2.58 HW3 Solutions

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Prob 2.6

For this problem, we only consider normal incidence:

(a) m_1 air
m_2 MgO
m_3 Pt

The reflection coefficient of a slab is given by:

$$r = \frac{r_{12} + r_{23} e^{2i\varphi_2}}{1 + r_{12} r_{23} e^{2i\varphi_2}}, \quad \text{where } r_{12} = \frac{m_1 - m_2}{m_1 + m_2}$$

$$r_{23} = \frac{m_2 - m_3}{m_2 + m_3}$$

$$\varphi_2 = \frac{2\pi m_2 d}{\lambda_0}$$

The reflectivity is: $R = |r|^2 = 0.614$

(b) For the average reflectivity, we can use eqn. (2.128)

$$R = \rho_{12} + \frac{\rho_{23}(1-\rho_{12})^2 e^{-2\kappa_2 d}}{1 - \rho_{12}\rho_{23} e^{-2\kappa_2 d}}$$

where $\rho_{12} = r_{12}^2$, $\rho_{23} = r_{23}^2$, $\kappa_2 = \frac{4\pi k_2}{\lambda_0}$

$$\Rightarrow \quad d = -\frac{\ln\left\{\frac{R - \rho_{12}}{\rho_{23}[(R-\rho_{12})\rho_{12} + (1-\rho_{12})^2]}\right\}}{2\kappa_2}$$

When $R = 0.4$, $d = 404.6\ \mu m$

For such a thick slab, the interference effects will rarely be observed.

Prob 3.31

For a single slab of glass, eqn. (3.89) gives

$$R_1 = \rho_{12} + \frac{\rho_{23}(1-\rho_{12})^2 \tau^2}{1-\rho_{12}\rho_{23}\tau^2}, \quad \text{where} \quad \rho_{12} = \rho_{23} = \left| \frac{m_{air} - m_{glass}}{m_{air} + m_{glass}} \right|^2$$

$$\tau = e^{-\frac{4\pi k_2 d}{\lambda_0}}$$

At $\lambda_0 = 0.6\ \mu m$, $\rho_{12} = \rho_{23} = 0.0422$, $\tau = 0.9387$

$\Rightarrow R_1 = 0.0764$

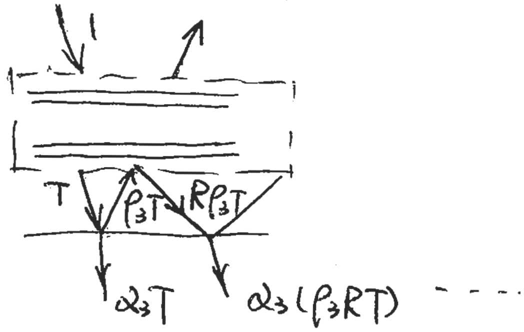
For the double-layer glass, apply eqns. 3.100 and 3.101

$$R = R_1 + \frac{T_1^2 R_1}{1-R_1^2}, \quad T = \frac{T_1^2}{1-R_1^2}$$

where $T_1 = \dfrac{(1-\rho_{12})(1-\rho_{23})\tau}{1-\rho_{12}\rho_{23}\tau^2} = 0.8625$

$$\Rightarrow R = 0.0764 + \frac{0.8625 \times 0.0764}{1 - 0.0764^2} = 0.1335$$

$$T = \frac{0.8625^2}{1-0.0764^2} = 0.7483$$



The effective absorptance of the solar collector is:

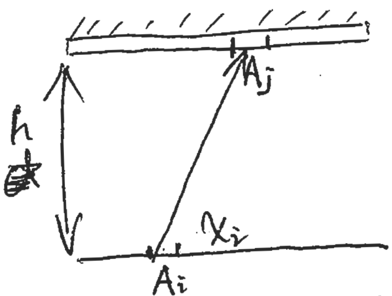
$$\alpha_{eff} = \alpha_3 T + (\rho_3 R)\alpha_3 T + (\rho_3 R)^2 \alpha_3 T + \cdots$$

$$= \frac{\alpha_3 T}{1-\rho_3 R} = \frac{0.90 \times 0.7483}{1 - 0.10 \times 0.1335} \approx 0.683$$

$\Rightarrow$ 68.3% of the normally incident solar radiation is absorbed.

