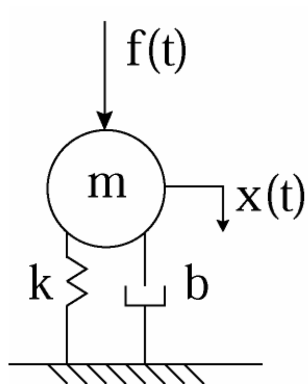
13.42 Homework #1

Spring 2005

Out: Thursday, February 3, 2005

Due: Thursday, February 10, 2005

Problem 1: A neutrally buoyant circular cylinder is mounted underwater in a strong current. The vortices shed from the cylinder generate a force on the cylinder and excite oscillations. The system can be modeled simply as the spring-mass-dashpot system shown below:



a) Draw a simple force-body diagram for this system
b) Derive from Newton's second law the equation of motion given a forcing function $f(t) = f_o \cos(wt)$. Explain the significance of each term.
c) Express the natural frequency of the system in terms of the given variables.
d) Assuming that this is a Linear-Time-Invariant system, the output will be harmonic in nature with the same frequency as the forcing function: $x(t) = x_o \cos(wt + y)$, where y is simply a phase shift between the input and output. Write an expression for the amplitude of the response in terms of the amplitude of the forcing and other given variables.

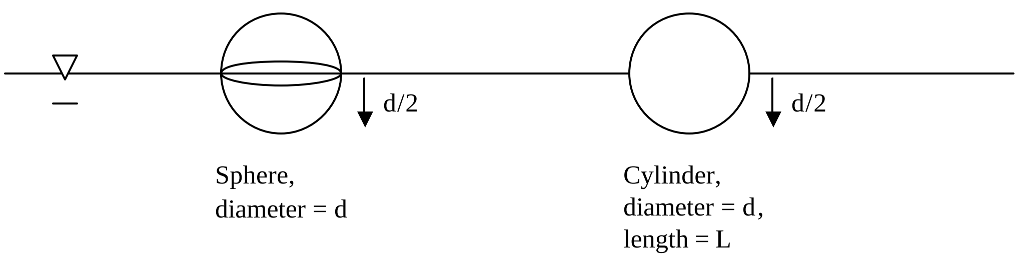
Problem 2: Calculate the magnitude and phase of the following complex numbers. Show your work.

a) $1 + i\sqrt{3}$ b) $\dfrac{1 + i\sqrt{3}}{\sqrt{3} + i}$ c) $-5 - 5i$ d) $i(1+i)e^{ip/6}$

Problem 3: Let z_o be a complex number with polar coordinates (r_o, q_o) and Cartesian coordinates (x_o, y_o). Determine expressions for the Cartesian coordinates of the following complex numbers in terms of x_o and y_o.

a) $z = r_o e^{-iq_o}$ b) $z = r_o e^{i(q+p)}$ c) $z = r_o e^{-i(q-p)}$ d) $z = r_o e^{i(q+2p)}$

Problem 4: Given a cylinder floating horizontally on the free surface and a sphere floating on the free surface. Both objects are in static equilibrium when submerged to half diameter depth. Show that for both objects the hydrostatic restoring coefficient in HEAVE (C_{33}) is approximately equal to $C_{33} \approx r g A_{wp}$, where A_{wp} is the area of the object at the water plane in static equilibrium.



Problem 5: Determine whether the following systems are Linear and/or Time-invariant. Show your work.

a. $$y(t) = \int_0^{t+a} u(s)ds$$

b. $$y(t) = \int_{t-a}^{t+a} [u(s)]^2 ds$$

c. $$y(t) = a \frac{du(t)}{dt} \left| \frac{du(t)}{dt} \right|$$

d. $$a\,\ddot{y}(t) + b\,\dot{y}(t) + g\,y(t) = u(t)$$