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VIBRATION

$$E_n = \hbar\omega\left(n+\frac{1}{2}\right) \quad , \quad n = 0, 1, 2 \ldots$$

RIGID ROTOR

$$E_\ell = \frac{\hbar^2}{2I_n}\,\ell(\ell+1)$$

COMBINED ROTATION – VIBRATION –

$$E_{n\ell} = \hbar\omega\left(n+\frac{1}{2}\right) + \frac{\hbar^2}{2I_n}\,\ell(\ell+1)$$

$\Delta n = 0 \implies$ ROTATION

$\Delta n = 1$

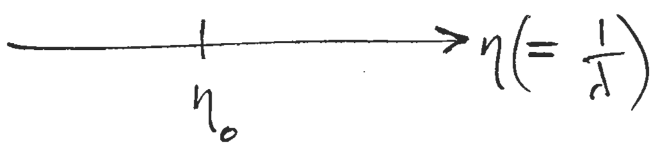
$\Delta \ell = -1$ ROTATION WITH EMISSION

$\Delta \ell = 0$

$\Delta \ell = 1$

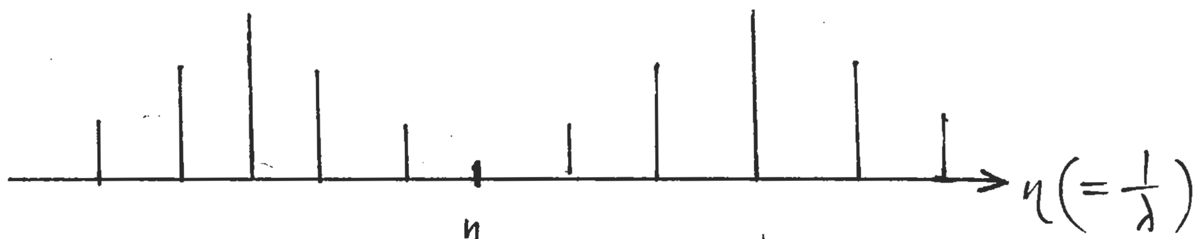
$$|m| \leq \ell$$

$$\hbar\omega_{Photon} = E_{n'\ell'} - E_{n\ell}$$



$$\eta_{\text{PHOTON}} = \eta_0 - (B_{n+1} + B_n)\ell + (B_{n+1} - B_n)\ell^2 \, , \quad \Delta \ell = -1$$

WHERE $B_n \propto \dfrac{\hbar^2}{2 I_n}$ $\left\{ \begin{array}{l} \text{USING } E = \hbar\omega \\ \eta = \dfrac{1}{\lambda} = \dfrac{\omega}{2\pi c_0} \end{array} \right\}$



INCREASES DUE TO...

DECREASES DUE TO LESSENING PROBABILITY

$\hookrightarrow P(E) \sim \exp\left(\dfrac{-E}{kT}\right)$

HARMONIC OSCILLATOR

$$\Delta x = x_0 e^{-i\omega t}$$

$$x_0 = \frac{E_0/m}{-\omega^2 + \omega_0^2 - i\gamma\omega}$$

$$\varepsilon_r = \frac{\varepsilon(\omega)}{\varepsilon_0} = 1 + \frac{Nq^2/m\varepsilon_0}{-\omega^2 + \omega_0^2 - i\gamma\omega} = \underbrace{\varepsilon_r'}_{\text{REAL}} + i \underbrace{\varepsilon_r''}_{\text{IMAG.}}$$

* SAY $\varepsilon_r' \approx 1$

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$$\varepsilon_r'' = \frac{Nq^2}{m\varepsilon_0} \frac{\gamma\omega}{(\omega^2-\omega_0^2)^2+(\gamma\omega)^2}$$

$$N = n + i\kappa = \sqrt{\varepsilon_r}$$

$$\Longrightarrow \quad 2n\kappa = \varepsilon_r''$$

$$\kappa = \frac{Nq^2}{2nm\varepsilon_0} \frac{\gamma\omega}{(\omega^2-\omega_0^2)^2+(\gamma\omega)^2}$$

$$\alpha = \frac{4\pi\kappa}{\lambda} = \frac{2\pi Nq^2}{mC_0\varepsilon_0} \frac{\gamma\omega^2}{\underbrace{(\omega^2-\omega_0^2)^2}+\gamma^2\omega^2}$$