Monte Carlo :

Prob 5.34



For this particular problem, it is relatively easy to set up the scheme of Monte Carlo simulation.

(1) First of all, both surfaces are black which eliminates the trouble of taking into account the spectral dependence.

(2) Secondly, the emission from both surfaces is diffuse which allows us to use simple correlation to generate a bundle:

$$\begin{cases} \theta = \sin^{-1}\sqrt{R_\theta} \\ \varphi = 2\pi R_\varphi \end{cases}$$

Then we can determine the location where the bundle hits:

$$X_j = X_i + r\sin\theta\cos\varphi = X_i + \frac{h}{\cos\theta}\sin\theta\cos\varphi = X_i + h\tan\theta\cos$$

(3) If we use N_{ij} to represent the numble of bundles emitted from segment i that reach segment j, we can write energy balance for each of the segment:

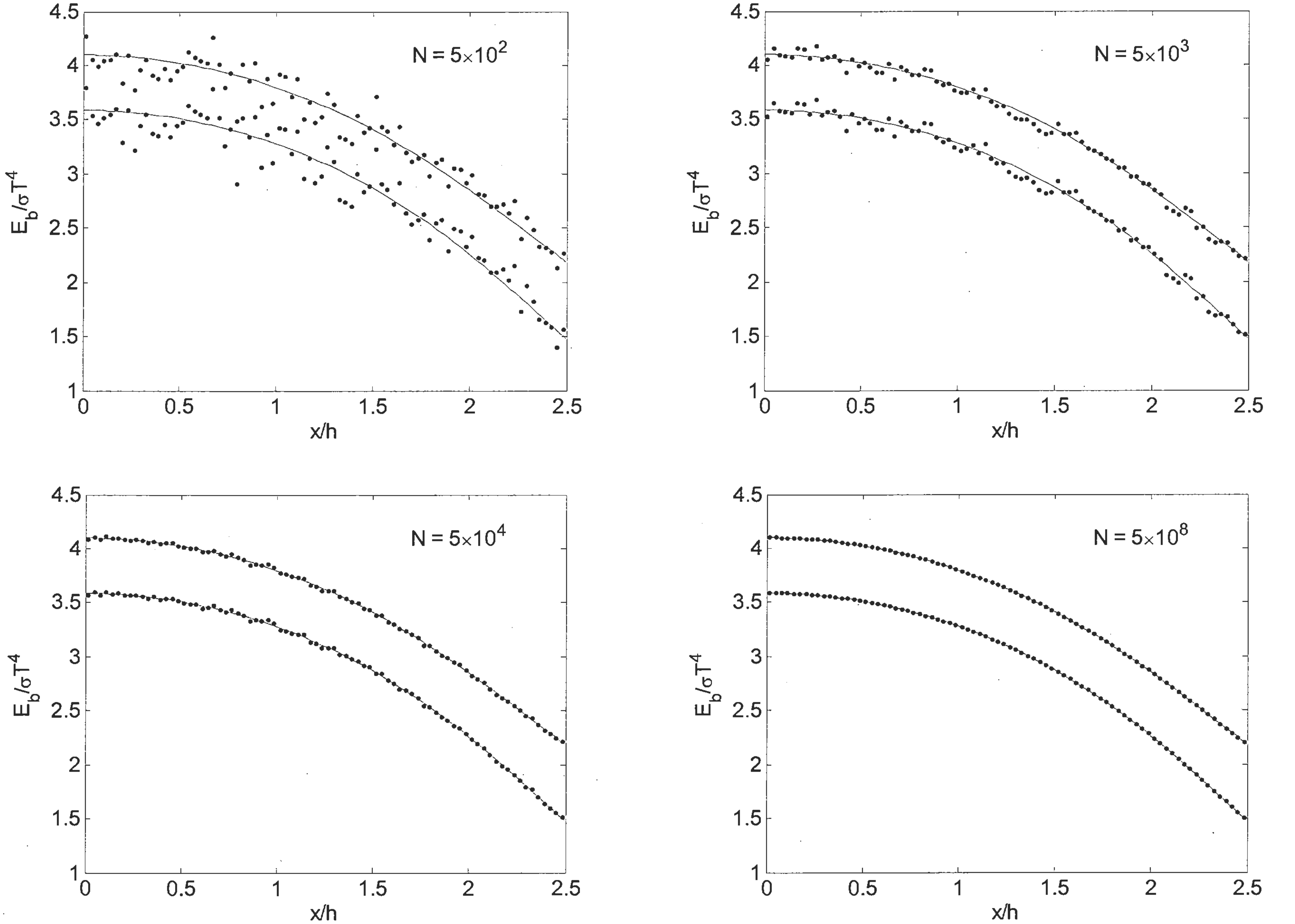
$$q_i = e_{bi} - \alpha_i \sum e_{bj}\frac{N_{ji}}{N_j}$$

