

## 2.14/2.140 Problem Set 2

**Assigned:** Thurs. Feb. 13, 2014

**Due:** Wed. Feb. 19, 2014, in class

**Reading:** FPE sections 3.2–3.5

The following problems are assigned to both 2.14 and 2.140 students.

**Problem 1** Archive Problem 17.9

**Problem 2** Archive Problem 17.18

**Problem 3** FPE 3.20

**Problem 4** FPE 3.28

**Problem 5** Consider a motor with the following parameters: torque constant  $K = 1$  Nm/A, rotor inertia  $J = 10^{-2}$  kg m<sup>2</sup>, and coil resistance  $R = 5$   $\Omega$ . A torque disturbance  $T_d$  acts on the rotor of the motor in opposition to the motor torque  $T_m = Ki$ , where  $i$  is the motor coil current. The voltage at the motor terminals is  $V_m = iR + K\omega$ , where  $\omega$  is the motor angular velocity.

- a) We consider the motor as a plant with input  $V_m$  and output  $\omega$ . Draw a block diagram for the motor system which shows these signals as well as  $i$ ,  $T_m$ ,  $T_d$ , and the back emf voltage  $v_b = K\omega$ .
- b) Now we ask you to design a proportional speed controller of the form

$$V_m = K_p(\omega_r - \omega).$$

Here  $K_p$  is the proportional gain of the controller, and  $\omega_r$  is the speed command. Choose  $K_p$  such that the closed-loop system has a time constant of  $\tau = 1$  msec. For this value of  $K_p$  make plots of  $V_m$  and  $\omega$  when the reference is a unit step  $\omega_r(t) = 1u_s(t)$  rad/sec.

- c) Now we ask you to design a proportional plus integral speed controller of the form

$$V_m(s) = K_p\left(1 + \frac{1}{T_i s}\right)(\omega_r(s) - \omega(s)).$$

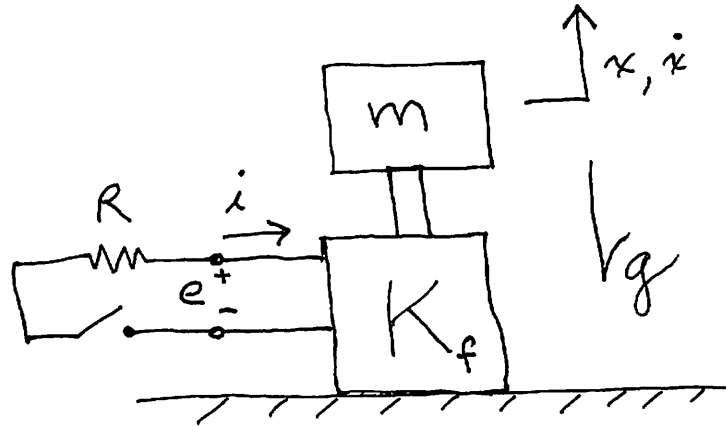
Here  $K_p$  is the proportional gain of the controller, and  $T_i$  is referred to as the integral time. Choose  $K_p$  and  $T_i$  such that the closed-loop system has a natural frequency of  $\omega_n = 1000$  rad/sec and a damping ratio of  $\zeta = 0.4$ . For these values make plots of  $V_m$  and  $\omega$  when the reference is a unit step  $\omega_r(t) = 1u_s(t)$  rad/sec.

**Problem 6** An ideal linear voice coil actuator is configured as shown below with the actuation axis oriented vertically. This ideal actuator has no internal mass or electrical resistance. Gravity acts downward as shown in the figure. For the purposes of this problem, assume that motion in the  $x$  direction is unlimited. The actuator has  $F = K_f i$  and  $e = K_f \dot{x}$ .

The actuator mechanical port is connected to a mass  $m$ . The actuator electrical port is connected to a resistance  $R$  in series with a switch. Consider the two cases: a) switch open and b) switch closed. At  $t = 0$ , the mass is released with  $x(0) = 0, \dot{x}(0) = 0$ .

