# 2.14/2.140 Problem Set 5

Assigned: Wed. Mar. 5, 2014
Due: Wed. March 12, 2014, in class
Reading: FPE sections 6.1–6.8; lecture notes on loop-shaping and the Nyquist test

The following problems are assigned to both 2.14 and 2.140 students. Except where noted, you are free to use Matlab or any other computational tools to aid your solutions. There are no additional problems for 2.140 students. Note that Quiz 1 will be held in class on Wed. 3/12, and is closed-book, but you may bring one page of notes, which can be two-sided. Only hand calculators may be used. Recognize that in Quiz 1, all requested plots will need to be hand-sketches, so you should work these problems accordingly.

Problem 1 A unity-gain feedback loop is shown below; note that the plant has a pair of poles located on the imaginary axis.
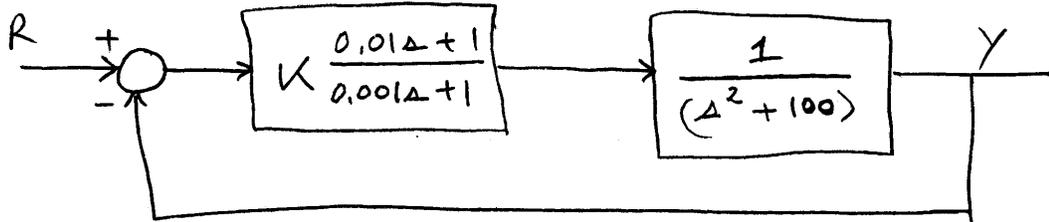
a) Sketch a root locus plot for $K > 0$.

b) Use a Bode plot of the return ratio $L(s)$ to choose the controller gain K such that the loop crossover frequency is 100 rad/sec. What gain K is required? What is the resulting phase margin ϕ_m?

c) Make a Nyquist plot for this loop, which is accurately drawn in the region where ϕ_m is defined. Be sure to label the corresponding points on the diagram. Note that your Nyquist analysis will require detours around the poles at $\pm j10$. Please clearly show the portion of the Nyquist contour corresponding to these detours.

**d)** Make a sketch of the closed-loop step response for your design.

Problem 2 A unity-gain feedback loop is shown below. The controller in this loop is a proportional gain K, but it is implemented on a computer which requires a delay time of T seconds to produce the control output.
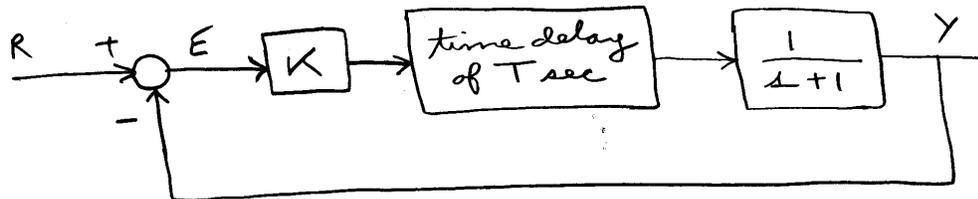
In all of this problem, assume that the time delay has a value of $T = 10$ msec.

a) Use a Bode plot of the return ratio $L(s)$ to determine the maximum controller gain K_{max} such that for $0 < K < K_{max}$ the loop is stable. For this maximal value of $K = K_{max}$ sketch a Nyquist plot for the loop.

b) What value of $K < K_{max}$ will result in a stable loop with a phase margin of $20°$? What is the resulting crossover frequency? What is the gain margin for this loop? For this value of K sketch a Nyquist plot for the loop which shows this information.

Problem 3 A feedback loop for a linearized magnetic suspension is shown below with a controller $G_c(s)$. The position of the suspended object is Y, the control reference value is R, and there is a disturbance D acting on the system.
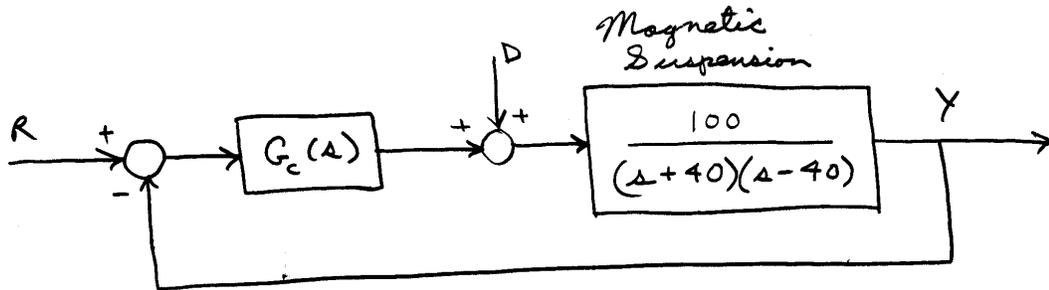
In this problem, assume the controller takes the form

$$G_c(s) = K\frac{10\tau s + 1}{\tau s + 1}.$$

a) Choose the values of K and τ such that the loop has a crossover frequency of 1000 rad/sec and a phase margin which is maximized for this controller. Sketch a Nyquist plot which confirms that the loop is stable.

b) For the value of K determined above, use Matlab to plot the response $Y(t)$ to a unit step in reference $R(t)$. Also plot the response $Y(t)$ to a unit step in disturbance $D(t)$.

c) This magnetic suspension is open-loop unstable, and thus there is a minimum controller gain $K > K_{min}$ required to stabilize the loop. Using the value of τ determined in part a), what is the numerical value of K_{min} at this marginally-stable condition? By what ratio can K be reduced with respect to the nominal design of part a) to bring the loop to the brink of instability? Note that this ratio can be defined as the *gain-reduction* margin, as distinguished over the standard gain margin which is defined as an allowable *gain-increase* margin.

Problem 4 Archive Problem 11.1 Circuit Bode Plots. It is important to review how to solve circuits for a particular transfer function. To solve for the requested transfer functions, use series/parallel and voltage/current divider relationships with impedances representing each circuit element. Sketch the indicated Bode plots by hand using the sketching rules presented in the lecture and notes. Do not use Matlab to generate these plots.

Problem 5 Archive Problem 19.12

Problem 6 FPE Problem 5.5 c,d

Problem 7 FPE Problem 5.7 c,d

2.14 / 2.140 Analysis and Design of Feedback Control Systems
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