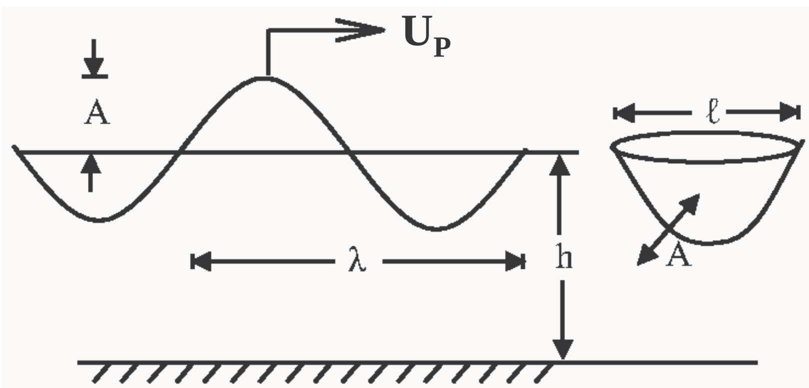
2.20 - Marine Hydrodynamics
Lecture 22

6.9 Wave Forces on a Body



$$U = \omega A$$

$$Re = \frac{U\ell}{\nu} = \frac{\omega A\ell}{\nu}$$

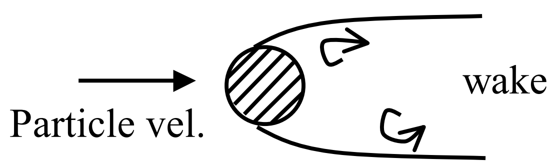
$$K_c = \frac{UT}{\ell} = \frac{A\omega T}{\ell} = 2\pi\frac{A}{\ell}$$

$$C_F = \frac{F}{\rho g A\ell^2} = f\left(\underbrace{\frac{A}{\lambda}}_{\substack{\text{Wave}\\ \text{steepness}}}, \underbrace{\frac{\ell}{\lambda}}_{\substack{\text{Diffraction}\\ \text{parameter}}}, R_e, \frac{h}{\lambda}, \text{ roughness}, \ldots \right)$$

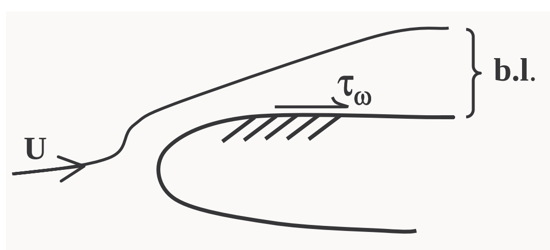
6.9.1 **Types of Forces**

1. **Viscous forces** Form drag, viscous drag $= f(R_e, K_c, \text{roughness}, \ldots)$.

   - *Form drag (C_D)*
     Associated primarily with flow separation - normal stresses.



   - *Friction drag (C_F)*
     Associated with skin friction τ_w, i.e., $\vec{F} \sim \iint\limits_{\substack{\text{body} \\ \text{(wetted surface)}}} \tau_w dS$.



2. **Inertial forces** Froude-Krylov forces, diffraction forces, radiation forces.

   Forces arising from potential flow wave theory,

$$\vec{F} = \underset{\substack{\text{body} \\ \text{(wetted surface)}}}{\iint} p\hat{n}dS, \text{ where } p = -\rho\Big(\frac{\partial \phi}{\partial t} + gy + \underbrace{\frac{1}{2}|\nabla\phi|^2}_{\substack{=0, \text{ for linear theory,} \\ \text{small amplitude waves}}}\Big)$$

   For linear theory, the velocity potential ϕ and the pressure p can be decomposed to

$$\phi = \underbrace{\phi_I}_{\substack{\text{Incident wave} \\ \text{potential } (a)}} + \underbrace{\phi_D}_{\substack{\text{Diffracted wave} \\ \text{potential } (b.1)}} + \underbrace{\phi_R}_{\substack{\text{Radiated wave} \\ \text{potential } (b.2)}}$$

$$-\frac{p}{\rho} = \frac{\partial \phi_I}{\partial t} + \frac{\partial \phi_D}{\partial t} + \frac{\partial \phi_R}{\partial t} + gy$$

   (a) **Incident wave potential**

   - *Froude-Krylov Force approximation* When $\ell << \lambda$, the incident wave field is not significantly modified by the presence of the body, therefore ignore ϕ_D and ϕ_R.