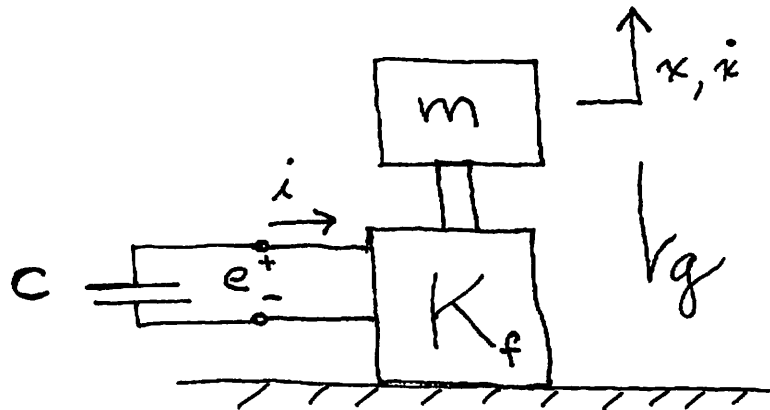
Write the equations of motion for the two cases. Take care with minus signs in your analysis: note that with the switch closed  $e = -iR$ . To facilitate this analysis, draw a block diagram for this system which includes at least the signals  $e, i, F, x, \dot{x}$ , and which has the acceleration of gravity  $g$  as an input. Use this block diagram to determine the transfer function from input  $g$  to output  $\dot{x}$ .

Solve for the time trajectory  $\dot{x}(t)$  for the two cases, and plot these as a function of time on the same axes. How are the initial ( $t = 0^+$ ) accelerations (slope) related? Why?



**Problem 7** An ideal linear voice coil actuator is configured as shown below with the actuation axis oriented vertically. This ideal actuator has no internal mass or electrical resistance. Gravity acts downward as shown in the figure. For the purposes of this problem, assume that motion in the  $x$  direction is unlimited. The actuator has  $F = K_f i$  and  $e = K_f \dot{x}$ . The actuator mechanical port is connected to a mass  $m$ . We consider two cases in which the electrical port is connected to a capacitor or an inductor.

a) The actuator electrical port is connected to a capacitance  $C$  as shown below.

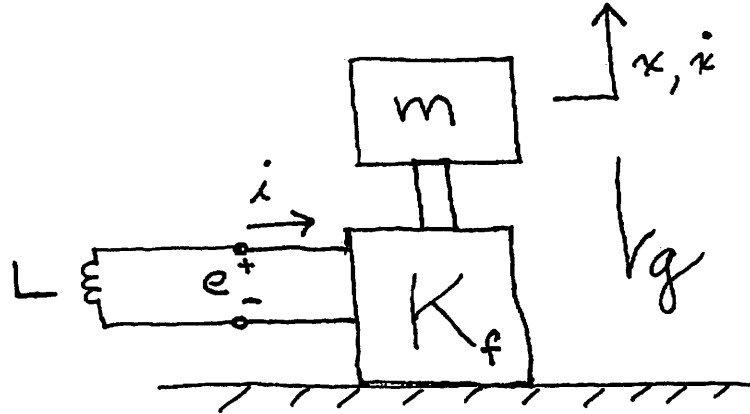


Write the equations of motion. Take care with minus signs in your analysis: note that

$i = -Cde/dt$ . To facilitate this analysis, draw a block diagram for this system which includes at least the signals  $e$ ,  $i$ ,  $F$ ,  $x$ ,  $\dot{x}$ , and which has the acceleration of gravity  $g$  as an input. Use this block diagram to determine the transfer function from input  $g$  to output  $\dot{x}$ .

