

13.42 Lecture:
Vortex Induced Vibrations



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21 April 2005

Offshore Platform



Courtesy of U.S. DOE.

Fixed Rigs

Tension Leg Platforms

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Spar Platforms

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Genesis Spar Platform

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Please See: <http://www.offshore-technology.com/projects/genesis/>.

VIV Catastrophe

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If neglected in design, vortex induced vibrations can prove catastrophic to structures, as they did in the case of the Tacoma Narrows Bridge in 1940.

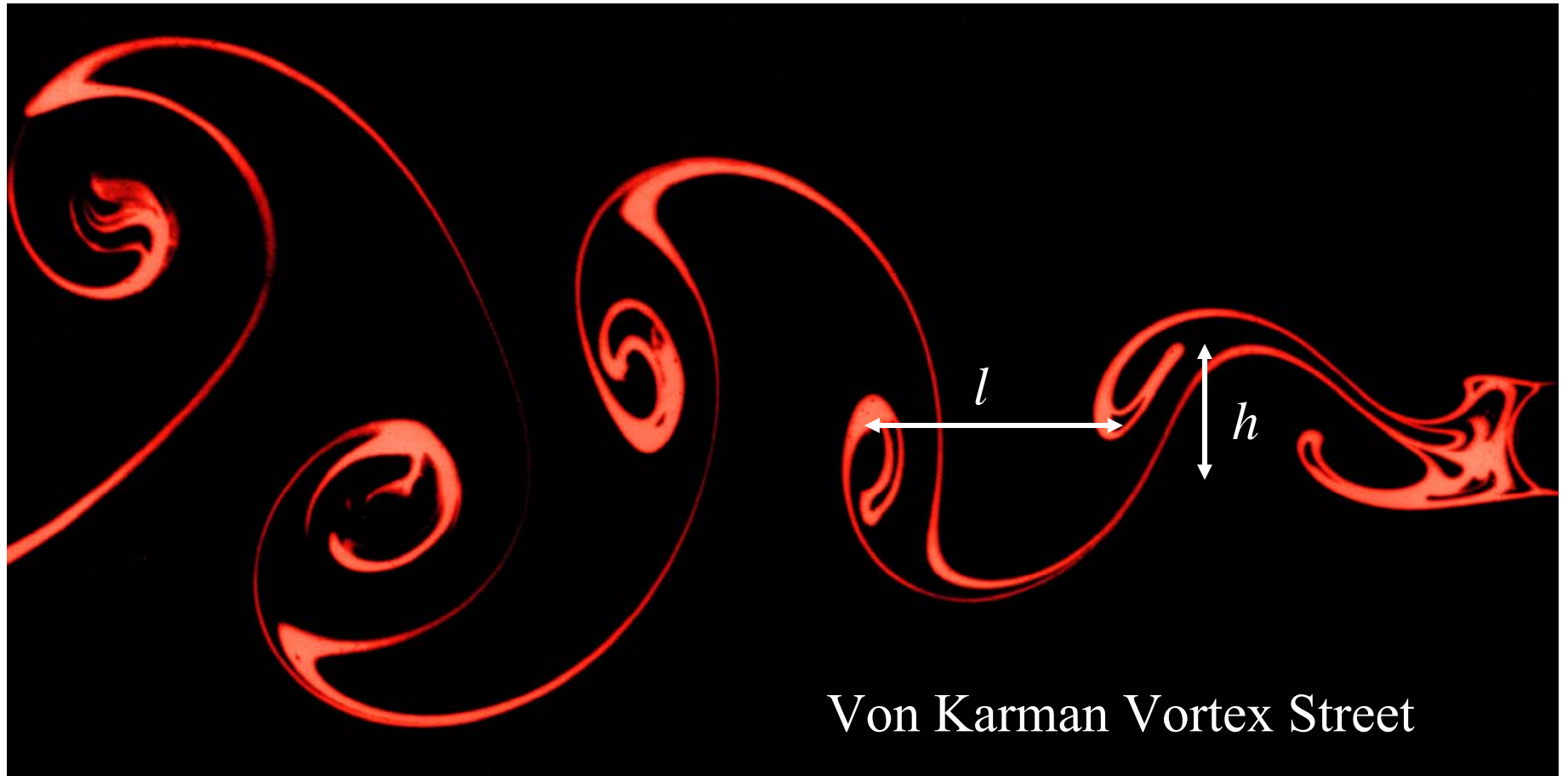
John Hancock Building

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“In another city, the John Hancock tower wouldn't be anything special -- just another reflective glass box in the crowd. But because of the way Boston and the rest of New England has grown up architecturally, this "70's modern" building stands out from the rest. Instead of being colonial, it breaks new ground. Instead of being quaint, it soars and imposes itself on the skyline. And instead of being white like so many buildings in the region, this one defies the local conventional wisdom and goes for black. For these reasons and more the people of Boston have fallen in love with the 790-foot monster looming as the tallest building in New England at the time of its completion. In the mid-1990's, The Boston Globe polled local architects who rated it the city's third best architectural structure. Much like Boston's well-loved baseball team, the building has had a rough past, but still perseveres, coming back stronger to win the hearts of its fans. The trouble began early on. During construction of the foundation the sides of the pit collapsed, nearly sucking Trinity Church into the hole. Then in late January, 1973 construction was still underway when a winter storm rolled into town and a 500-pound window leapt from the tower and smashed itself to bits on the ground below. Another followed. Then another. Within a few weeks, more than 65 of the building's 10,344 panes of glass committed suicide, their crystalline essence piling up in a roped-off area surrounding the building. The people of Bean Town have always been willing to kick a brother when he's down, and started calling the tower the Plywood Palace because of the black-painted pieces of wood covering more than an acre of its façade. Some people thought the building was swaying too much in the wind, and causing the windows to pop out. Some thought the foundation had shifted and it was putting stress of the structural geometry. It turns out the culprit was nothing more than the lead solder running along the window frame. It was too stiff to deal with the kind of vibrations that happen every day in thousands of office buildings around the world. So when John Hancock Tower swayed with the wind, or sighed with the temperature, the windows didn't and eventually cracked and plummeted to Earth. It cost \$7,000,000.00 to replace all of those panes of glass. The good news is, you can own a genuine piece of the skyscraper. According to the Globe, the undamaged sheets were sold off for use as table tops, so start combing those garage sales. For any other skyscraper, the hardship would end there. But the Hancock building continued to suffer indignities. The last, and most ominous, was revealed by Bruno Thurlimann, a Swiss engineer who determined that the building's natural sway period was dangerously close to the period of its torsion. The result was that instead of swaying back-and-forth like a in the wind like a metronome, it bent in the middle, like a cobra. The solution was putting a pair of 300-ton tuned mass dampeners on the 58-th floor. The same engineer also determined that while the \$3,000,000.00 mass dampeners would keep the building from twisting itself apart, the force of the wind could still knock it over. So 1,500 tons of steel braces were used to stiffen the tower and the Hancock building's final architectural indignity was surmounted.”

