13.42 HW #3 SOLUTIONS

1. THE TOTAL NUMBER OF POSSIBLE 5-CARD HANDS IS:

$$\binom{52}{5} = \frac{52!}{5!(52-5)!} = \frac{52!}{5!\,47!} = 2,598,900$$

THE NUMBER OF POSSIBLE 5-CARD HANDS THAT ARE ALL HEARTS IS:

$$\underbrace{\binom{13}{5}\binom{39}{\emptyset}} = \frac{13!}{5!\,8!}\,\frac{39!}{0!\,39!} = 1,287$$

(# OF WAYS WHICH 5 HEARTS MAY BE SELECTED FROM 13 HEARTS × # OF WAYS ∅ NON-HEARTS MAY BE SELECTED FROM 39 NON-HEARTS.)

THEN p(5-CARD HAND W/ EVERY CARD A HEART) =

$$= \frac{1,287}{2,598,900}$$

$$= \underline{4.952 \times 10^{-4}}.$$

2. a. $p(A \cup B) = p(A) + p(B) - p(B \cap A)$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

$$= \underline{\frac{4}{13}}$$

b. $p(B \cap D) = p(\emptyset) = \underline{0.}$

c. $p(A \cap C) = \underline{\frac{3}{52}}$

d. $p(B \cup D) = p(B) + p(D) - p(B \cap D)$

$= \frac{13}{52} + \frac{26}{52} - 0$

$= \underline{\frac{39}{52}}$

e. $p(C \cup D) = p(C) + p(D) - p(C \cap D)$

$= \frac{39}{52} + \frac{26}{52} - \frac{26}{52}$

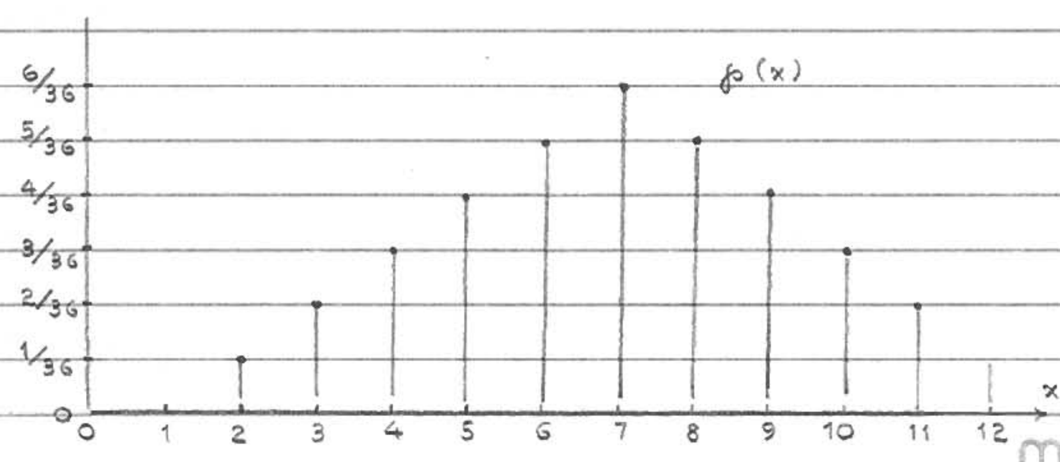
$= \underline{\frac{39}{52}}$

3. POSSIBLE OUTCOMES:

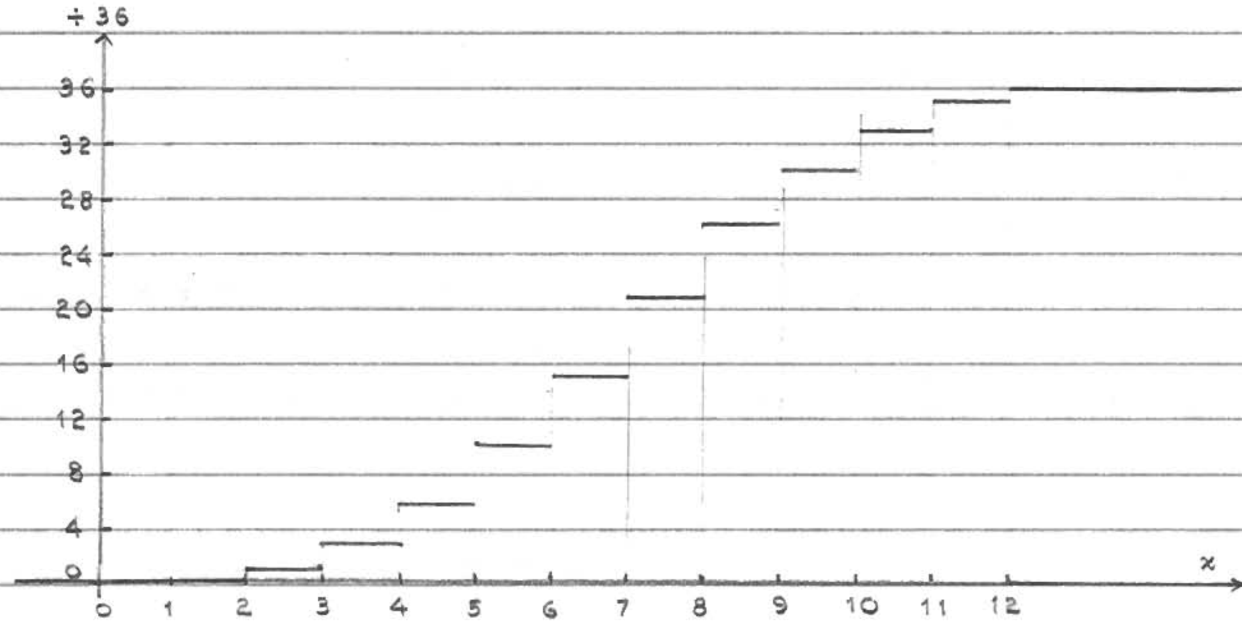
$$\begin{matrix} (1,1) & (1,2) & \cdots & (1,6) \\ (2,1) & (2,2) & \cdots & (2,6) \\ \vdots & \vdots & \ddots & \vdots \\ (6,1) & (6,2) & \cdots & (6,6) \end{matrix} \Longrightarrow X \longrightarrow \begin{pmatrix} 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 6 & 7 & 8 & 9 \\ 5 & 6 & 7 & 8 & 9 & 10 \\ 6 & 7 & 8 & 9 & 10 & 11 \\ 7 & 8 & 9 & 10 & 11 & 12 \end{pmatrix}$$