At  $t = 0$ , the mass is released with  $x(0) = 0$ ,  $\dot{x}(0) = 0$ . Solve for the time trajectory  $\dot{x}(t)$  and plot as a function of time. Write an expression for the acceleration; how does changing the value of  $C$  affect the acceleration? From the viewpoint of the mass, what mechanical element is it connected to?

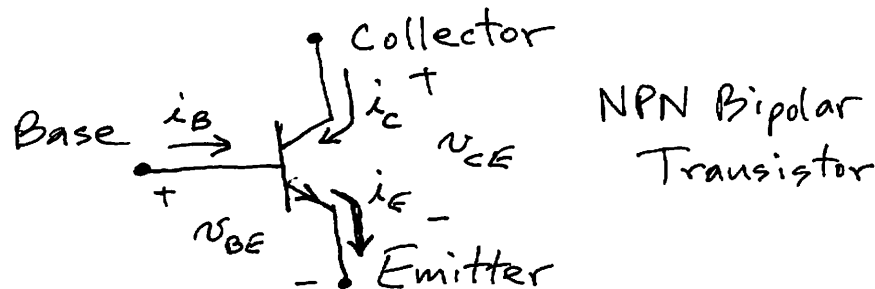
- b) The actuator electrical port is connected to an inductance  $L$  as shown below.



Write the equations of motion. Take care with minus signs in your analysis: note that  $e = -Ldi/dt$ . To facilitate this analysis, draw a block diagram for this system which includes at least the signals  $e$ ,  $i$ ,  $F$ ,  $x$ ,  $\dot{x}$ , and which has the acceleration of gravity  $g$  as an input. Use this block diagram to determine the transfer function from input  $g$  to output  $\dot{x}$ .

At  $t = 0$ , the mass is released with  $x(0) = 0$ ,  $\dot{x}(0) = 0$ . Solve for the time trajectory  $\dot{x}(t)$  and plot as a function of time, with the axes clearly labeled and dimensioned. How does changing the value of  $L$  affect the trajectory? From the viewpoint of the mass, what mechanical element is it connected to?

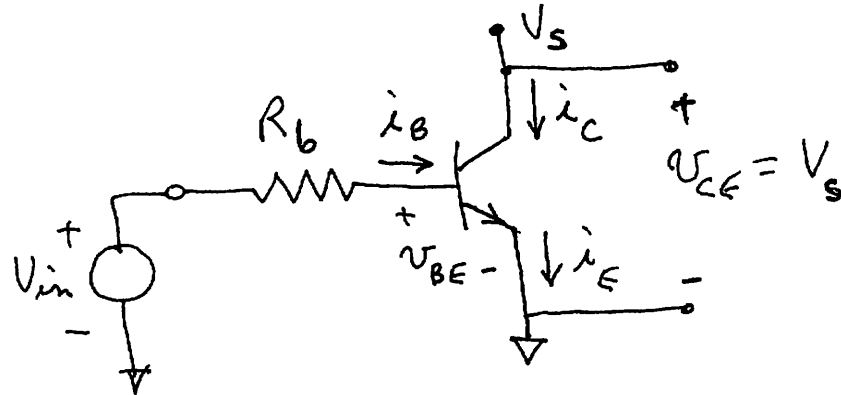
**Problem 8** This problem considers an electrical/thermal model of a transistor circuit using an NPN bipolar junction transistor. This circuit is used in Labs 2 and 3 to give you experience with a temperature control system. Here we look at the details of the electrical and thermal models for the circuit. As shown below, an NPN bipolar transistor is a three-terminal device (base, emitter, collector), with correspondingly defined voltages and currents.