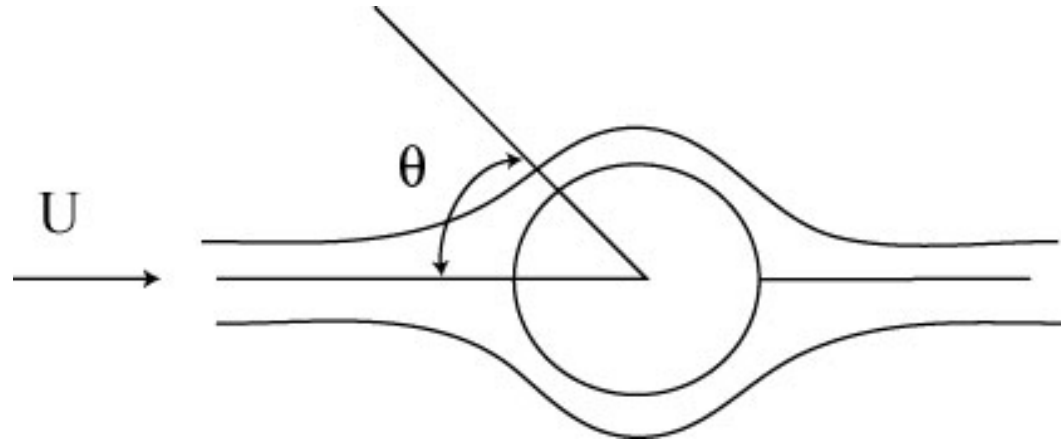
Courtesy of Artefaqs Corporation. Used with permission.
Source: Glass Steel and Stone, <http://www.glassteelandstone.com>.

Classical Vortex Shedding



Alternately shed opposite signed vortices

Potential Flow

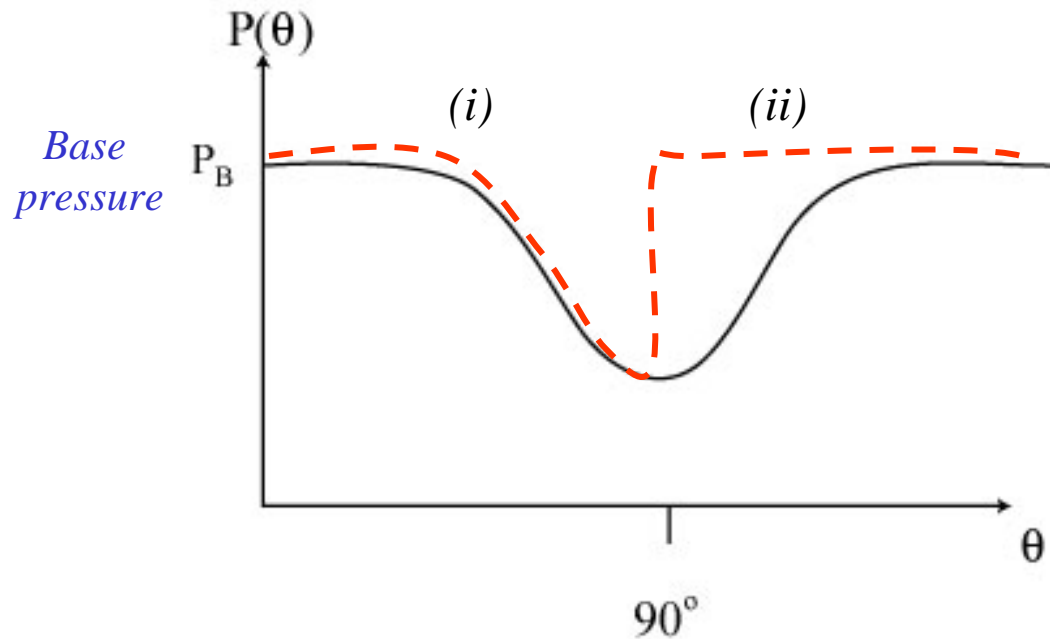


$$U(\theta) = 2U_{\infty} \sin\theta$$

$$P(\theta) = \frac{1}{2} \rho U(\theta)^2 = P_{\infty} + \frac{1}{2} \rho U_{\infty}^2$$

$$C_p = \frac{P(\theta) - P_{\infty}}{\frac{1}{2} \rho U_{\infty}^2} = 1 - 4\sin^2\theta$$

Axial Pressure Force



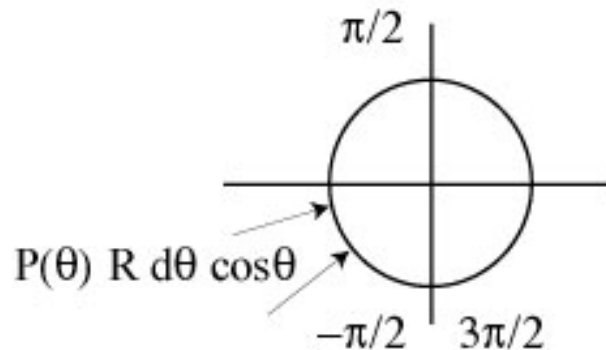
i) Potential flow:

$$-\pi/w < \theta < \pi/2$$

ii) $P \sim P_B$

$$\pi/2 \leq \theta \leq 3\pi/2$$

(for LAMINAR flow)

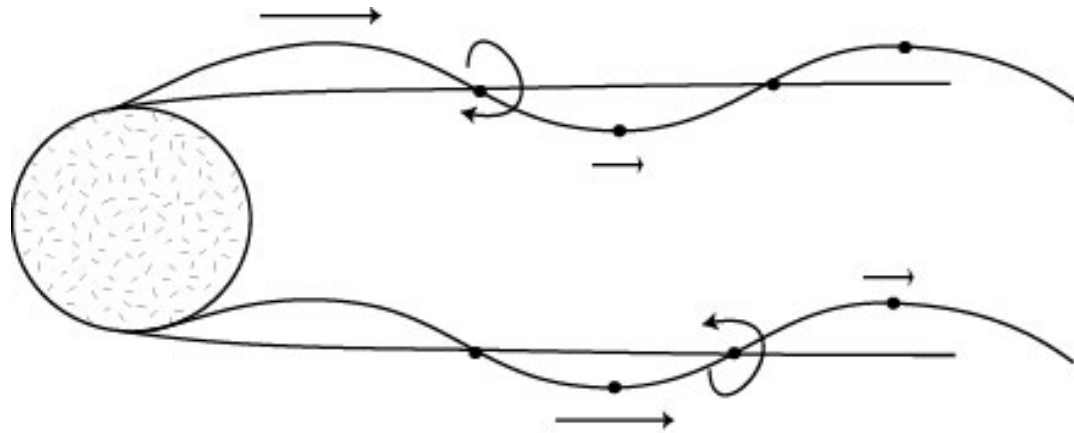


$$F_{\text{axial}} = \int P(\theta) R d\theta \cos\theta$$

Wake Instability

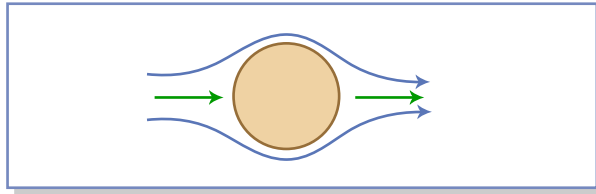
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Shear layer instability causes vortex roll-up