     Froude-Krylov approximation:

$$\left.\begin{array}{l} \phi \approx \phi_I \\ p \approx -\rho\Big(\dfrac{\partial \phi_I}{\partial t} + gy\Big) \end{array}\right\} \Rightarrow \vec{F}_{FK} = \underset{\substack{\text{body} \\ \text{surface}}}{\iint} \underbrace{-\rho\left(\frac{\partial \phi_I}{\partial t} + gy\right)}_{\equiv\, p_I} \hat{n}dS \leftarrow \substack{\text{can calculate knowing (incident)} \\ \text{wave kinematics (and body geometry)}}$$

   - *Mathematical approximation* After applying the divergence theorem, the $\vec{F}_{FK}$ can be rewritten as $\vec{F}_{FK} = -\underset{\substack{\text{body} \\ \text{surface}}}{\iint} p_I\hat{n}dS = -\underset{\substack{\text{body} \\ \text{volume}}}{\iiint} \nabla p_I d\forall$.

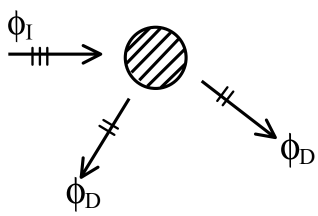
     If the body dimensions are very small comparable to the wave length, we can *assume* that ∇p_I is *approximately* constant through the body volume $\forall$ and 'pull' the ∇p_I out of the integral. Thus, the $\vec{F}_{FK}$ can be approximated as

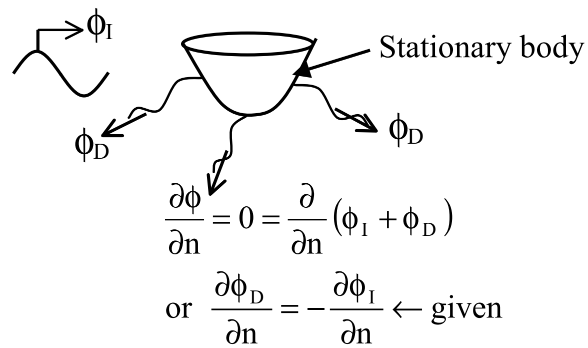
$$\vec{F}_{FK} \cong \Big(-\nabla p_I\Big)\bigg|_{\substack{\text{at body} \\ \text{center}}} \underset{\substack{\text{body} \\ \text{volume}}}{\iiint} d\forall = \underbrace{\forall}_{\substack{\text{body} \\ \text{volume}}} \Big(-\nabla p_I\Big)\bigg|_{\substack{\text{at body} \\ \text{center}}}$$

     The last relation is particularly useful for small bodies of non-trivial geometry - for 13.021, that is all bodies that do not have a rectangular cross section.

(b) **Diffraction and Radiation Forces**

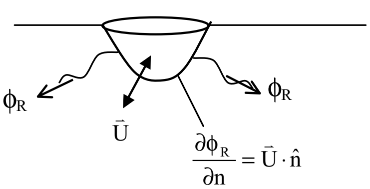
(b.1) **Diffraction or scattering force** When $\ell \not\ll \lambda$, the wave field near the body will be affected even if the body is stationary, so that no-flux B.C. is satisfied.





$$\vec{F}_D = \underset{\substack{\text{body}\\\text{surface}}}{\iint} -\rho \left(\frac{\partial \phi_D}{\partial t} \right) \hat{n} dS$$

(b.2) **Radiation Force - added mass and damping coefficient** Even in the absence of an incident wave, a body in motion creates waves and hence inertial wave forces.



$$\vec{F}_R = \underset{\substack{\text{body}\\\text{surface}}}{\iint} -\rho \left(\frac{\partial \phi_R}{\partial t} \right) \hat{n} dS = -\underbrace{m_{ij}}_{\substack{\text{added}\\\text{mass}}} \dot{U}_j - \underbrace{d_{ij}}_{\substack{\text{wave}\\\text{radiation}\\\text{damping}}} U_j$$

6.9.2 **Important parameters**

$$\left.\begin{array}{l}(1) K_c = \frac{UT}{\ell} = 2\pi\frac{A}{\ell} \\ \\ (2) \text{diffraction parameter } \frac{\ell}{\lambda}\end{array}\right\} \begin{array}{c}\text{Interrelated through maximum wave steepness} \\ \frac{A}{\lambda} \leq 0.07 \\ \left(\frac{A}{\ell}\right)\left(\frac{\ell}{\lambda}\right) \leq 0.07\end{array}$$