What makes a transistor useful is that it *amplifies*. For this NPN device, a basic model of

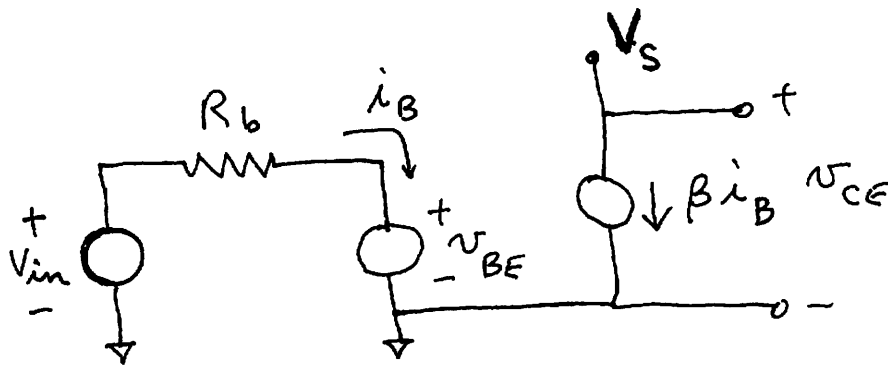
this amplification is the relationship  $i_C = \beta i_B$ , where in typical devices  $\beta \approx 100$ . Thus a small input current at the base terminal can be amplified into a much larger current at the collector terminal. Since the sum of currents entering the device must equal zero, we also have  $i_E = i_C + i_B$ . Further, when  $i_B > 0$ , we can assume  $v_{BE} \approx 0.6$  V. (This problem only models the transistor operation in the *forward-active* region, and omits internal dynamics. There is significantly more to learn about these devices for general use.)

The particular circuit of interest



drives the transistor with an input  $V_{in}$  through a base resistor  $R_b$ . Note also in this circuit that  $v_{CE} = V_s > 0$ .

An equivalent circuit for the transistor is



$$i_B = (V_{in} - v_{BE}) / R_b$$

$$i_C = \beta i_B = \beta (V_{in} - v_{BE}) / R_b$$

$$P_{diss} \approx i_C v_{CE} = V_s \beta (V_{in} - v_{BE}) / R_b$$

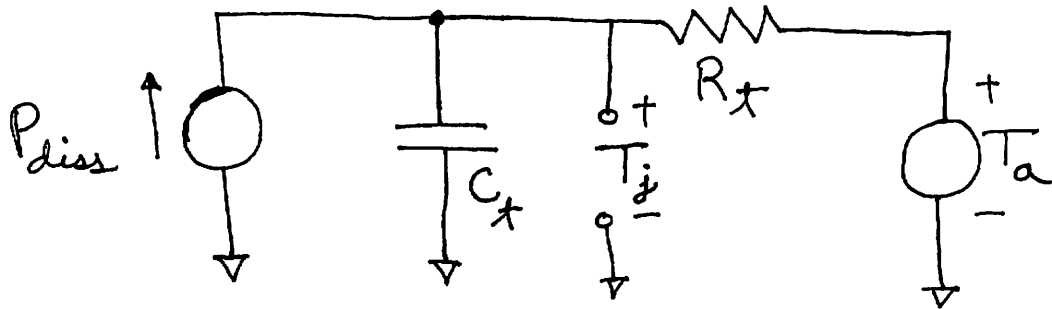
We will assume that when  $i_B$  is some positive value,  $v_{BE} = 0.6$  V, and thus require  $V_{in} \geq 0.6$ . Look at the circuit equations to follow the derivation which shows that the power dissipated in the device is  $P_{diss} \approx V_s \beta (V_{in} - v_{BE}) / R_b$ . (The approximation comes in because we ignore the very small dissipation  $i_B v_{BE}$ . Why is this reasonable?) Thus we see that the input  $V_{in}$  can directly control the device dissipation, via the base current, at relatively low input power. That is the key idea of amplification: large power or signals somewhere can be controlled by small power input signals. Note that we inherently also assume  $V_{in} > v_{BE}$ , since power dissipation must always be positive. (If you want negative and positive heat flows from one device, get a thermoelectric cooler, and go learn more about thermodynamics!)