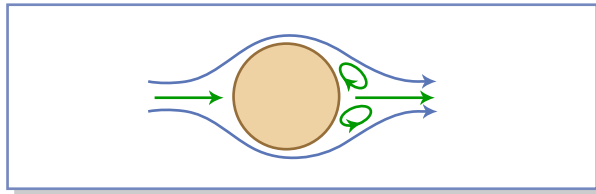
- **Flow speed outside wake is much higher than inside**
- **Vorticity gathers at downcrossing points in upper layer**
- **Vorticity gathers at upcrossings in lower layer**
- **Induced velocities (due to vortices) causes this perturbation to amplify**

Reynolds Number Dependency



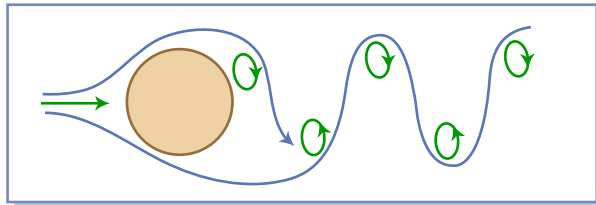
Regime of Unseparated Flow

$$R_d < 5$$



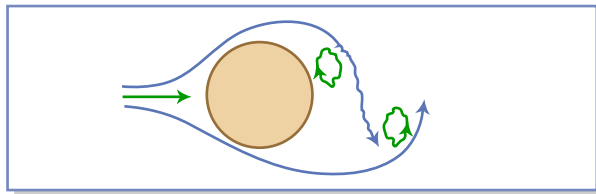
A Fixed Pair of Foppl Vortices in Wake

$$5-15 < R_d < 40$$



Two Regimes in which Vortex Street is Laminar

$$40 < R_d < 150$$



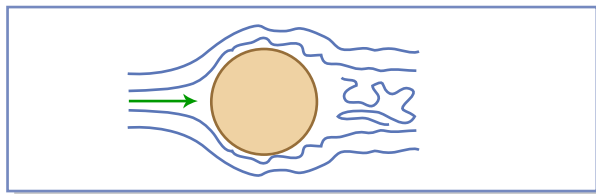
Transition Range to Turbulence in Vortex

$$150 < R_d < 300$$

Transition to turbulence

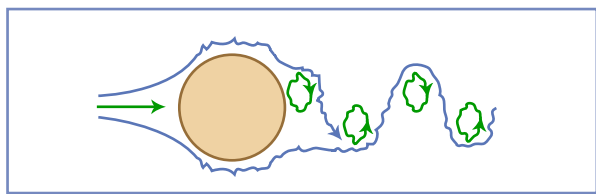
Vortex Street is Fully Turbulent

$$300 < R_d < 3 * 10^5$$



Laminar Boundary Layer has Undergone Turbulent Transition and Wake is Narrower and Disorganized

$$3 * 10^5 < R_d < 3.5 * 10^6$$



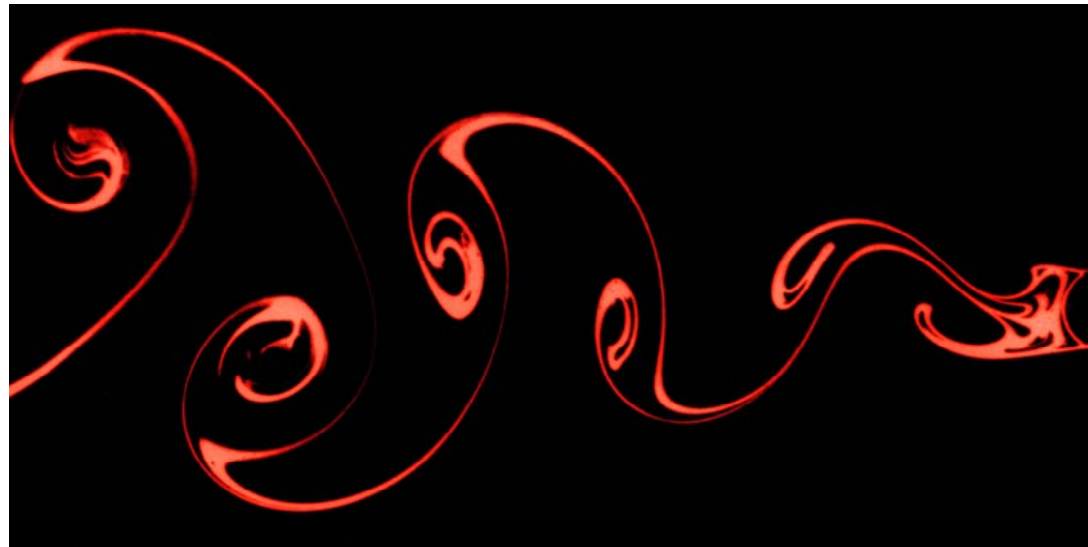
Re-establishment of Turbulent Vortex Street

$$3.5 * 10^6 < R_d$$

Regimes of fluid flow across smooth circular cylinders (Lienhard, 1966).

Figure by MIT OCW.

Vortex shedding dictated by the Strouhal number



$$S_t = f_s d / U$$

f_s is the shedding frequency, d is diameter and U inflow speed

Additional VIV Parameters

- *Reynolds Number*

$$Re = \frac{UD}{\nu} \approx \frac{\text{inertial effects}}{\text{viscous effects}}$$

- subcritical ($Re < 10^5$) (laminar boundary)