Where $\alpha_i = 1$ and N_j is the total bundle emitted from j.

The next step is to solve the group of equations we obtain in the above, which can either be done by iteration or matrix elimination (Gaussian, LU, etc.)

```
program mc1
implicit none
integer(2), parameter::Nd=80
integer(4),parameter::ns=50000
integer(2)::i,j,nic,fid1,fid2
integer(4)::S12(Nd,Nd),iseed1=323421,n
real(8), parameter::PI=3.14159265358979d0,q1=1.d0,q2=0.d0,L=5.D0,EPS=1.D-8
real(8)::x1,x2,xrand,xic,xmin,xmax,dx,eb1(Nd),eb2(Nd),tan_theta, &
         sin_theta2,cos_phi,ratio(Nd,Nd),ebtemp,delta,deltatemp

xmin=0.d0
xmax=L/2.d0
dx=(xmax-xmin)/dfloti(Nd)
S12=0
fid1=7
fid2=9

open(unit=fid1,file='ebmc1.txt',status='replace',action='write',&
     access='sequential')
open(unit=fid2,file='ebmc2.txt',status='replace',action='write',&
     access='sequential')

do i=1,Nd
   x1=xmin+dx*dfloti(i-1)
   do n=1,ns
      xrand=x1+dx*ran(iseed1)
      sin_theta2=ran(iseed1)
      tan_theta=dsqrt(sin_theta2/(1.d0-sin_theta2))
      cos_phi=dcos(2.d0*PI*ran(iseed1))
      xic=dabs(tan_theta*cos_phi+xrand)
      if(xic.le.xmax) then
        nic=floor(sngl(xic/dx))+1
        if(nic.le.Nd) S12(i,nic)=S12(i,nic)+1
      end if
   end do
end do
ratio=dflotj(S12)/dflotj(ns)

eb1=1.d0
eb2=0.5D0
ebtemp=10.d0
delta=1.D0
do while(delta>EPS)
  delta=0.d0
  do i=1,Nd
     ebtemp=eb1(i)
     eb1(i)=q1
     do j=1,Nd
        eb1(i)=eb1(i)+eb2(j)*ratio(j,i)
     end do
     deltatemp=dabs(ebtemp-eb1(i))
     if(deltatemp>delta) delta=deltatemp
  end do
  do i=1,Nd
     ebtemp=eb2(i)
     eb2(i)=q2
     do j=1,Nd
        eb2(i)=eb2(i)+eb1(j)*ratio(j,i)
     end do
     deltatemp=dabs(ebtemp-eb2(i))
     if(deltatemp>delta) delta=deltatemp
  end do
end do

do i=1,Nd
   write(fid1,100) xmin+dx*dfloti(i)-dx/2.d0,eb1(i)
   write(fid2,100) xmin+dx*dfloti(i)-dx/2.d0,eb2(i)
end do

close(fid1)
close(fid2)
100 format(2F8.4)
end program mc1
```

Monte Carlo Simulation Results (L/h = 5)



Prob 4.