Now for the thermal side of the model. Assume that the transistor is a lump of material at uniform temperature. Further assume a linear power flow to ambient proportional to the difference between transistor  $T_j$  and ambient  $T_a$  temperatures. The thermal capacitance of the lump  $C_t$  has units of  $[J/^\circ K]$ , and the thermal resistance  $R_t$  has units of  $[^\circ K/W]$ . The power dissipation  $P_{diss}$  injects heat into the transistor lump, which connects to ambient via the thermal resistance.

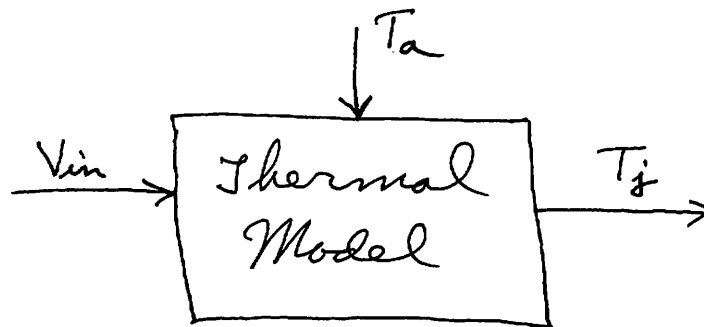
The thermal capacitance is described by

$$C_t \frac{dT_j}{dt} = \sum P = P_{diss} - \frac{(T_j - T_a)}{R_t}.$$

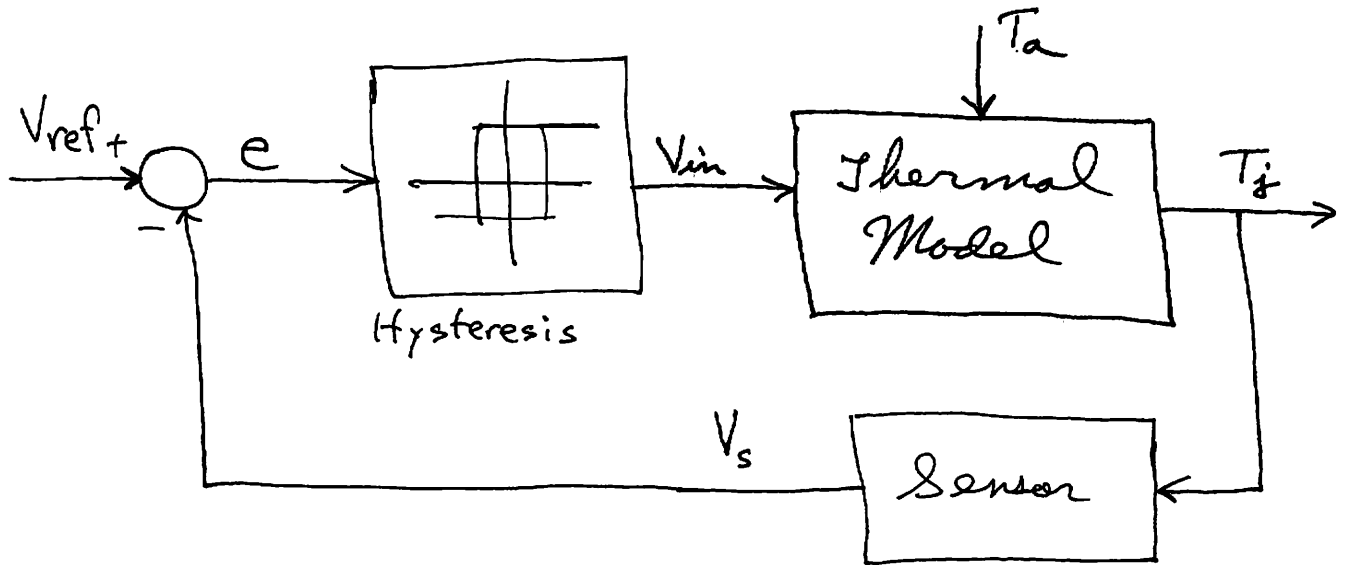
This relationship can be shown in the equivalent thermal circuit, where we equate electrical current with power flow, and electrical voltage with temperature.