- *Reduced Velocity*

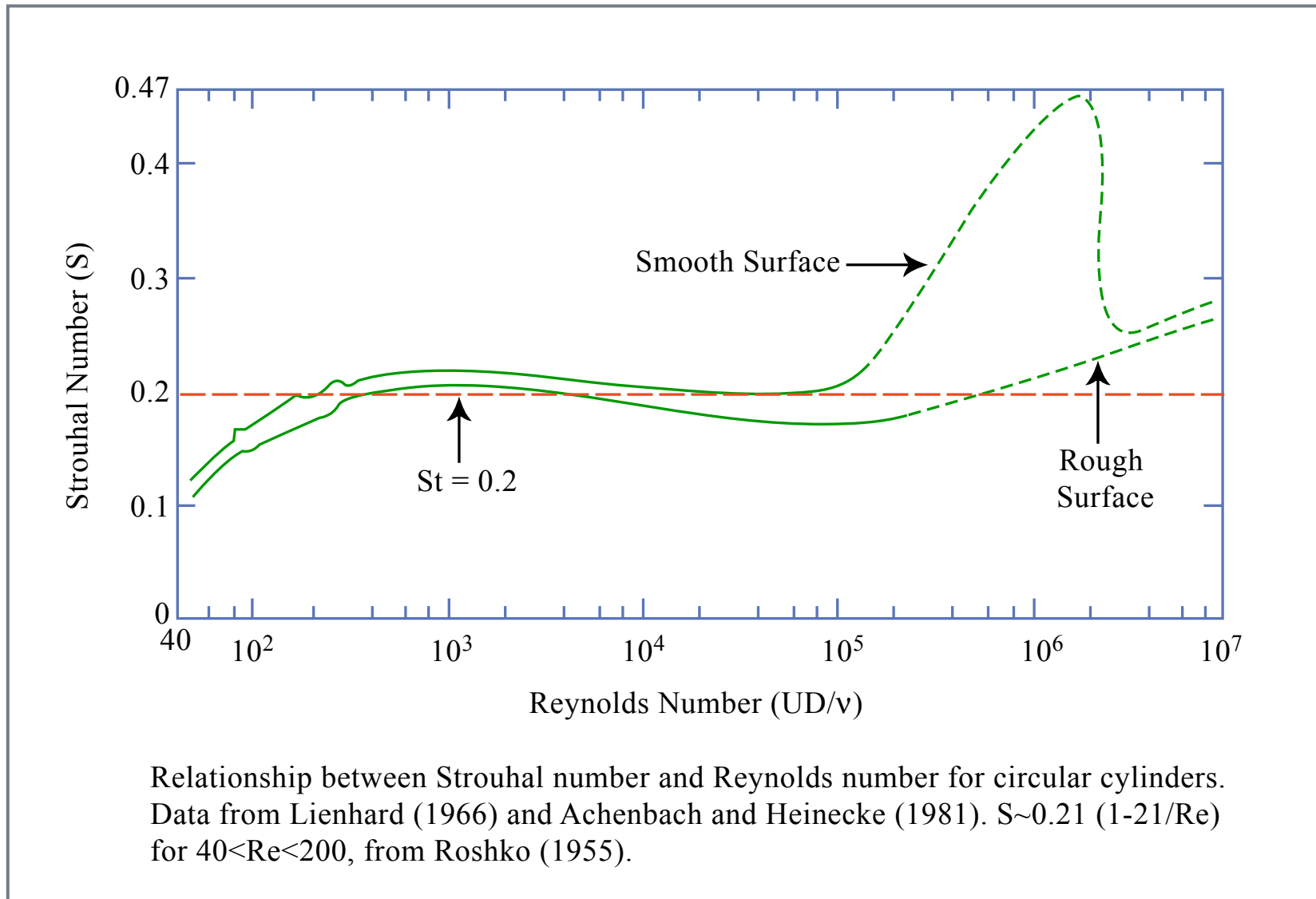
$$V_{rn} = \frac{U}{f_n D}$$

- *Vortex Shedding Frequency*

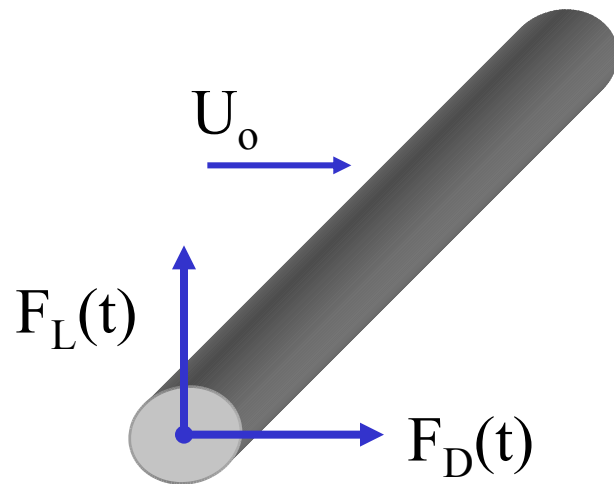
$$f_s = \frac{SU}{D}$$

- $S \approx 0.2$ for subcritical flow

Strouhal Number vs. Reynolds Number



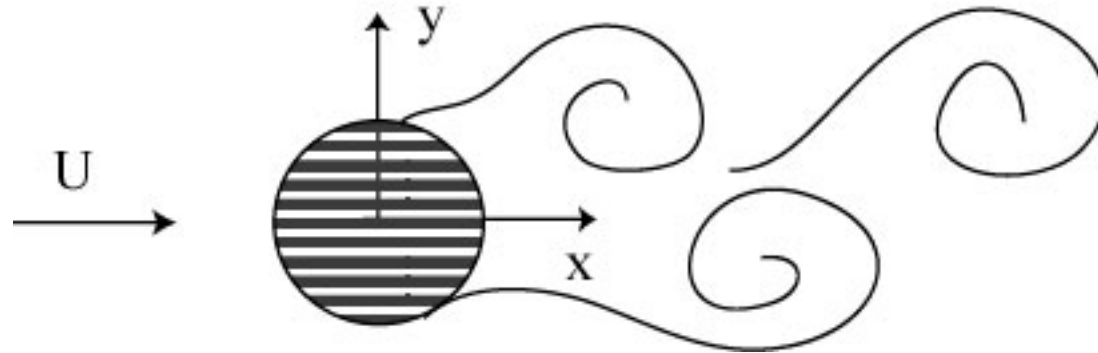
Vortex Shedding Generates forces on Cylinder



Both Lift and Drag forces persist on a cylinder in cross flow. Lift is perpendicular to the inflow velocity and drag is parallel.

Due to the alternating vortex wake (“Karman street”) the oscillations in lift force occur at the vortex shedding frequency and oscillations in drag force occur at *twice* the vortex shedding frequency.