THE PROBABILITY OF EACH OUTCOME IS $\frac{1}{36}$.
THEN, FOR EXAMPLE, $p(x = 5) = p(X = 5) = 4 \cdot \frac{1}{36}$.

a. $p(x)$ IS THEN:



b. AND $P(x)$ IS:



c. $\mu_x = E\{X\} = \sum x\,p(x)$

$$= 2\left(\tfrac{1}{36}\right) + 3\left(\tfrac{2}{36}\right) + 4\left(\tfrac{3}{36}\right) + 5\left(\tfrac{4}{36}\right) +$$

$$+ 6\left(\tfrac{5}{36}\right) + 7\left(\tfrac{6}{36}\right) + 8\left(\tfrac{5}{36}\right) + 9\left(\tfrac{4}{36}\right) +$$

$$+ 10\left(\tfrac{3}{36}\right) + 11\left(\tfrac{2}{36}\right) + 12\left(\tfrac{1}{36}\right)$$

$$= \underline{7.0000}$$

$$\sigma_x^2 = E\{(X - \mu_x)^2\}$$

$$= \sum (x - \mu_x)^2 p(x)$$

$$= \underline{5.8333}$$

$$\sigma_x = \underline{2.4152}$$

4. a. $p(1 \le X < 1.5) = P(1.5) - P(1)$

$$= \tfrac{1}{2}(1.5) - \tfrac{1}{2}(1)$$

$$= \underline{0.25}$$

b. $p(X \le 1/4) = \left(\tfrac{1}{4}\right)^2 = \underline{1/16}.$

c. $p(X \ge 4) = p(4 \le X < \infty) = P(\infty) - P(4) = 1 - 1 = \underline{0}.$

5. $\mu_x = E\{X\} = \int_{-\infty}^{\infty} x\, p(x)\, dx$

$$= \int_0^1 x\, 2x\, dx$$

$$= 2\left[\tfrac{1}{3}x^3\right]_0^1$$

$$\underline{= 2/3}$$

$\sigma_x^2 = E\{(X - \mu_x)^2\} = \int_{-\infty}^{\infty} (x - \mu_x)^2\, p(x)\, dx$

$$= \int_0^1 (x - 2/3)^2\, 2x\, dx$$

$$= \int_0^1 \left(x^2 - \tfrac{4}{3}x + 4/9\right) 2x\, dx$$

$$= 2\int_0^1 \left(x^3 - \tfrac{4}{3}x^2 + 4/9\, x\right) dx$$

$$= 2\left[\tfrac{1}{4}x^4 - 4/9\, x^3 + 2/9\, x^2\right]_0^1$$

$$= 1/18.$$

$\sigma_x = \underline{\sqrt{1/18}}\,.$