2.20 Problem Set 2A **Name: ____________________**

1. The velocity vector field $\vec{V}(x,y,z,t)=(u,v,w)$ is given by $\vec{V}=2xy\hat{i}+t^2\hat{k}$. At the point (-1,2,0) evaluate the following:

(a) $(\vec{\nabla}\times\vec{V})\times\vec{V}$

(b) $(\vec{V}\cdot\vec{\nabla})\vec{V}$

2. Stokes' Theorem states that for a 2-sided surface *S* in three dimensions having a closed curve *C* as its boundary,

$$\int_S \hat{n}\cdot(\vec{\nabla}\times\vec{V})dS=\oint_C \vec{V}\cdot d\vec{l}\ .$$

where $\vec{V}$ is a continuously differentiable vector function, $\hat{n}$ is the unit normal to the chosen positve side of *S*, and $d\vec{l}$ is in the corresponding positive direction along *C*.

Verify the theorem using direct integration for the case where $\vec{V}=y^2\hat{i}+x^2\hat{j}$ and *S* is a circular disk of radius *R* in the *x-y* plane having $\hat{n}=\hat{k}$.

3. Suppose a fluid is rotating about an axis fixed in space at a constant angular velocity $\vec{\omega}$. Choose the origin of a rectangular coordinate system to be on this axis. The position vector to any point in the fluid is $\vec{r}=x\hat{i}+y\hat{j}+z\hat{k}$. Knowing that the velocity of any point is then $\vec{V}=\vec{\omega}\times\vec{r}$, determine the following:

(a) $\vec{\nabla}\cdot\vec{V}$ and (b) $\vec{\nabla}\times\vec{V}$. *Hint*: Consider $\vec{\nabla}\cdot\vec{r}$, $\vec{\nabla}\times\vec{r}$ and $\vec{u}\cdot\vec{\nabla}r$ ($\vec{u}$ any vector)

4. Using the divergence theorem, evaluate $\iint_S \vec{F}\cdot\hat{n}dA$ over the closed surface of the cube bounded by the planes $x=\pm1$, $y=\pm1$, $z=\pm1$ where $\vec{F}=x\hat{i}+y\hat{j}+z\hat{k}$.