N} w_i f(x_i) = h\left[\tfrac{1}{2}f_1 + f_2 + \cdots f_{N-1} + \tfrac{1}{2}f_N\right]$$



SIMPSON'S RULE ALSO CAN USE.

MORE ACCURATELY

GAUSS QUADRATURE

$$\int_a^b w(x) f(x) \approx \sum_{i=1}^{N} w_i f(x_i)$$

w_i: WEIGHTING FACTOR

x_i: LEGENDRE POLYNOMIAL ROOT

$$\phi(x) = f(x) + \int_a^b K(x,x')\,\phi(x')\,dx'$$

$$\phi(x) = f(x) + \sum_{i=1}^{N} w_i\,\phi(x_i)\,K(x,x_i)$$