$$(\omega^2-\omega_0^2)(\omega^2-\omega_0^2) = (\omega^2-\omega_0^2)(\omega+\omega_0)(\omega-\omega_0)$$

$$= (\omega-\omega_0)^2\underbrace{(\omega+\omega_0)^2}_{\omega\approx\omega_0 \;\approx (2\omega)^2}$$

$$=$$

$$= \frac{2\pi Nq^2}{4mC_0\varepsilon_0} \frac{\gamma}{(\omega-\omega_0)^2+(\gamma/2)^2}$$

$$\omega \to \eta, \quad \alpha \to \kappa_\eta$$

$$\Longrightarrow \boxed{\kappa_\eta = \frac{S}{\pi}\frac{b_c}{(\eta-\eta_0)^2+b_c^2}}$$

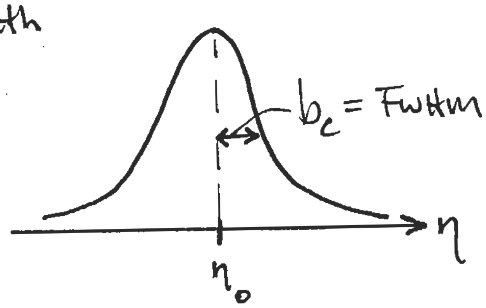
LORENTZIAN PROFILE

$S \equiv$ line strength

$b_c \equiv$ line width

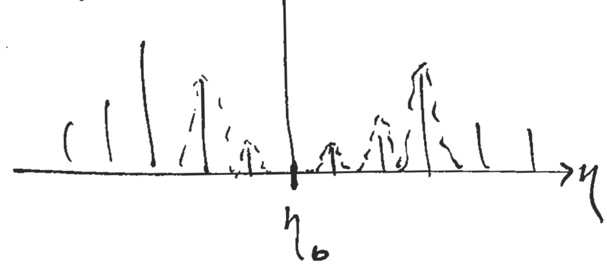
* RESULTS IN SPREADING

** CAN HAVE OVERLAP



"LORENTZIAN PROFILE"

$\Longrightarrow$



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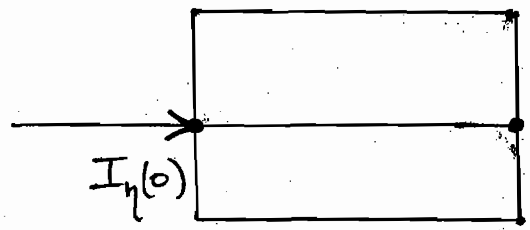
$$\frac{dI_\eta}{dX} = -\kappa_\eta I_\eta$$

$$I_\eta = I_\eta(0)\, e^{-\kappa_\eta X}$$



$$A_\eta = \frac{I_\eta(0) - I_\eta(s)}{I_\eta(0)} = 1 - e^{-\kappa_\eta X} \equiv \varepsilon_\eta$$

EMISSIVITY

(GAS EMISSIVE POWER)

$$W = \frac{\int_0^\infty \varepsilon_\eta I_{b\eta}\, d\eta}{\int_0^\infty I_{b\eta}\, d\eta} \cong \int_0^\infty \varepsilon_\eta\, d\eta = \underbrace{\int_0^\infty \left(1 - e^{-\kappa_\eta X}\right) d\eta}$$

DOESN'T CHANGE MUCH

... CAN BE DIFFICULT TO INTEGRATE IF ONE USES LORENTZ PROFILE

$X \equiv$ OPTICAL PATH LENGTH

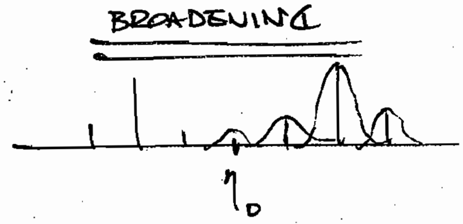
κ_η / ρ_a

SPECIES gas "a"