For TM wave,

$$R_{TM} = \left| \frac{n_1 \cos\theta_2 - n_2 \cos\theta_1}{n_1 \cos\theta_2 + n_2 \cos\theta_1} \right|^2,$$

$$T_{TM} = 1 - R_{TM}$$

$$\varepsilon_{TM} = 1 - R_{TM}$$

For TE wave,

$$R_{TE} = \left| \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} \right|^2$$

$$T_{TE} = 1 - R_{TE}$$

$$\varepsilon_{TE} = 1 - R_{TE}$$

Snell's law gives

$$\sin\theta_2 = \frac{n_1}{n_2} \sin\theta_1$$

(a) ~~At~~ $\theta_1 = 0°$

$R_{TM} = R_{TE} = 0.3485$,

$T_{TM} = T_{TE} = 0.6515$

$\varepsilon_{TM} = \varepsilon_{TE} = 0.6515$

(b) $\theta_1 = 30°$

$R_{TM} = 0.2964$, $T_{TM} = 0.7036$, $\varepsilon_{TM} = 0.7036$

$R_{TE} = 0.4003$, $T_{TE} = 0.5997$, $\varepsilon_{TE} = 0.5997$

(C) $\theta_1 = 60°$

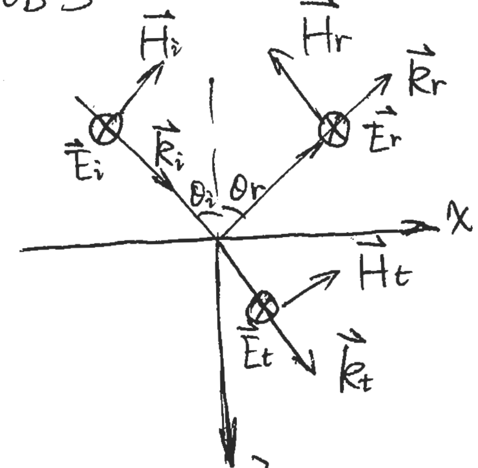
$R_{TM} = 0.1098$, $T_{TM} = 0.8902$, $\varepsilon_{TM} = 0.8902$

$R_{TE} = 0.5877$, $T_{TE} = 0.4123$, $\varepsilon_{TE} = 0.4123$

The penetration depth:

$$l = \frac{\lambda_0}{4\pi k} = \frac{0.63}{4\pi \times 0.019} \approx 2.64\ \mu m$$

Prob 5



$$\vec{H}_i = \vec{H}_{io} \exp\left[-i\omega\left(t - \frac{N_1 x\sin\theta_i + N_1 z\cos\theta_i}{C_o}\right)\right]$$

$$\vec{H}_r = \vec{H}_{ro} \exp\left[-i\omega\left(t - \frac{N_1 x\sin\theta_r - N_1 z\cos\theta_r}{C_o}\right)\right]$$

$$\vec{H}_t = \vec{H}_{to} \exp\left[-i\omega\left(t - \frac{N_2 x\sin\theta_t + N_2 z\cos\theta_t}{C_o}\right)\right]$$

$$\begin{cases} \vec{E}_i = -\dfrac{\vec{k}_i \times \vec{H}_i}{\omega\varepsilon_1}, \quad \vec{E}_r = -\dfrac{\vec{k}_r \times \vec{H}_r}{\omega\varepsilon_1} \\ \vec{E}_t = -\dfrac{\vec{k}_t \times \vec{H}_t}{\omega\varepsilon_2} \end{cases}$$

Match the BC for $\vec{H}$ field at $z=0$: $\hat{z}\times[(\vec{H}_i+\vec{H}_r)-\vec{H}_t]=0$

$$\Rightarrow \quad H_{io}\cos\theta_i \exp\left(i\omega\frac{N_1 x\sin\theta_i}{C_o}\right) - H_{ro}\cos\theta_r \exp\left(i\omega\frac{N_1 x\sin\theta_r}{C_o}\right)$$

$$= H_{to}\cos\theta_t \exp\left(i\omega\frac{N_2 x\sin\theta_t}{C_o}\right) \quad \text{--- (1)}$$

