

$$\epsilon_r = 1 + \frac{\omega_p^2}{\omega^2 - \omega_0^2 + i\gamma\omega} = \frac{\vec{P}}{\vec{D}}$$

lorentz model

$$\frac{1}{\mu} \nabla \times \vec{B} = \frac{\partial \vec{D}}{\partial t} + \vec{J}$$

$$\vec{D} = \vec{D}_0 e^{-i\omega t}$$

$$\vec{J} = \sigma \vec{E}$$

$$\frac{\nabla \times \vec{B}}{\mu} = \frac{\partial}{\partial t} \left\{ \epsilon_0 \left(1 + \chi - \frac{\sigma}{i\omega\epsilon_0} \right) \vec{E} \right\}$$

Free electron

ϵ_r

$$A = \frac{-eE_0/m}{-\omega^2 + i\gamma\omega} \quad \text{No } \omega_0$$

$$\vec{J} = -ne(-i\omega) A e^{-i\omega t}$$

$$= i\omega \frac{e^2 n/m}{\omega^2 + i\gamma\omega} \vec{E}$$

$$\epsilon_r = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega} \quad \text{due to damping}$$

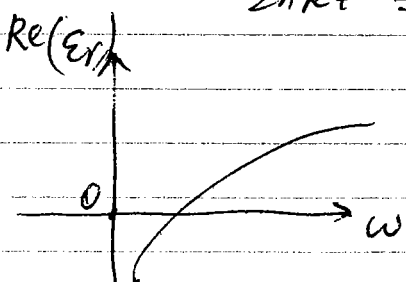
Drude model

$$\omega_p^2 = \frac{e^2 n_e}{m \epsilon_0} \quad \text{— plasma frequency}$$

$$N = n + ik = \sqrt{\epsilon_r}$$

$$n^2 = k^2 = \text{Re}(\epsilon_r)$$

$$2nk\phi = \text{Im}(\epsilon_r)$$



$\epsilon_r < 0$

$\omega \uparrow \quad \epsilon_r \rightarrow 1$

$R = 0$

If N is imaginary number

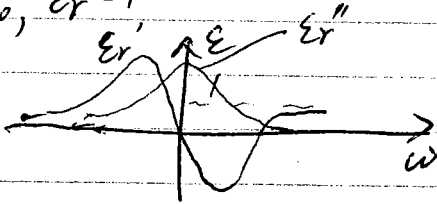
$$R = \left| \frac{N-1}{N+1} \right|^2$$

$$= \left| \frac{ai - 1}{ai + 1} \right|^2 = 1 \quad \text{for metal}$$

Lorentz model

$$\omega \ll \omega_0, \epsilon_r = 1 - \frac{\omega_p^2}{\omega_0^2}$$

$$\omega \gg \omega_0, \epsilon_r = 1$$



Kramer-Kronig relation $\epsilon_r', \epsilon_r''$

$$\epsilon_r = \epsilon_0 + \left(\sum_j \frac{\omega_{pj}^2}{\omega^2 - \omega_{0j}^2 + i\gamma_j \omega} \right) - \frac{\omega_p^2}{\omega^2 + i\gamma \omega} = \epsilon_0 + () + ()$$

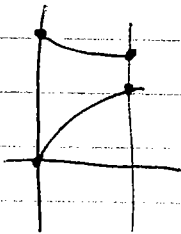
↑ oscillator ↑ free electron

FTIR

Fourier transform IR

If $\omega_{01}, \omega_{02}, \omega_{03}$ are separated.

Not plasma frequency, $\omega_p = \frac{Ne^2}{\epsilon_0 m}$ related to N in Lorentz model



$$1 + \frac{\omega_{p1}^2}{\omega^2 - \omega_{01}^2} + \frac{\omega_{p2}^2}{\omega^2 - \omega_{02}^2} - \frac{\omega_p^2}{\omega_{01}^2}$$

$\omega \rightarrow 0$

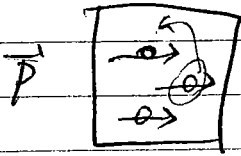
$\epsilon_0 \neq 1$

Local field

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

↑
external

\vec{E}_{local}



$$\vec{P} \sim \vec{E}_{local}$$

$$\vec{E}_{local} = \vec{E}_{ex} + \vec{E}_{induced}$$

Superposition ↑ other dipoles

$$\vec{E}_{induced} = \vec{P} / 3\epsilon_0$$

↑
local

$$\vec{E}_{local} = \frac{\epsilon_0 \epsilon_x + 2\epsilon_0}{3\epsilon_0} \vec{E}_{ex}$$

⇓

$$\frac{\epsilon_x - 1}{\epsilon_x + 2} = \frac{1}{3} \sum_j \frac{\omega_{pj}^2}{\omega^2 - \omega_{oj}^2 + i\gamma_j \omega}$$

Clausius - moso ~~ei~~ relation

Gas properties:

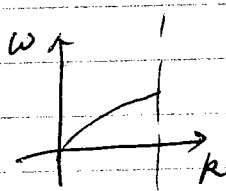
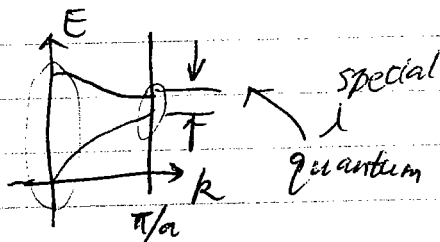
atom { translation $kT \sim \hbar \omega_p$
 electronic $\frac{13.6 \text{ eV}}{n^2}$ spin (l, m, \uparrow)

molecules { vibration $E_n = \hbar \omega_v (n + \frac{1}{2})$, $\omega_v = \sqrt{\frac{k}{m}}$
 or rotation $E_l = \frac{\hbar^2}{2I} l(l+1)$, $|m| \leq l$

two degrees

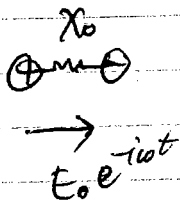
two ~~number~~ number
 m, l

Solid { electronic
phonon



$$\frac{P}{\hbar} = \frac{1}{\lambda}$$

metal, short λ , transparent



$$m \frac{d^2 x}{dt^2} = -e E_0 - k(x - x_0) - \beta \frac{dx}{dt}$$

↑ sign corrected

$$\Delta x = x - x_0 = A e^{-i\omega t}$$

↓ substitute by itself

$$\frac{d^2 \Delta x}{dt^2} + \gamma \frac{d\Delta x}{dt} + \omega_0^2 \Delta x = -\frac{e E_0}{m} e^{-i\omega t}$$

$$A = \frac{-e E_0 / m}{-\omega^2 + \omega_0^2 - i r \omega}$$

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P}$$

$$= \epsilon_0 (1 + \chi) \vec{E}$$

↓
 ϵ_r

$$\vec{P} = \frac{e N}{m} \frac{e E_0}{\omega^2 - \omega_0^2 + i r \omega} \vec{E}$$

$$= \epsilon_0 \chi \vec{E}$$