$X \cdot \rho_a$ — MASS OPTICAL PATH LENGTH

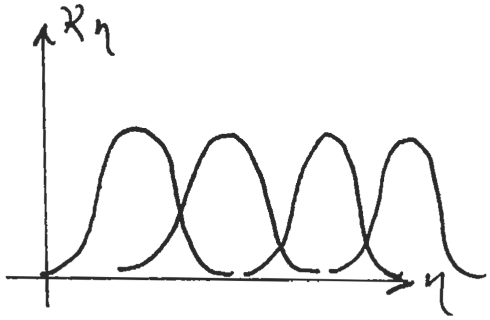
$$W = \begin{cases} S \cdot X & (\kappa_\eta X \ll 1) \\ 2\sqrt{S \cdot X \cdot b_c} & (\kappa_\eta X \gg 1) \end{cases}$$

BROADENING

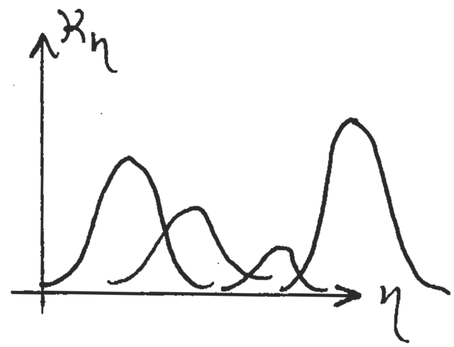


MODELS FOR SMOOTH FUNCTIONS OF EMISSIVITY

$\bar{\kappa}_\eta$ – NARROW BAND MODELS

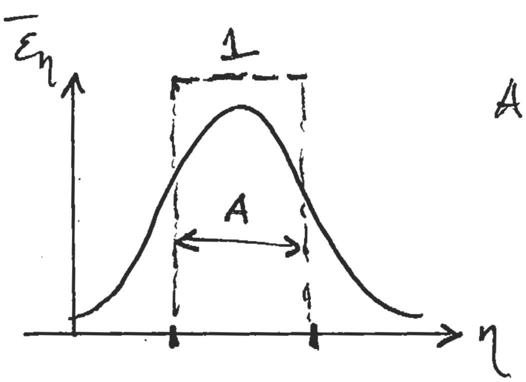


ELASSER MODEL



GOODY MODEL

"ADDING UP EACH LINE"



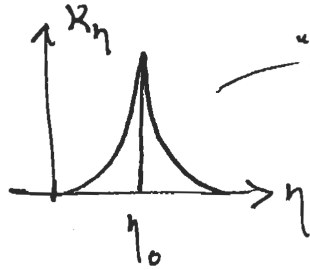
$A \equiv$ EFFECTIVE BANDWIDTH

$$A = \int_{\text{BAND}} \bar{\varepsilon}_\eta \, d\eta$$

$$\bar{\varepsilon}_\eta = 1 - e^{-\bar{\kappa}_\eta X}$$

"WIDE-BAND MODEL"

WHERE $\bar{\kappa}_\eta$



"EDWARDS EXPONENTIAL FUNCTION"

** MOST OF THESE MODELS CARRY ~20% UNCERTAINTY

BROADENING MECHANISMS

(a) COLLISIONAL BROADENING

(b) NATURAL BROADENING $\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$

(c) DOPPLER EFFECT ("DOPPLER SHIFT") $\eta_{o,b} = \eta_{em}\left(1 + \frac{\vec{V}\cdot\hat{s}}{c}\right)$

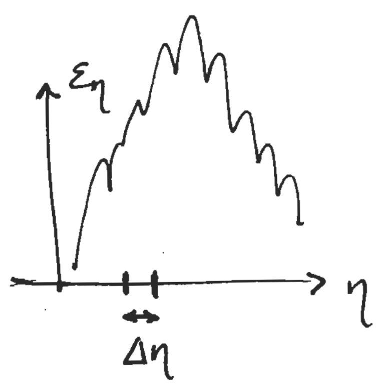
$$P(v) = \left(\frac{m}{2\pi kT}\right)^{3/2} 4\pi v^2 \exp\left(-\frac{mv^2}{2kT}\right)$$

* DOPPLER PROFILE IS EXPONENTIAL DECAY, NOT LORENTZIAN SHA

ABSORPTION COEFF. $K_\eta = \sum_j K_{\eta_i}$

"PATH LENGTH"

$$\varepsilon_\eta = 1 - \exp\left(-X \cdot \sum_j K_{\eta_j}\right)$$



! SMOOTH OUT

AVG. VALUE: $\overline{\varepsilon_\eta} = \frac{1}{\Delta\eta}\int_{\Delta\eta} \varepsilon_\eta \, d\eta$