Capstan—a mechanical amplifier

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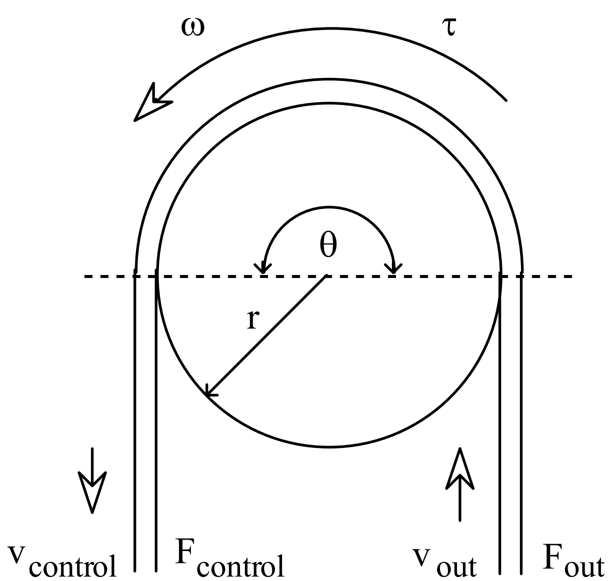
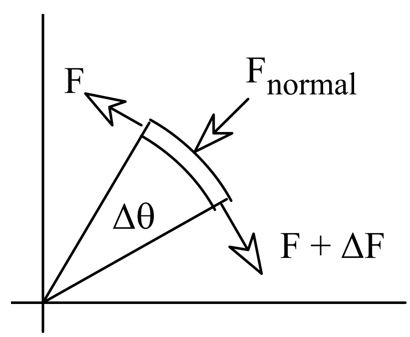




A schematic diagram of a basic capstan and a force diagram for a small segment of the rope are shown in the figures.

$F_{normal} = F \sin \Delta\theta/2 + (F+\Delta F) \sin \Delta\theta/2$

In the limit of small angles

$F_{normal} = F\, d\theta/2 + (F+dF)\, d\theta/2$

Assuming continuous slip and Coulomb friction between rope and drum,

$$dF = \mu F_{normal} = \mu \frac{2F+dF}{2} \; d\theta$$

$dF = F\, d\theta$ or $d \ln F = \mu \, d\theta$

Integrating from 0 to θ

$F_{out} = e^{\mu\theta} F_{control}$

Note that this relation is only valid if $\omega r \geq v_{control}$

From continuity: $v_{control} = v_{out}$

Torque required of capstan drive:

$\tau = (F_{out} - F_{control})\, r = (e^{\mu\theta} - 1)\, r\, F_{control}$

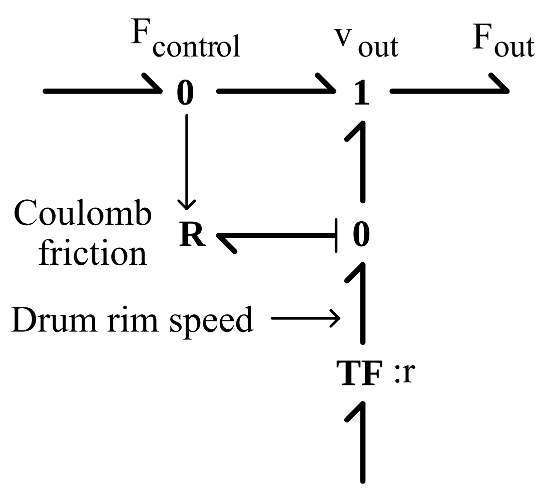
Power dissipated:

$P_{dissipated} = \tau\, \omega + F_{control}\, v_{control} - F_{out}\, v_{out}$

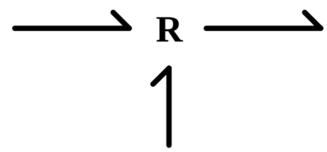
$P_{dissipated} = (e^{\mu\theta} - 1)\, r\, F_{control}\, \omega - (e^{\mu\theta} - 1)\, F_{control}\, v_{control}$

Note that $P_{dissipated} \geq 0$ with $P_{dissipated} = 0$ if $\omega r = v_{control}$

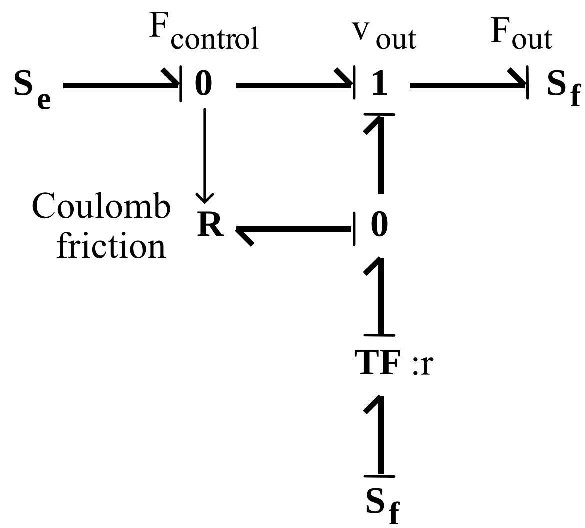
A bond graph follows:



This is a three-port resistor.



Typical boundary conditions result in the following causality.



Constitutive equations are

$$\begin{bmatrix} F_{out} \\ \tau \\ v_{control} \end{bmatrix} = \begin{bmatrix} 0 & 0 & e^{\mu\theta} \\ 0 & 0 & (e^{\mu\theta}-1)r \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} v_{out} \\ \omega \\ F_{control} \end{bmatrix}$$