The above equation holds for arbitrary x

$$\Rightarrow \quad N_1\sin\theta_i = N_1\sin\theta_r = N_2\sin\theta_t$$

$$\Rightarrow \quad \theta_i = \theta_r, \quad N_1\sin\theta_i = N_2\sin\theta_t \quad \text{--(2)} \quad \text{(Snell's law)}$$

Substitute (2) into (1) to yield

$$H_{io}\cos\theta_i - H_{ro}\cos\theta_i = H_{to}\cos\theta_t \quad \text{--- (3)}$$

Similaly, we can apply BC for $\vec{E}$ field at $z=0$: $\hat{z}\times(\vec{E}_i+\vec{E}_r - \vec{E}_t)=$

$$\Rightarrow \quad E_{io}+E_{ro} = E_{to} \quad \text{--- (4)}$$

Since $\vec{E} = \dfrac{\vec{k}\times\vec{H}}{\omega\varepsilon}$, $H = \dfrac{\omega\varepsilon E}{k}$, eqn. (3) can be rewritten as :

$$N_1 E_{io}\cos\theta_i - N_1 E_{ro}\cos\theta_i = N_2 E_{to}\cos\theta_t \quad \text{--(5)}$$

Equations (4) and (5) yield

$$r_{TE} = \frac{E_{ro}}{E_{io}} = \frac{N_1 \cos\theta_i - N_2 \cos\theta_t}{N_1 \cos\theta_i + N_2 \cos\theta_t}$$

$$\tau_{TE} = \frac{E_{to}}{E_{io}} = \frac{2N_1 \cos\theta_i}{N_2 \cos\theta_t + N_1 \cos\theta_i}$$

The reflectivity and transmissivity are defined as the ratio as energy flux:

$$R_{TE} = \left| \frac{\frac{1}{2}\mathrm{Re}(\vec{P}_r \cdot \hat{z})}{\frac{1}{2}\mathrm{Re}(\vec{P}_i \cdot \hat{z})} \right| = \left| \frac{E_{ro} H_{ro}^*}{E_{io} H_{io}^*} \right| = \left| \frac{E_{ro}}{E_{io}} \right|^2 = |r_{TE}|^2$$

$$T_{TE} = \left| \frac{\frac{1}{2}\mathrm{Re}(\vec{P}_t \cdot \hat{z})}{\frac{1}{2}\mathrm{Re}(\vec{P}_i \cdot \hat{z})} \right| = \left| \frac{\mathrm{Re}(E_{to} H_{to}^* \cos\theta_t)}{\mathrm{Re}(E_{io} H_{io}^* \cos\theta_i)} \right|$$

$$= \left| \frac{\mathrm{Re}(E_{to} E_{to}^* (N_2 \cos\theta_t))}{\mathrm{Re}(E_{io} E_{io}^* (N_1 \cos\theta_i)^*)} \right| = \frac{\mathrm{Re}(N_2 \cos\theta_t)}{\mathrm{Re}(N_1 \cos\theta_i)} |\tau_{TE}|^2$$

Prob.6

$$T = 1 - R = 1 - \left| \frac{r_{12} + r_{23} e^{2i\varphi_2}}{1 + r_{12} r_{23} e^{2i\varphi_2}} \right|^2$$

$$\varphi_2 = \frac{2\pi n_2 d \cos\theta_2}{\lambda_0}$$

For TM wave, $r_{12} = -r_{23} = \dfrac{n_2 \cos\theta_i - n_1 \cos\theta_2}{n_2 \cos\theta_i + n_1 \cos\theta_2}$

$\sin\theta_2 = \dfrac{n_1}{n_2} \sin\theta_1$, $n_1 = 1.46$, $n_2 = 1$