2.161 Signal Processing: Continuous and Discrete
Fall 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING

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Problem Set 6: The z-Transform and Linear Filters

Assigned: October 23, 2008 **Due:** October 30, 2008

Problem 1: A linear time-invariant filter is used to process sampled data (with sampling interval T), and is described by the the difference equation:

$$y_n = -0.5y_{n-1} + 0.5u_n + u_{n-1}$$

(a) Determine the transfer function $H(z)$ for the system. Express $H(z)$ as a ratio of polynomials in z^{-1}, and also as a ratio of polynomials in z.

(b) Plot the poles and zeros of $H(z)$ in the z-plane.

(c) Is this a stable system?

(d) From $H(z)$, find the system frequency response function, and show that this is an "all-pass" system, that is $|H^*(j\omega)| = 1$ for all $|\omega| < \pi/T$. Determine the system phase response at frequencies $\omega = 0$ and $\omega = \pi/T$.

Problem 2: Find causal, stable digital pulse response of discrete time systems with the following transfer functions:

(a)

$$H_a(z) = \frac{1 - z^{-1}}{1 + 0.77z^{-1}}$$

(b)

$$H_b(z) = \frac{1 + z^{-1}}{1 + 0.9z^{-1} + 0.81z^{-2}}$$

In each case specify the region-of-convergence that you assumed in deriving your answers.
Hint:

$$\begin{aligned} \mathcal{Z}\{r^n \cos(an)\} &= \frac{z(z - r\cos(a))}{z^2 - 2r\cos(a)z + r^2} \\ \mathcal{Z}\{r^n \sin(an)\} &= \frac{r\sin(a)z}{z^2 - 2r\cos(a)z + r^2}, \end{aligned}$$

Problem 3: Proakis and Manolakis: Problem 3.8 (p. 215)

Problem 4: Use a partial fraction expansion to find the impulse response of a discrete-time system with a transfer function. (You may use MATLAB's `residuez()` function if you need to.)

$$H(z) = \frac{1}{1 - \frac{5}{6}z^{-1} + \frac{1}{6}z^{-2}}$$

Then use MATLAB's `impz()` function (from the Signal Processing Toolbox) to compute the impulse response and plot it.

Problem 5: An *elliptic* filter, also known as the Chebyshev-Cauer filter, allows a sharper cut-off in the transition-band by allowing ripples in both the pass-band and in the stop-band. A digital elliptic low-pass digital filter has been designed in MATLAB using the `ellip()` function, giving the transfer function

$$H(z) = \frac{0.0798(1 + z^{-1} + z^{-2} + z^{-3})}{1 - 1.556z^{-1} + 1.272z^{-2} - 0.398z^{-3}}.$$

(a) Write the recursive difference equation you would use to implement the filter.

(b) Use MATLAB's function `roots` to find the system poles and zeros. Make a sketch of the system pole-zero plot using MATLAB's `zplane()` function.

(c) Use MATLAB's function `freqz()` to compute and plot the frequency response of the filter (magnitude and phase). Note that the standard function call plots the magnitude on a logarithmic (decibel scale) - make a separate plot with magnitude on a linear scale. Rationalize the behavior of the magnitude plot in terms of the pole-zero plot.

(d) If this filter was used in a real-time operation with $\Delta T = 0.1$ msec, what would the -3 dB cut-off frequency be? (You may use the plots from (c) as a starting point.)