Vortex Induced Forces



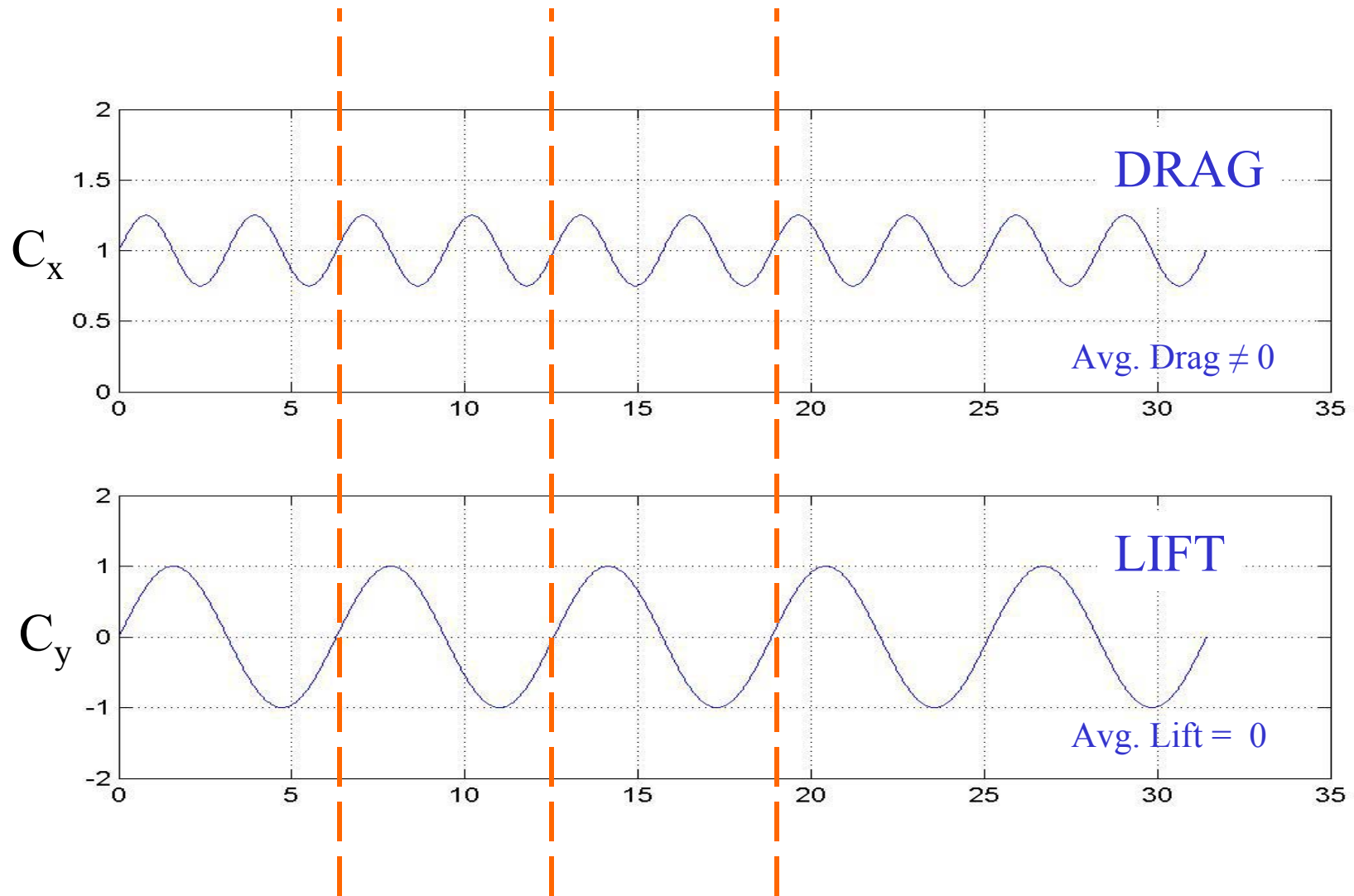
Due to unsteady flow, forces, $X(t)$ and $Y(t)$, vary with time.

Force coefficients:

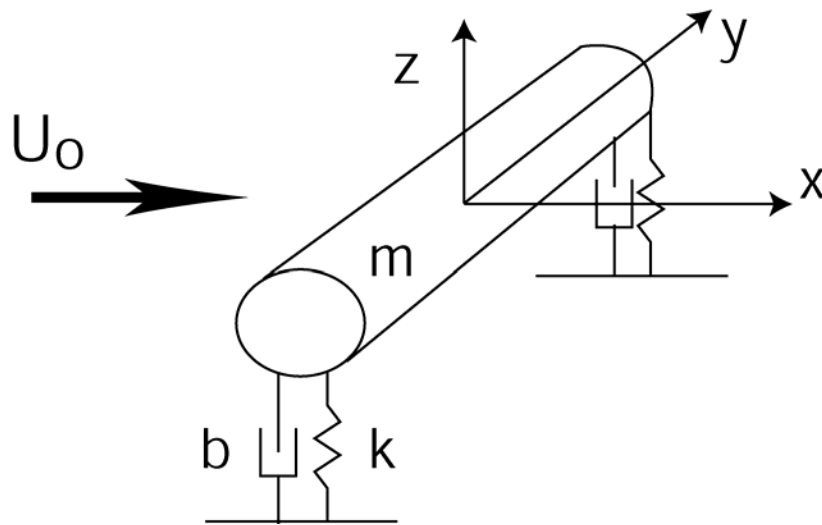
$$C_x = \frac{D(t)}{\frac{1}{2} \rho U^2 d}$$

$$C_y = \frac{L(t)}{\frac{1}{2} \rho U^2 d}$$

Force Time Trace



Alternate Vortex shedding causes oscillatory forces which induce structural vibrations



Rigid cylinder is now similar to a spring-mass system with a harmonic forcing term.

Heave Motion $z(t)$

$$z(t) = z_o \cos \omega t$$

$$\dot{z}(t) = -z_o \omega \sin \omega t$$

$$\ddot{z}(t) = -z_o \omega^2 \cos \omega t$$

$$\text{LIFT} = L(t) = L_o \cos (\omega_s t + \psi)$$

$$\text{DRAG} = D(t) = D_o \cos (2\omega_s t + \psi)$$

$$\omega_s = 2\pi f_s$$

“Lock-in”

A cylinder is said to be “locked in” when the frequency of oscillation is equal to the frequency of vortex shedding. In this region the largest amplitude oscillations occur.

Shedding
frequency

$$\omega_v = 2\pi f_v = 2\pi S_t (U/d)$$

Natural frequency
of oscillation

$$\omega_n = \sqrt{\frac{k}{m + m_a}}$$

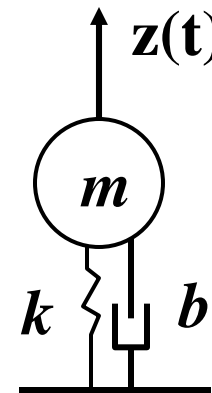
Equation of Cylinder Heave due to Vortex shedding

$$m\ddot{z} + b\dot{z} + kz = L(t)$$

$$L(t) = -L_a \ddot{z}(t) + L_v \dot{z}(t)$$

$$m\ddot{z}(t) + b\dot{z}(t) + kz(t) = -L_a \ddot{z}(t) + L_v \dot{z}(t)$$

$$\underbrace{(m + L_a)}_{\text{Added mass term}} \ddot{z}(t) + \underbrace{(b - L_v)}_{\text{Damping}} \dot{z}(t) + \underbrace{kz(t)}_{\text{Restoring force}} = 0$$



Added mass term

Damping

Restoring force

If $L_v > b$ system is UNSTABLE

Lift Force on a Cylinder

Lift force is sinusoidal component and residual force. Filtering the recorded lift data will give the sinusoidal term which can be subtracted from the total force.