- Draw a block diagram including at least input  $V_{in}$ , input  $T_a$ ,  $P_{diss}$ ,  $T_j$ .
- Now (finally!) we consider closed-loop control issues. For this purpose,  $T_j$  is the output to be controlled and  $V_{in}$  is the input for control. In this context,  $T_a$  is a disturbance signal.

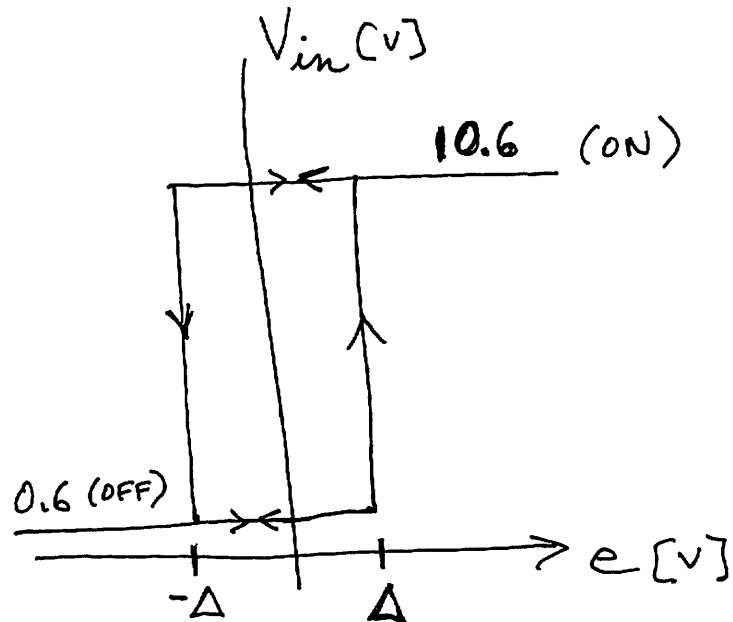


This thermal plant will be controlled by an on/off controller with hysteresis, as shown



Temperature is measured with a sensor which converts  $^{\circ}K$  to volts, via  $V_s = 0.1(T_j - 273)$  [V]. That is, the sensor measures in degrees Celsius, with a scale factor of 1 Volt per 10 degrees Celsius.

The hysteresis block has the form



where the switching points are at  $\pm\Delta$ . For the remainder of this problem, assume the following values of the parameters:  $V_{ref} = 5$  V,  $\Delta = 0.1$  V,  $R_b = 20$  k $\Omega$ ,  $\beta = 100$ ,  $V_s = 5$  V,  $C_t = 1$  J/ $^{\circ}K$ , and  $R_t = 400$   $^{\circ}K/W$ . Use these numbers in your dynamic model developed above.

- i) Assume  $T_a = 25\text{ }^\circ\text{C}$ , and that the system has been off for a long time, such that  $T_j(0^+) = T_a$ . At  $t = 0$  the control is engaged and begins operating. Solve for and plot  $T_j(t)$  and  $V_{in}(t)$ . What is the steady-state switching frequency and duty cycle (ratio of on time to cycle period)?
- i) Now assume  $T_a = 0\text{ }^\circ\text{C}$ , and that the system has been off for a long time, such that  $T_j(0^+) = T_a$ . At  $t = 0$  the control is engaged and begins operating. Solve for and plot  $T_j(t)$  and  $V_{in}(t)$ . What is the steady-state switching frequency and duty cycle? Compare with the earlier case.
- Does this lower ambient temperature cause any error in the average value of  $T_j$ ? You can answer this question qualitatively on the basis of your time plots. You do not need to formally compute this average.

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