- If $K_c \leq 1$: no appreciable flow separation, viscous effect confined to boundary layer (hence small), solve problem via potential theory. In addition, depending on the value of the ratio $\frac{\ell}{\lambda}$,
  - If $\frac{\ell}{\lambda} << 1$, ignore diffraction , wave effects in radiation problem (i.e., $d_{ij} \approx 0, m_{ij} \approx m_{ij}$ infinite fluid added mass). F-K approximation might be used, calculate $\vec{F}_{FK}$.
  - If $\frac{\ell}{\lambda} >> 1/5$, must consider wave diffraction, radiation ($\frac{A}{\ell} \leq \frac{0.07}{\ell/\lambda} \leq 0.035$).

- If $K_c >> 1$: separation important, viscous forces can not be neglected. Further on if

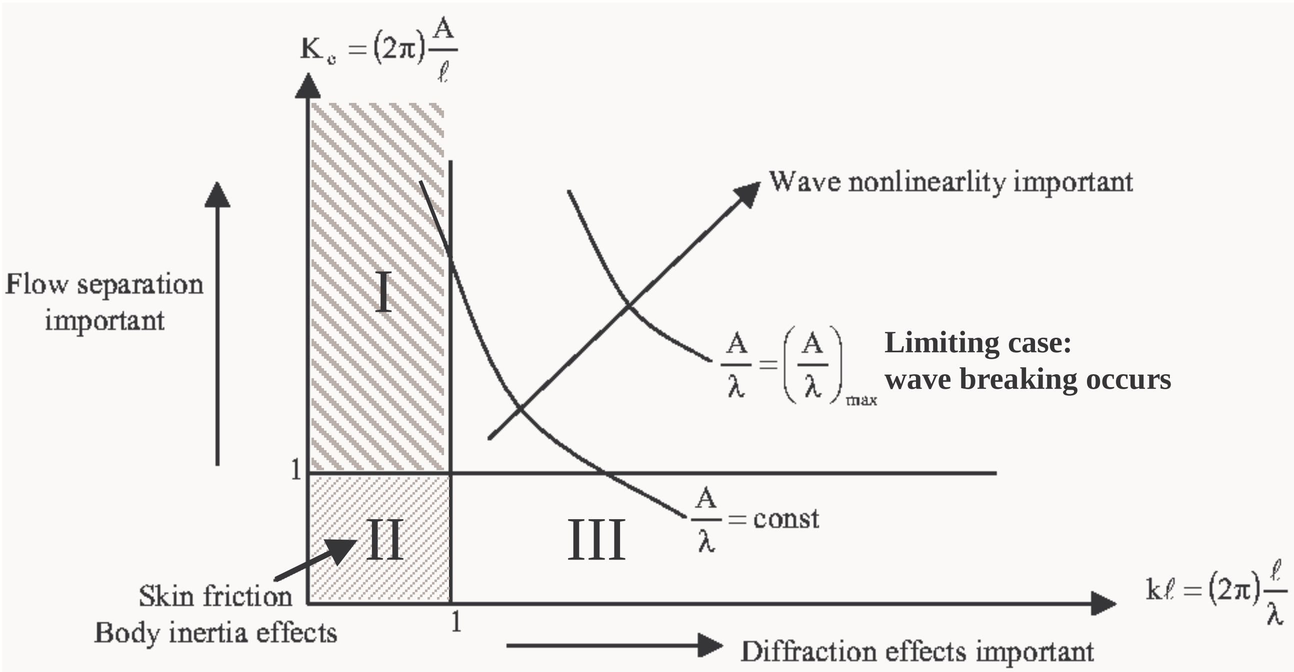
$$\frac{\ell}{\lambda} \leq \frac{0.07}{A/\ell} \quad \text{so } \frac{\ell}{\lambda} << 1 \text{ ignore diffraction, i.e., the Froude-Krylov approximation is valid.}$$

$$F = \frac{1}{2}\rho\ell^2 \underbrace{U(t)}_{\substack{\text{relative}\\\text{velocity}}} |U(t)| C_D(R_e)$$

- Intermediate K_c - both viscous and inertial effects important, use Morrison's formula.

$$F = \frac{1}{2}\rho\ell^2 U(t)|U(t)|C_D(R_e) + \rho\ell^3\dot{U}C_m(R_e, K_c)$$

- Summary



I. Use: C_D and $F - K$ approximation.

II. Use: C_F and $F - K$ approximation.

III. C_D is not important and $F - K$ approximation is not valid.