LIFT FORCE: $L(t) = L_o \cos(\omega t + \psi_o)$ if $\omega < \omega_v$

$$L(t) = L_o \cos \omega t \cos \psi_o - L_o \sin \omega t \sin \psi_o$$

$$L(t) = \frac{-L_o \cos \psi_o}{z_o \omega^2} \ddot{z}(t) + \frac{L_o \sin \psi_o}{z_o \omega} \dot{z}(t)$$

where ω_v is the frequency of vortex shedding

Lift Force Components:

Two components of lift can be analyzed:

Lift in phase with acceleration (added mass):

$$M_a(\omega, a) = \frac{L_o}{a\omega^2} \cos \psi_o$$

Lift in-phase with velocity:

$$L_v = -\frac{L_o}{a\omega} \sin \psi_o$$

Total lift:

$$L(t) = -M_a(\omega, a) \ddot{z}(t) + L_v(\omega, a) \dot{z}(t)$$

($a = z_o$ is cylinder heave amplitude)

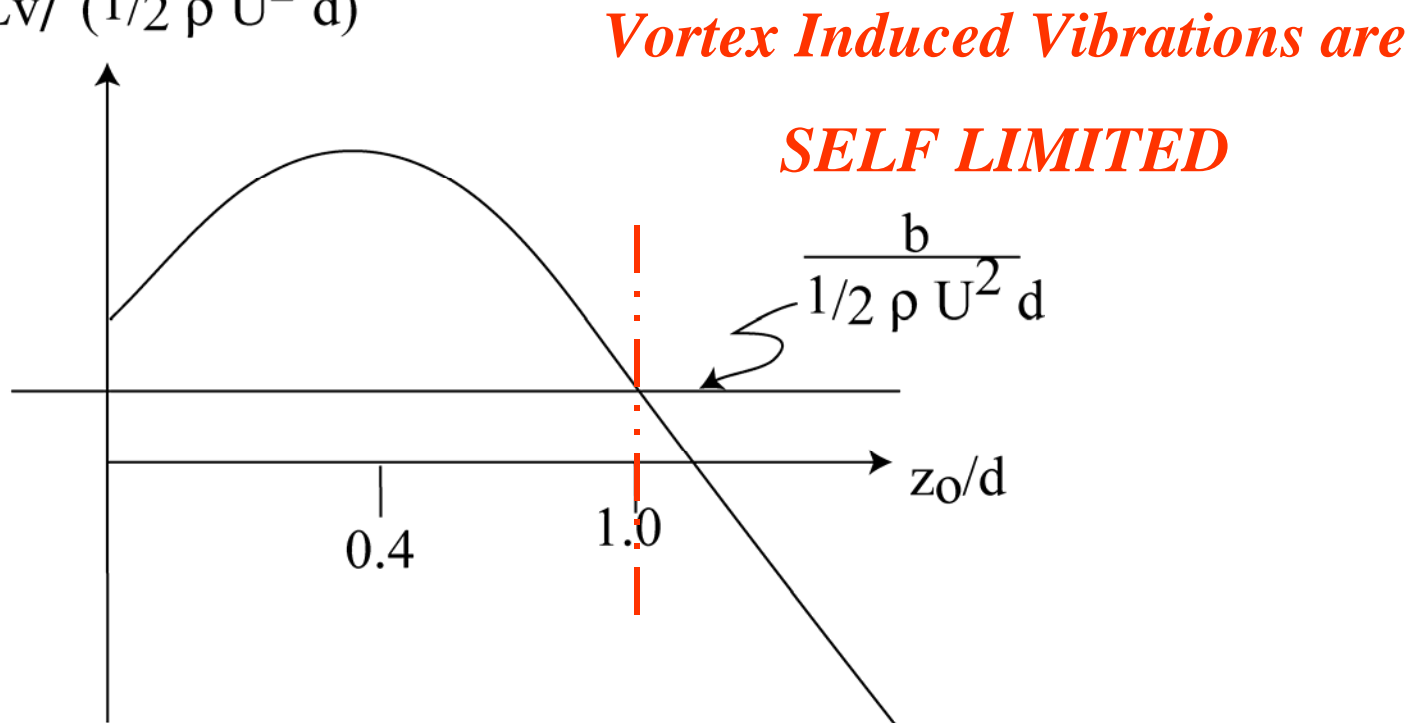
Total Force:

$$\begin{aligned} L(t) &= -M_a(\omega, a) \ddot{z}(t) + L_v(\omega, a) \dot{z}(t) \\ &= -\left(\frac{\pi}{4} \rho d^2\right) C_{ma}(\omega, a) \ddot{z}(t) \\ &\quad + \left(\frac{1}{2} \rho d U^2\right) C_{Lv}(\omega, a) \dot{z}(t) \end{aligned}$$

- If $C_{Lv} > 0$ then the fluid force amplifies the motion instead of opposing it. This is self-excited oscillation.
- C_{ma} , C_{Lv} are dependent on ω and a .

Coefficient of Lift in Phase with Velocity

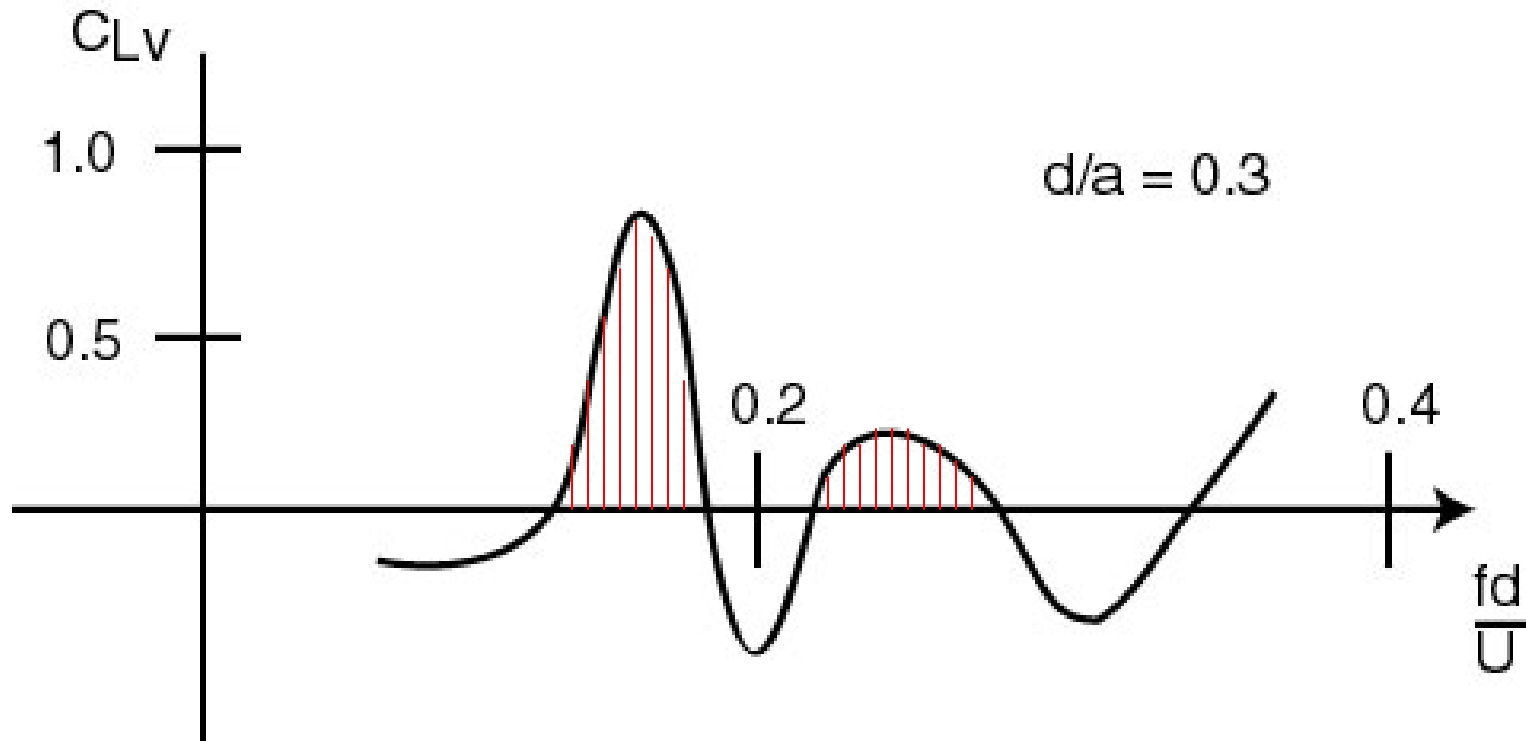
$$C_{lv} = L_v / (1/2 \rho U^2 d)$$



In air: $\rho_{\text{air}} \sim \text{small}$, $z_{\text{max}} \sim 0.2$ diameter

In water: $\rho_{\text{water}} \sim \text{large}$, $z_{\text{max}} \sim 1$ diameter

Lift in phase with velocity



Gopalkrishnan (1993)

Amplitude Estimation

Blevins (1990)

$$a/d \simeq 1.29 / [1 + 0.43 S_G]^{3.35}$$

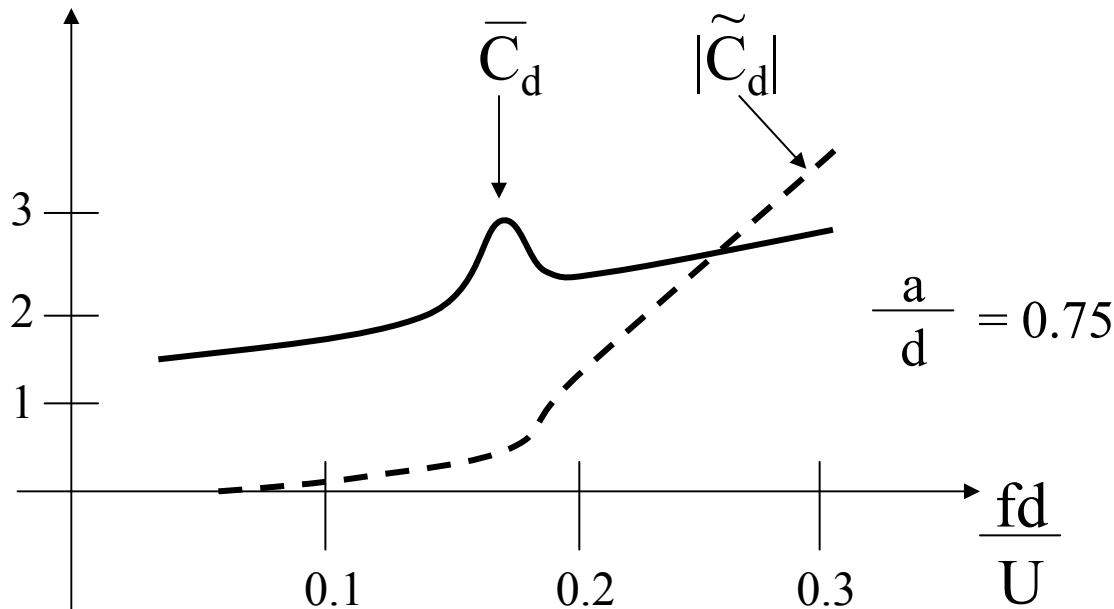
$$S_G = 2 \pi \hat{f}_n^2 \frac{2\bar{m} (2\pi\zeta)}{\rho d^2} ; \hat{f}_n = f_n/f_s ; \bar{m} = m + m_a^*$$

$$\zeta = \frac{b}{2\sqrt{k(m+m_a^*)}}$$

$$m_a^* = \rho \forall C_{ma} ; \text{ where } C_{ma} = 1.0$$

Drag Amplification

VIV tends to increase the effective drag coefficient. This increase has been investigated experimentally.



Gopalkrishnan (1993)

Mean drag:

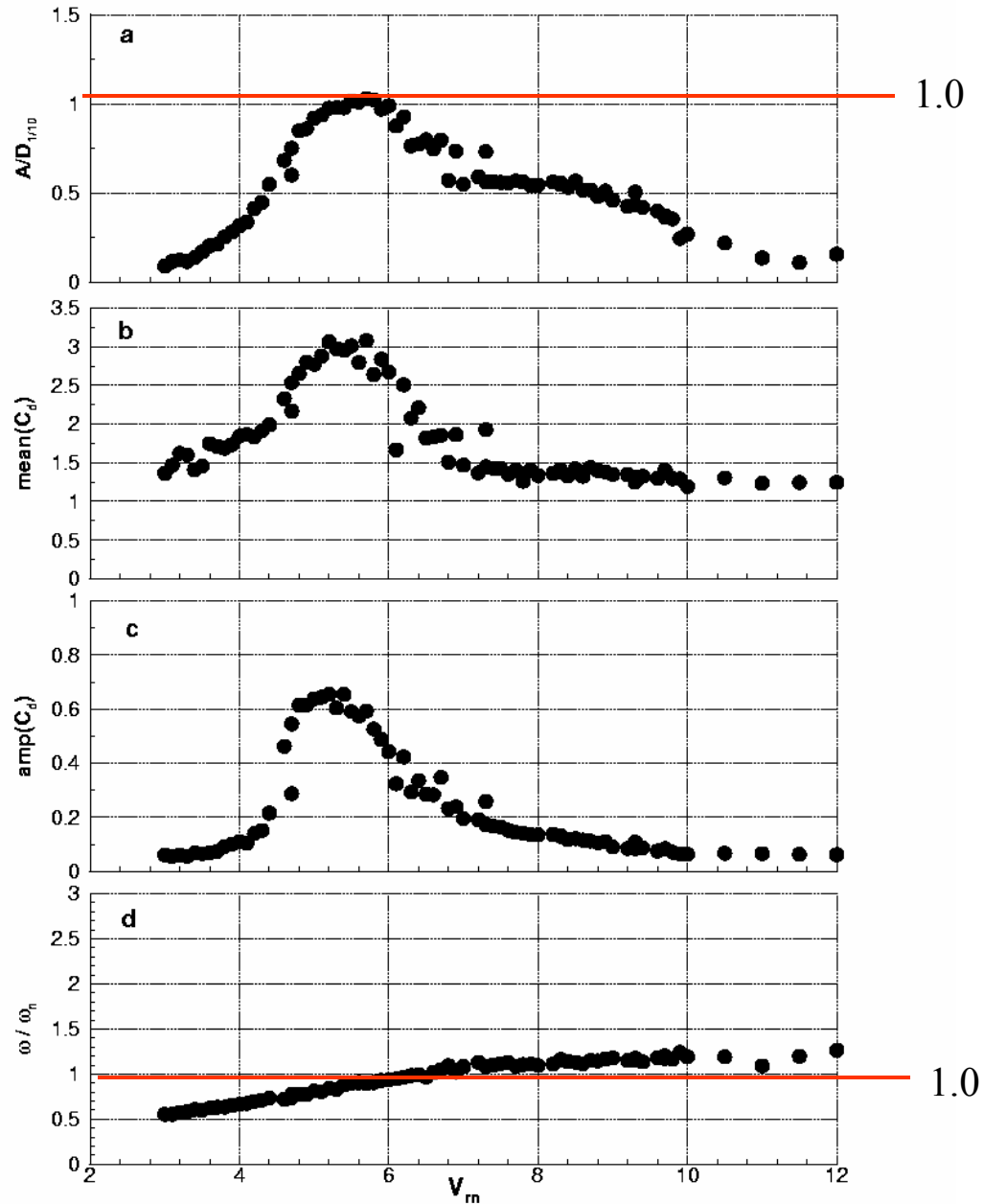
$$\bar{C}_d = 1.2 + 1.1\left(\frac{a}{d}\right)$$

Fluctuating Drag:

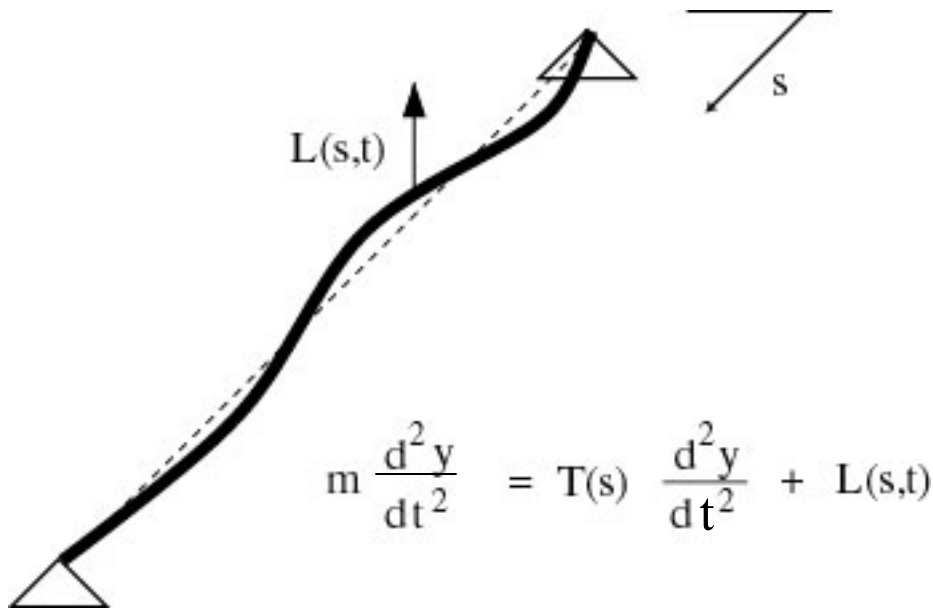
\tilde{C}_d occurs at twice the shedding frequency.

Single Rigid Cylinder Results

- a) One-tenth highest transverse oscillation amplitude ratio
- b) Mean drag coefficient
- c) Fluctuating drag coefficient
- d) Ratio of transverse oscillation frequency to natural frequency of cylinder



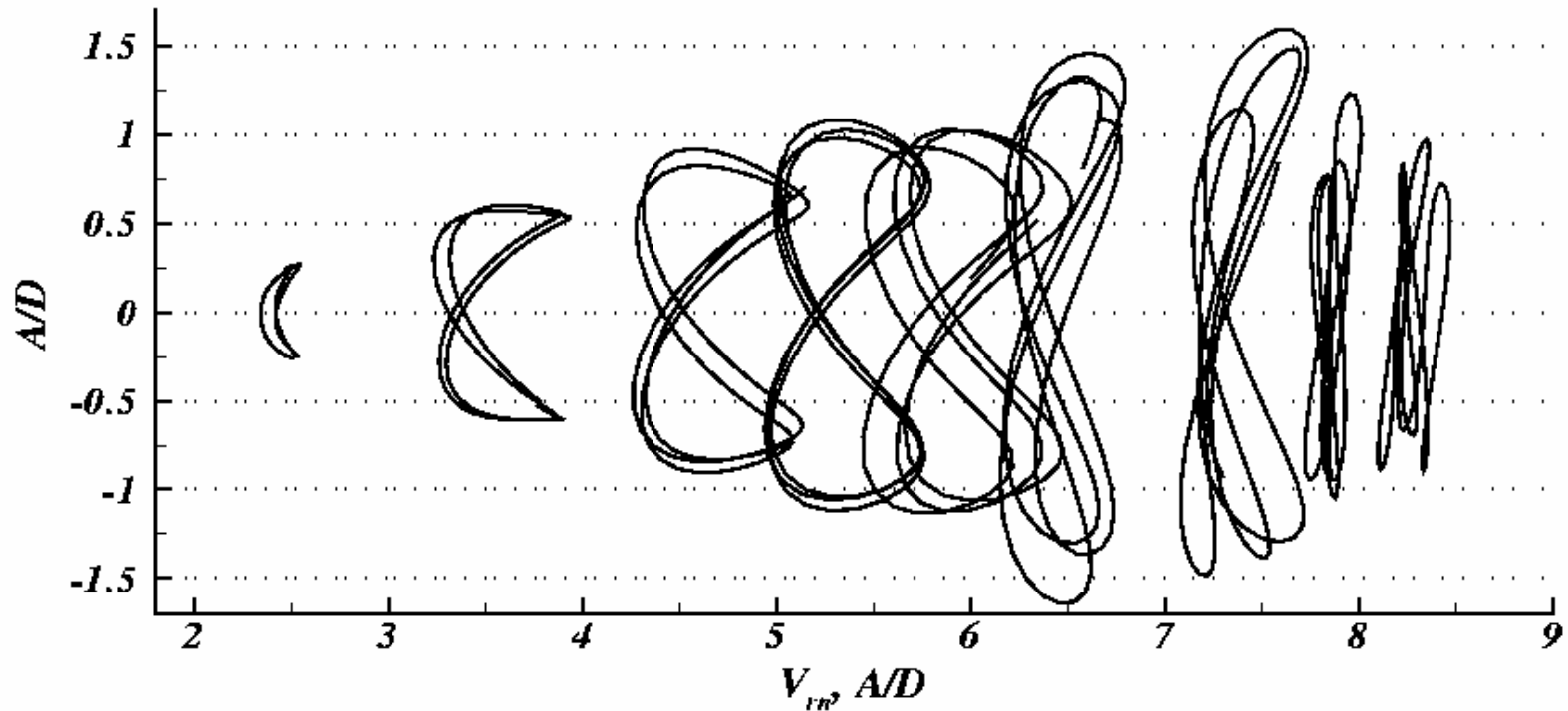
Flexible Cylinders



Mooring lines and towing cables act in similar fashion to rigid cylinders except that their motion is not spanwise uniform.

Tension in the cable must be considered when determining equations of motion

Flexible Cylinder Motion Trajectories



Long flexible cylinders can move in two directions and tend to trace a figure-8 motion. The motion is dictated by the tension in the cable and the speed of towing.